Composition

Announcements

Linked Lists

Linked List Structure

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`3 , 4 , 5`

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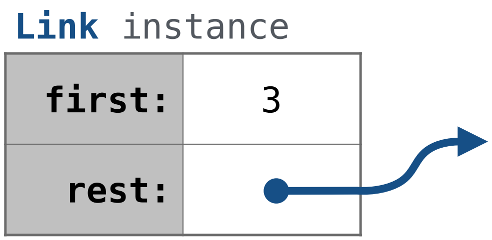

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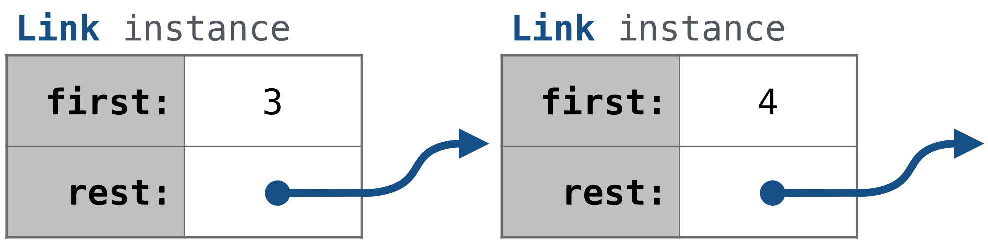

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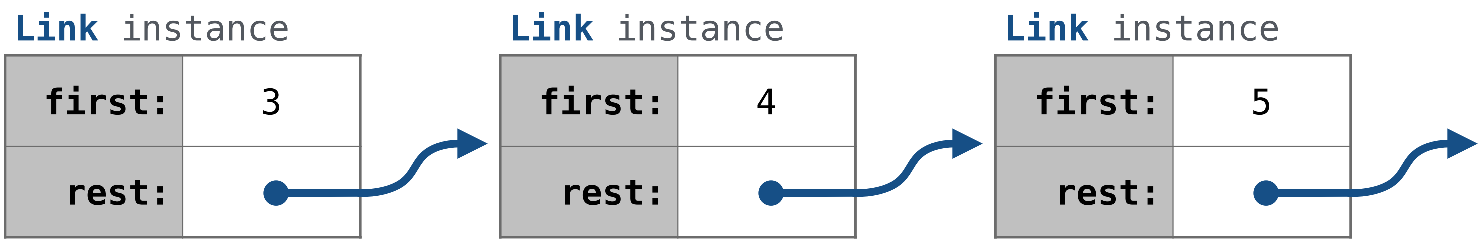

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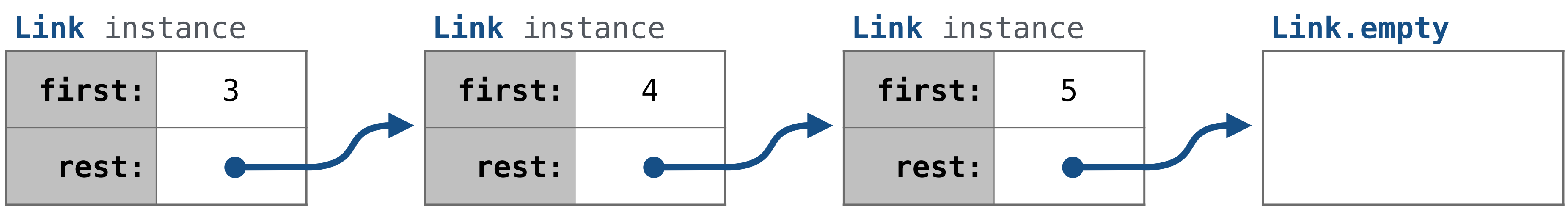

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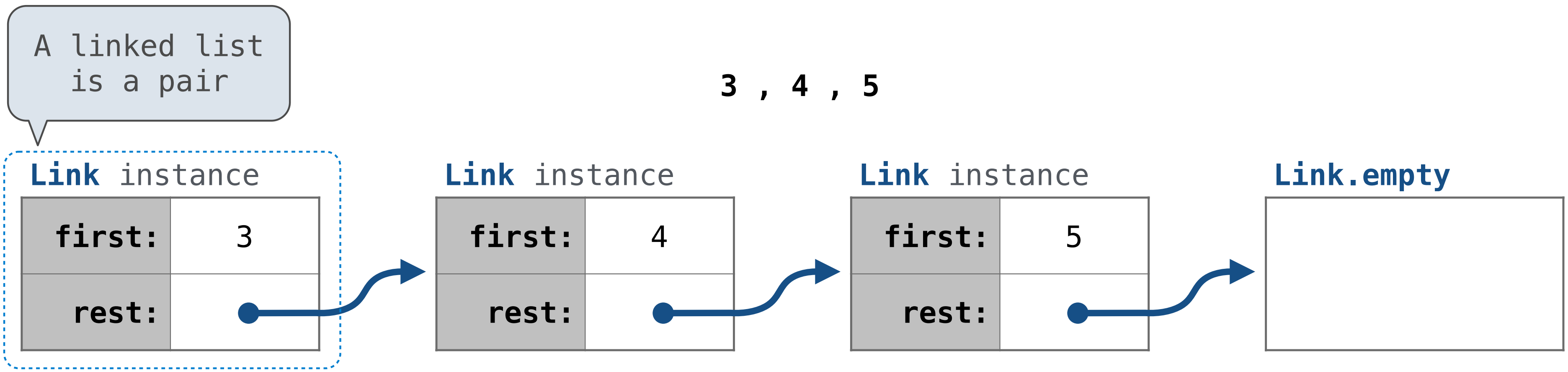

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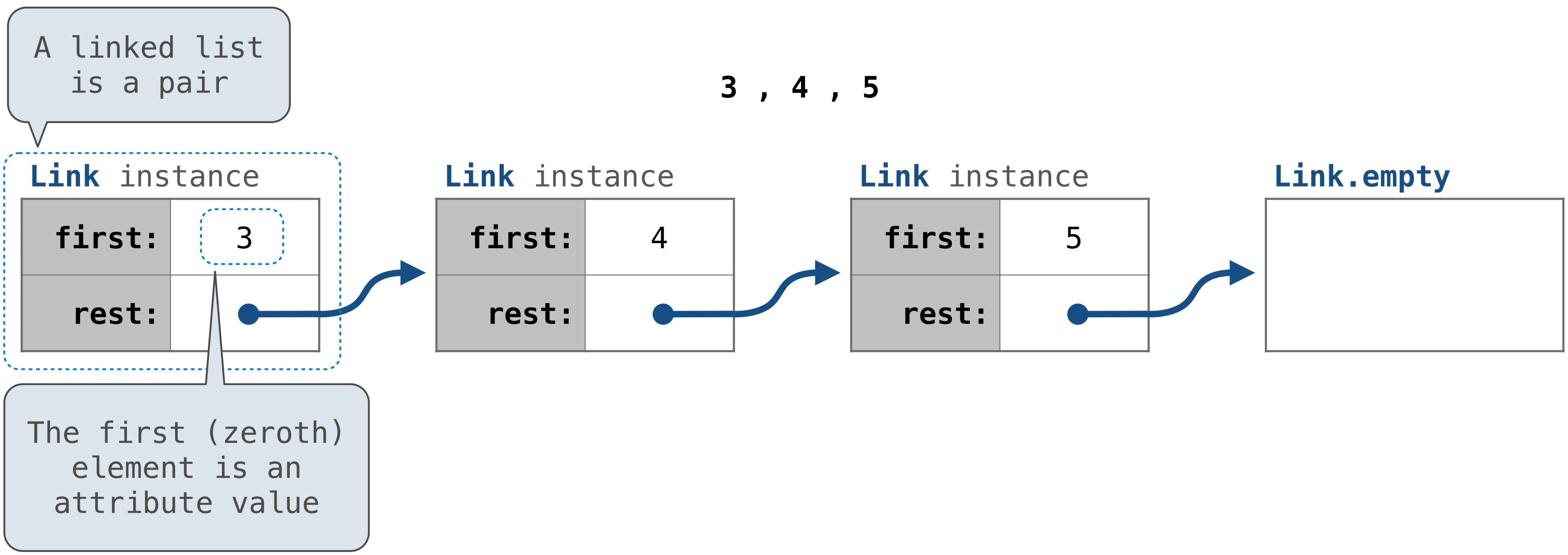

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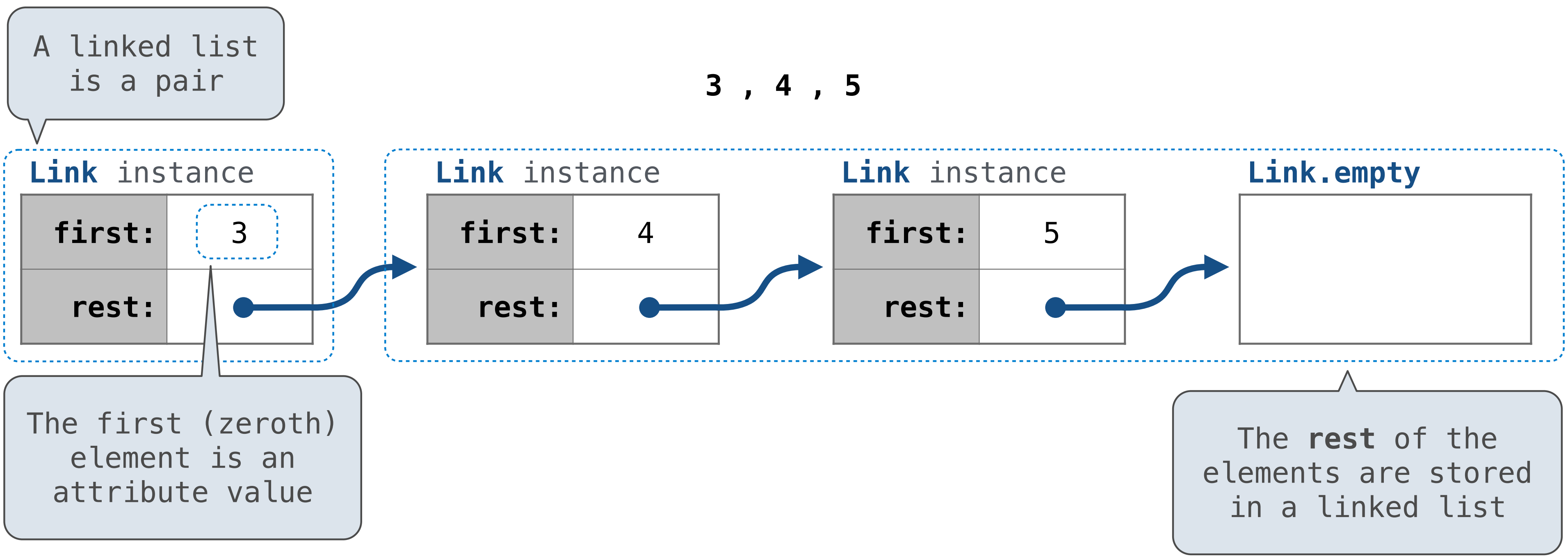

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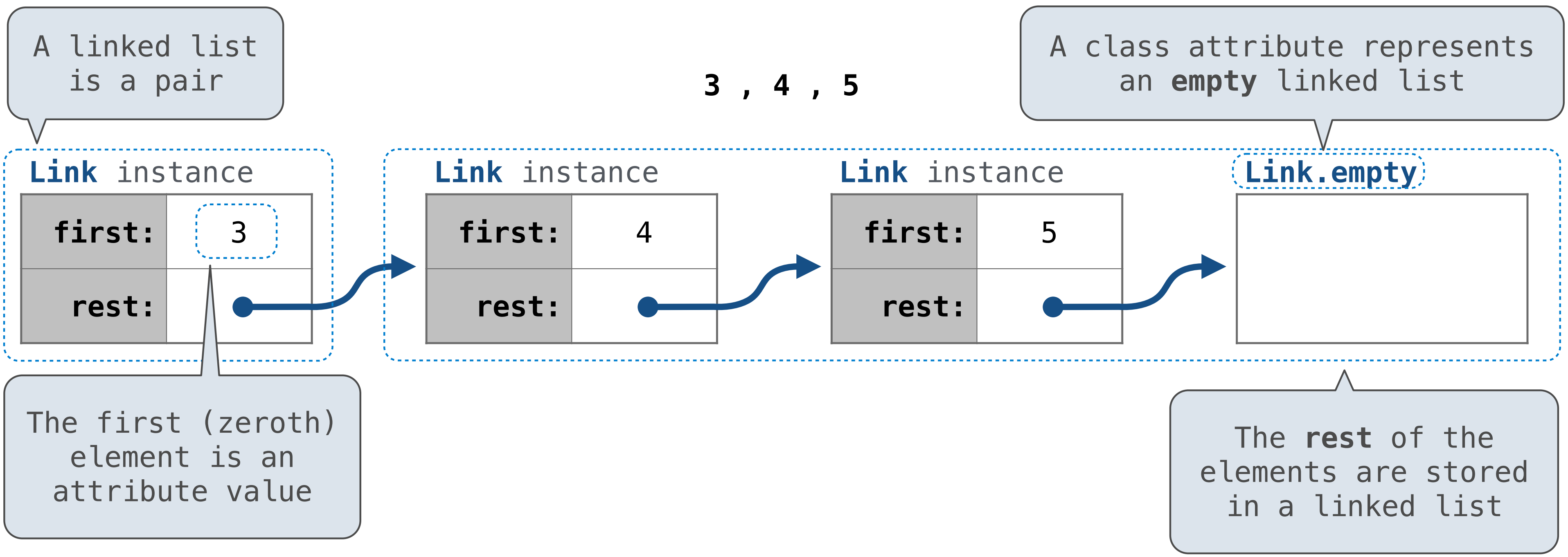

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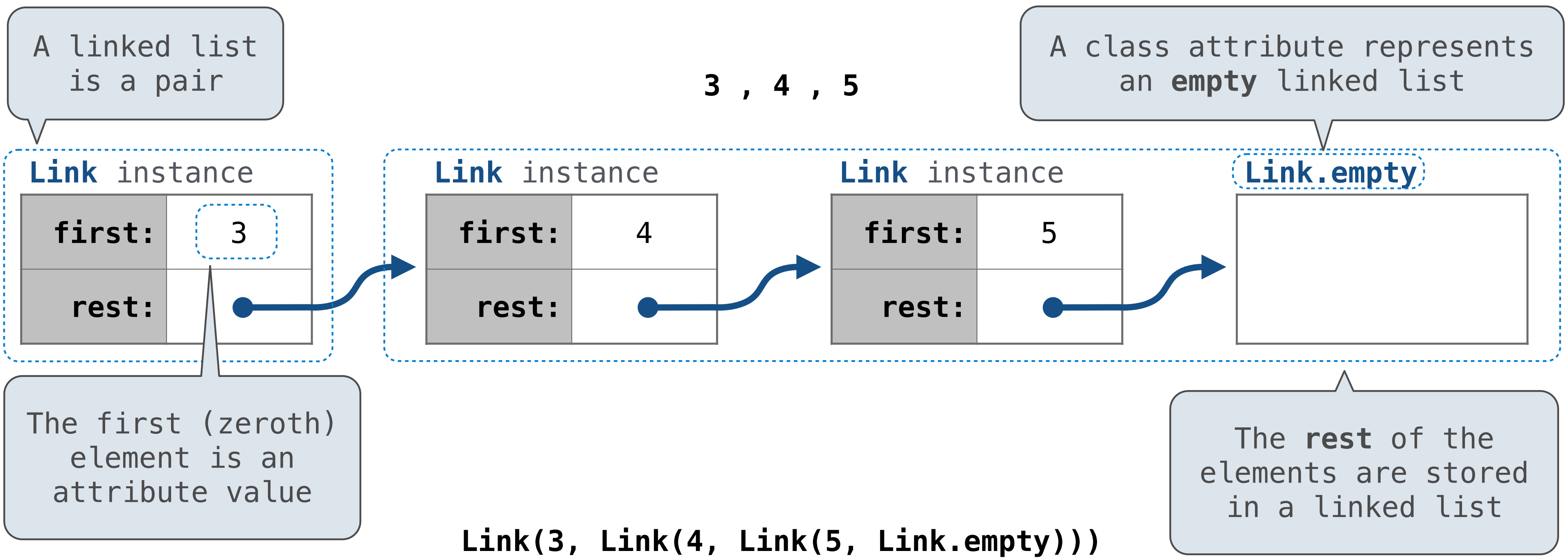

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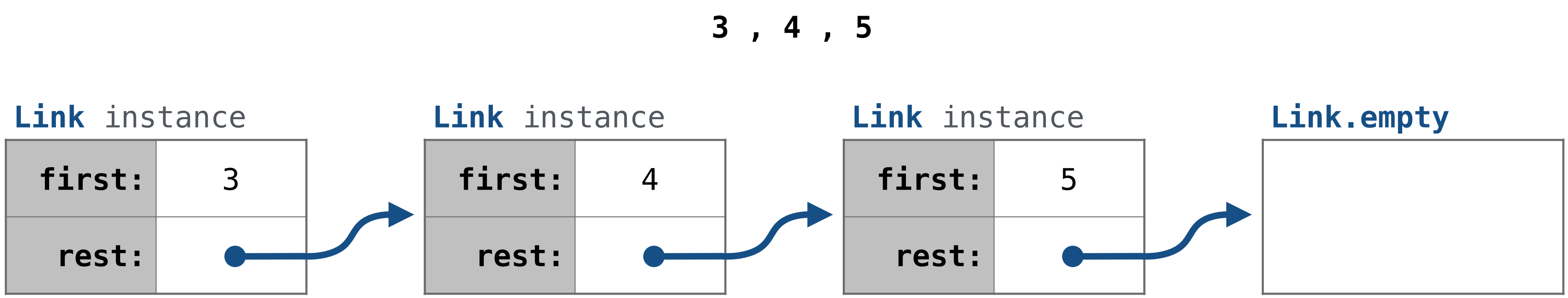


```
Link(3, Link(4, Link(5, Link.empty)))
```

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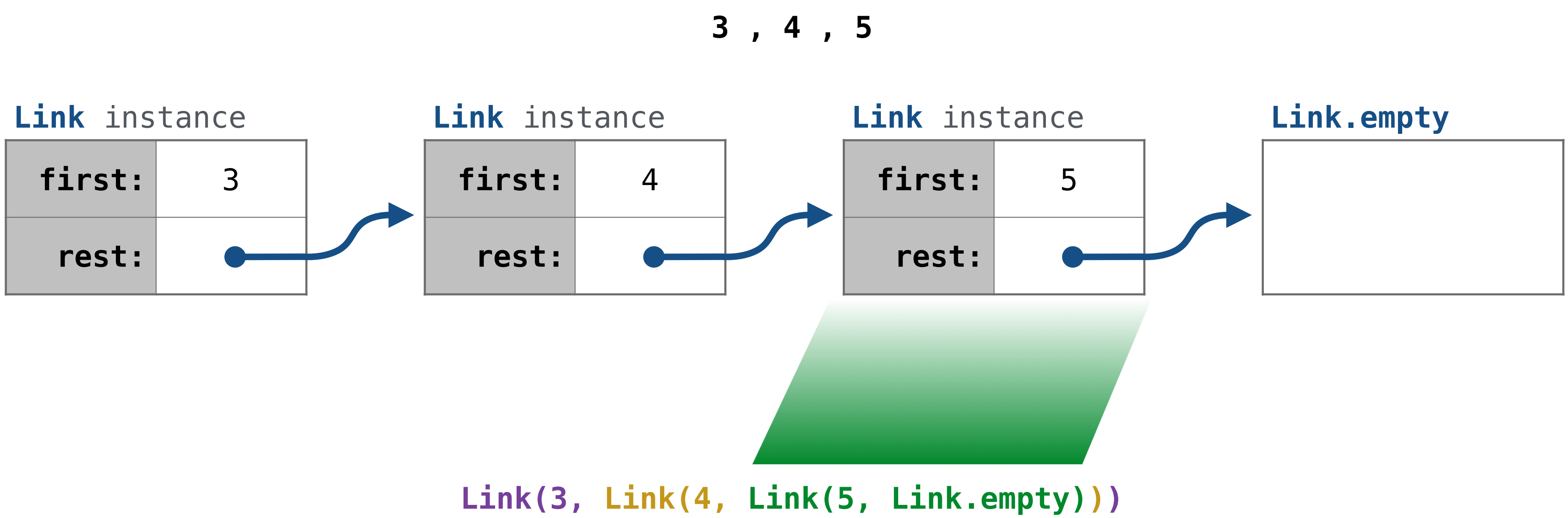

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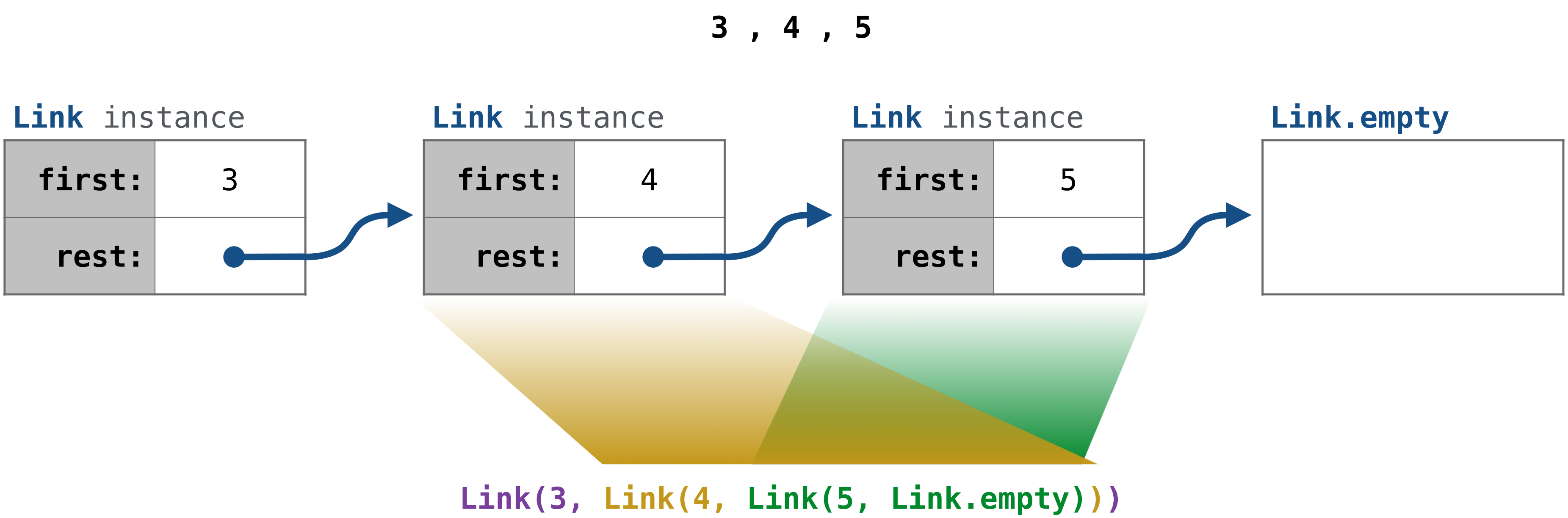

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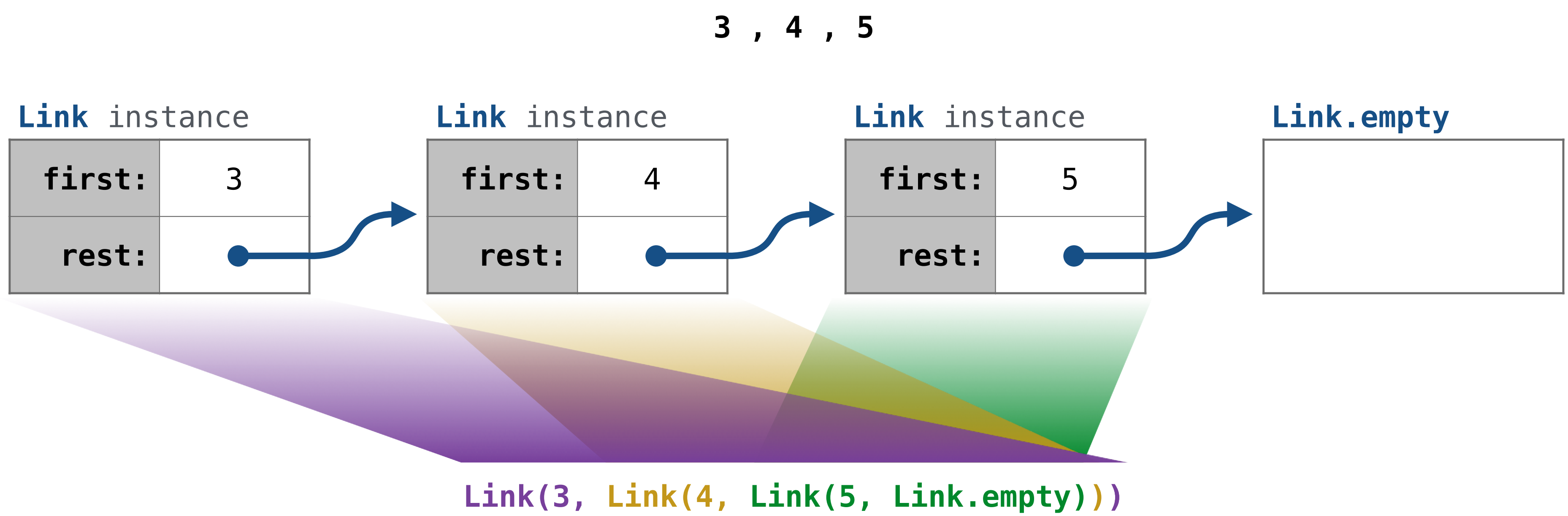

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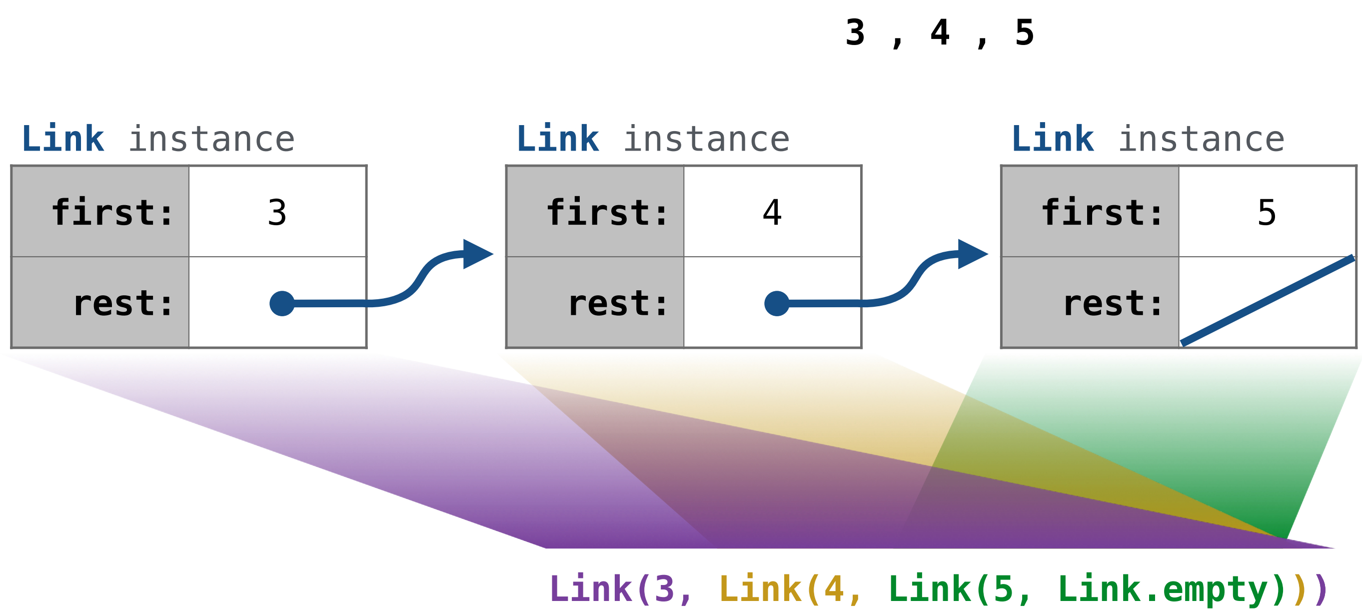

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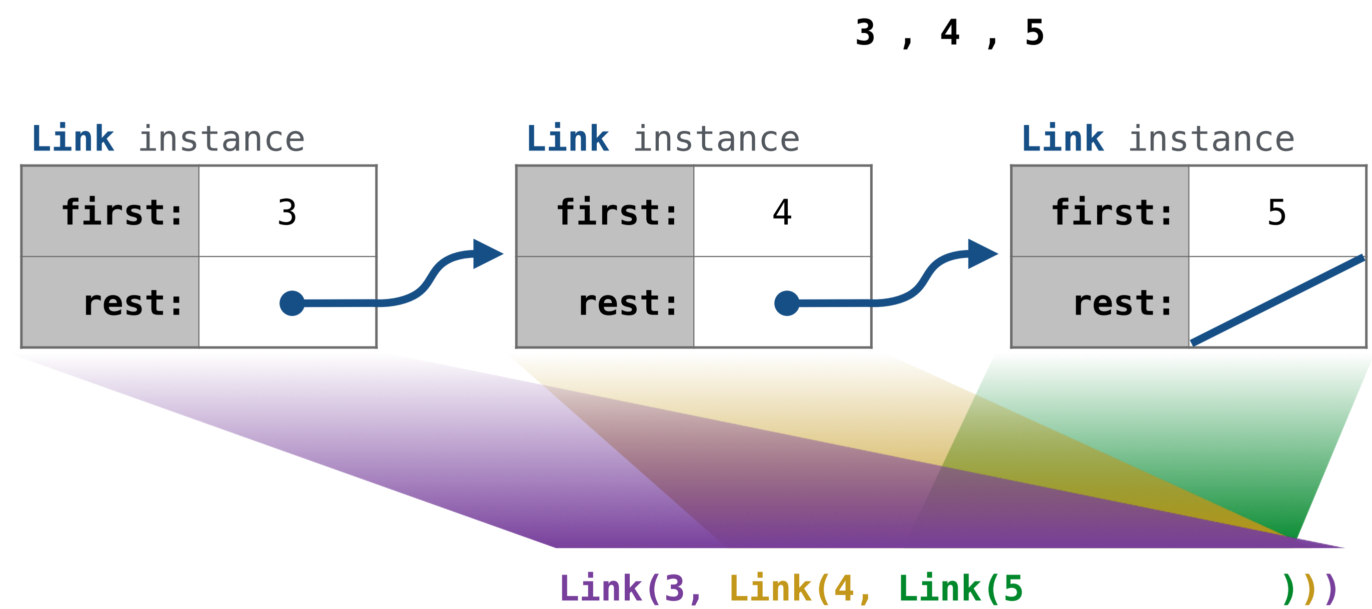

Linked List Class

```
Link(3, Link(4, Link(5          )))
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Linked List Class

`Linked list class: attributes are passed to __init__`

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Linked list class: attributes are passed to `__init__`

```
class Link:
```

```
Link(3, Link(4, Link(5          )))
```

Linked List Class

Linked list class: attributes are passed to __init__

```
class Link:

    def __init__(self, first, rest=empty):
```

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Link(3, Link(4, Link(5          )))
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Linked List Class

Linked list class: attributes are passed to `__init__`

```
class Link:


    def __init__(self, first, rest=empty):
        assert rest is Link.empty or isinstance(rest, Link)
```

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Link(3, Link(4, Link(5          )))
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Linked List Class

Linked list class: attributes are passed to `__init__`

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class Link:

    def __init__(self, first, rest=empty):
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        self.first = first
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Linked list class: attributes are passed to `__init__`

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Returns whether rest is a Link

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help(isinstance): Return whether an object is an instance of a class or of a subclass thereof.

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(Demo)

Linked List Processing

Example: Range, Map, and Filter for Linked Lists

```
square, odd = lambda x: x * x, lambda x: x % 2 == 1
list(map(square, filter(odd, range(1, 6))))          # [1, 9, 25]
map_link(square, filter_link(odd, range_link(1, 6)))  # Link(1, Link(9, Link(25)))

def range_link(start, end):
    """Return a Link containing consecutive integers from start to end.

    >>> range_link(3, 6)
    Link(3, Link(4, Link(5)))
    """


def map_link(f, s):
    """Return a Link that contains f(x) for each x in Link s.

    >>> map_link(square, range_link(3, 6))
    Link(9, Link(16, Link(25)))
    """


def filter_link(f, s):
    """Return a Link that contains only the elements x of Link s for which f(x)
    is a true value.

    >>> filter_link(odd, range_link(3, 6))
    Link(3, Link(5))
    """
```

Linked Lists Mutation

Linked Lists Can Change

```
Attribute assignment statements can change first and rest attributes of a Link
```

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>>> s = Link(1, Link(2, Link(3)))
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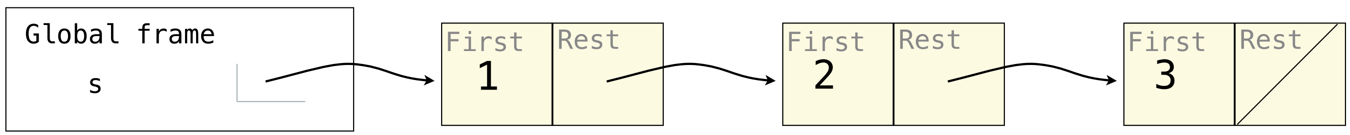

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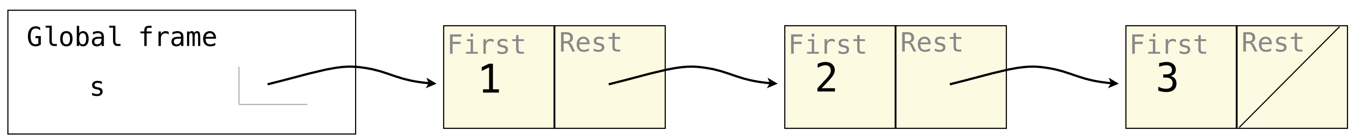

Note: The actual environment diagram is much more complicated.

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>>> s.first = 5
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```
>>> s = Link(1, Link(2, Link(3)))
>>> s.first = 5
>>> t = s.rest
```

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>>> t = s.rest
>>> t.rest = s
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```
>>> s = Link(1, Link(2, Link(3)))
>>> s.first = 5
>>> t = s.rest
>>> t.rest = s
>>> s.first
5
>>> s.rest.rest.rest.rest.rest.first
```

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The rest of a linked list can contain the linked list as a sub-list

```
>>> s = Link(1, Link(2, Link(3)))
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>>> t = s.rest
>>> t.rest = s
>>> s.first
5
>>> s.rest.rest.rest.rest.rest.first
2
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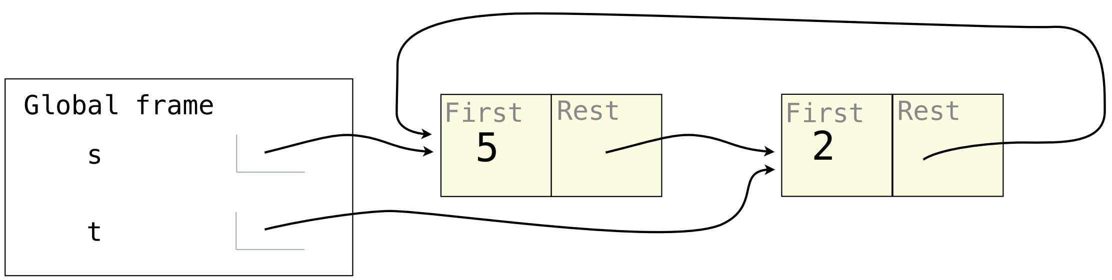
```
>>> s = Link(1, Link(2, Link(3)))
>>> s.first = 5
>>> t = s.rest
>>> t.rest = s
>>> s.first
5
>>> s.rest.rest.rest.rest.rest.first
2
```



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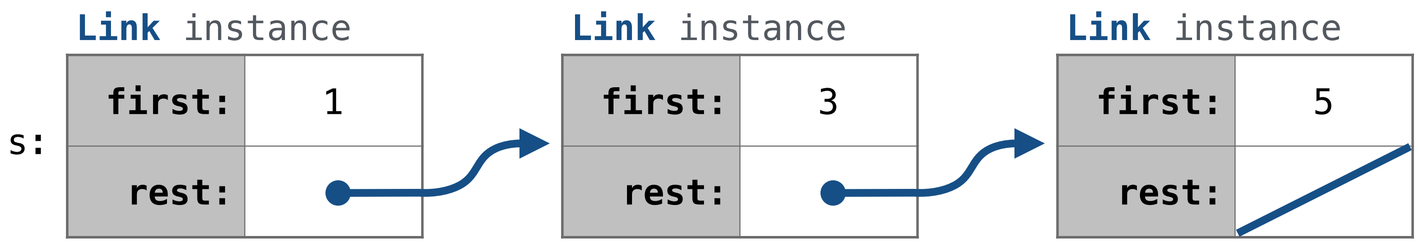
Linked List Mutation Example

Adding to an Ordered List



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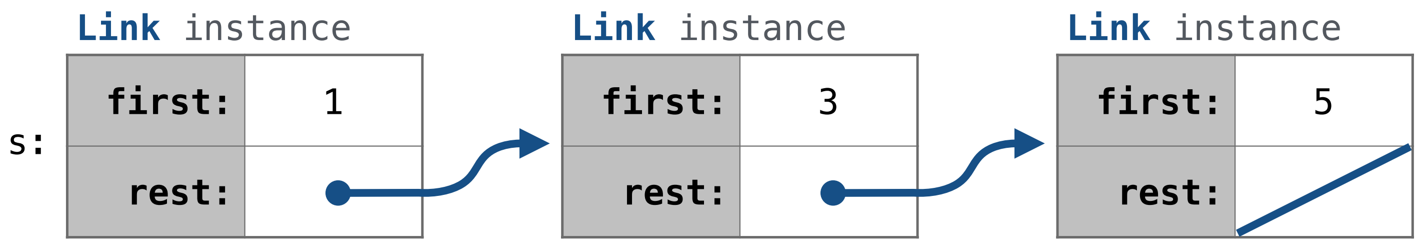


```
def add(s, v):
    """Add v to an ordered list s with no repeats, returning modified s."""
```

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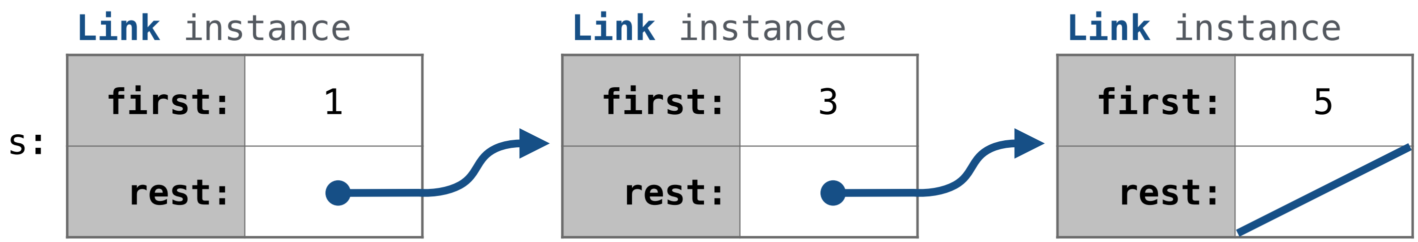


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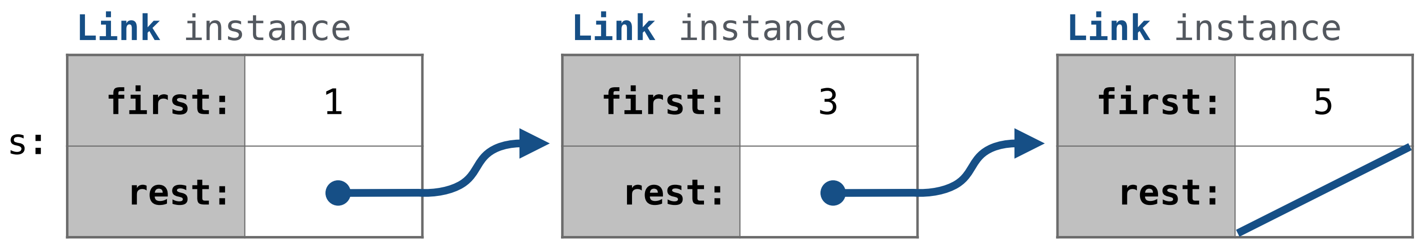
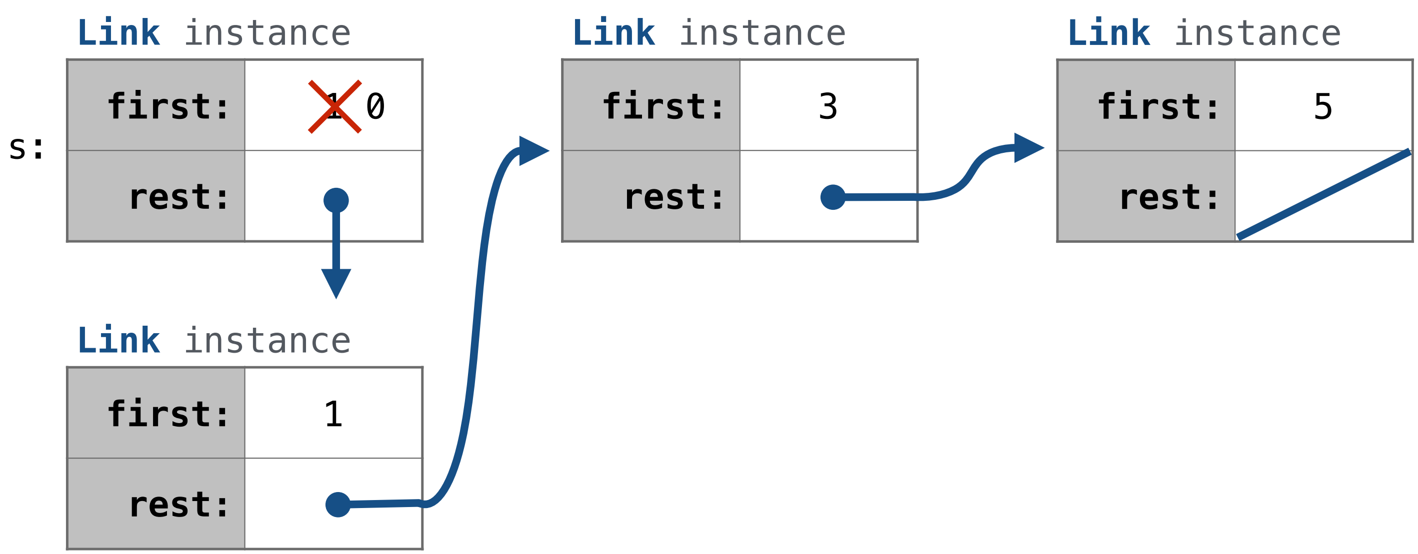



```
def add(s, v):
    """Add v to an ordered list s with no repeats, returning modified s."""
    (Note: If v is already in s, then don't modify s, but still return it.)

        add(s, 0)
```

Adding to an Ordered List

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        add(s, 0)
```

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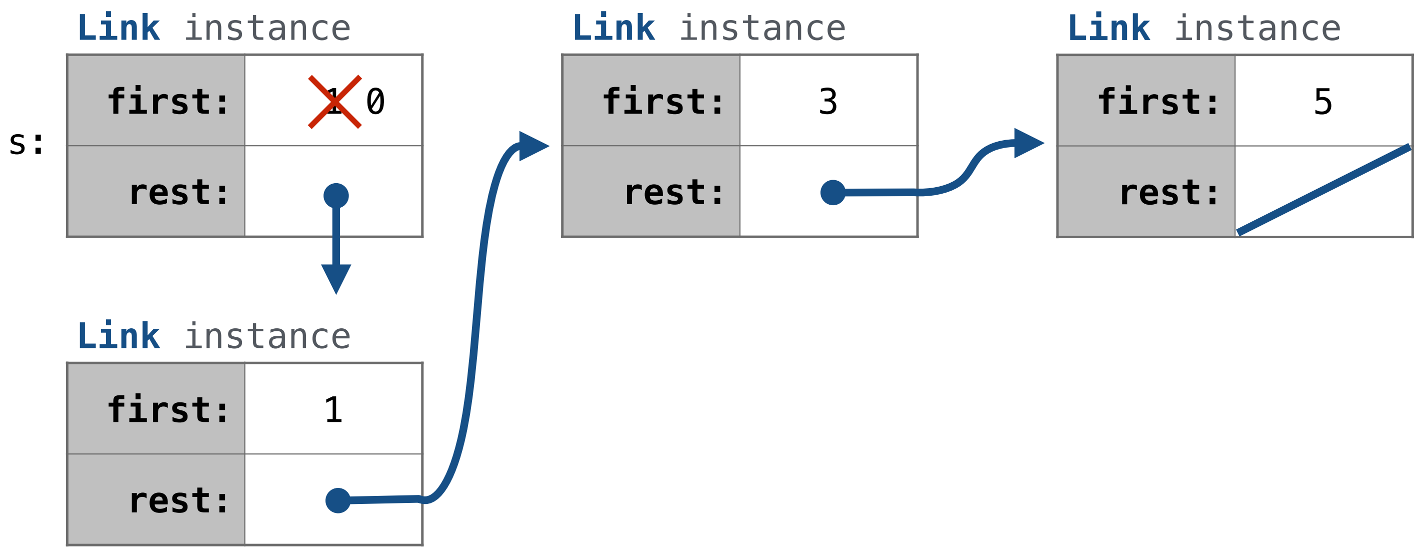


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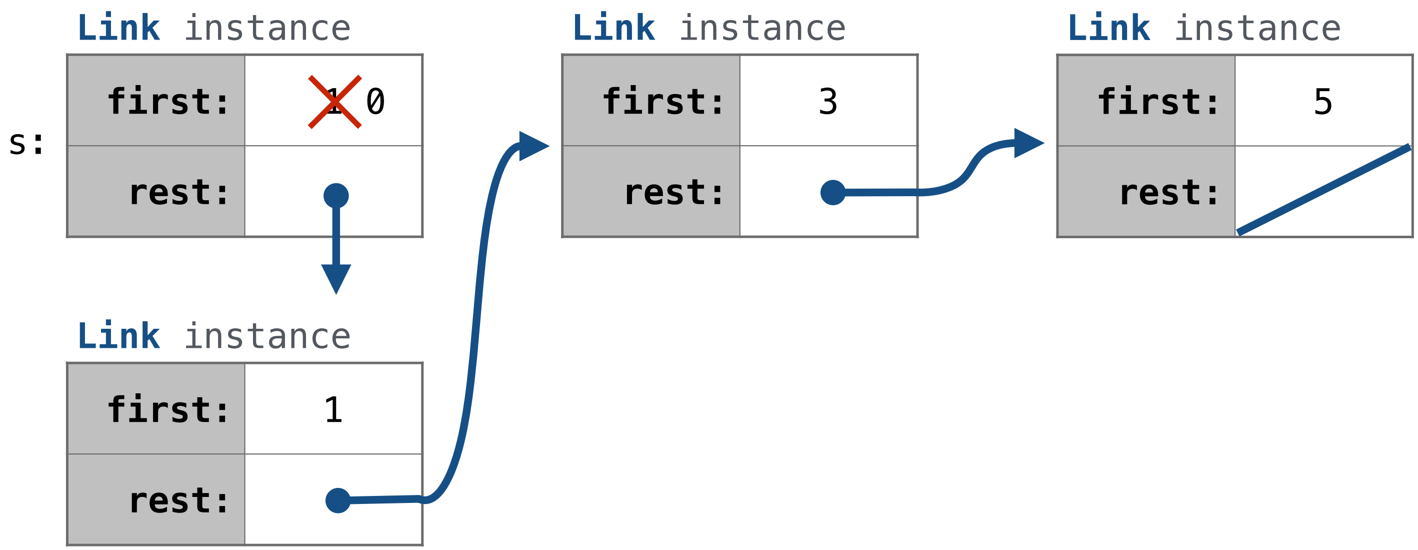
        add(s, 0)      add(s, 3)
```

Adding to an Ordered List

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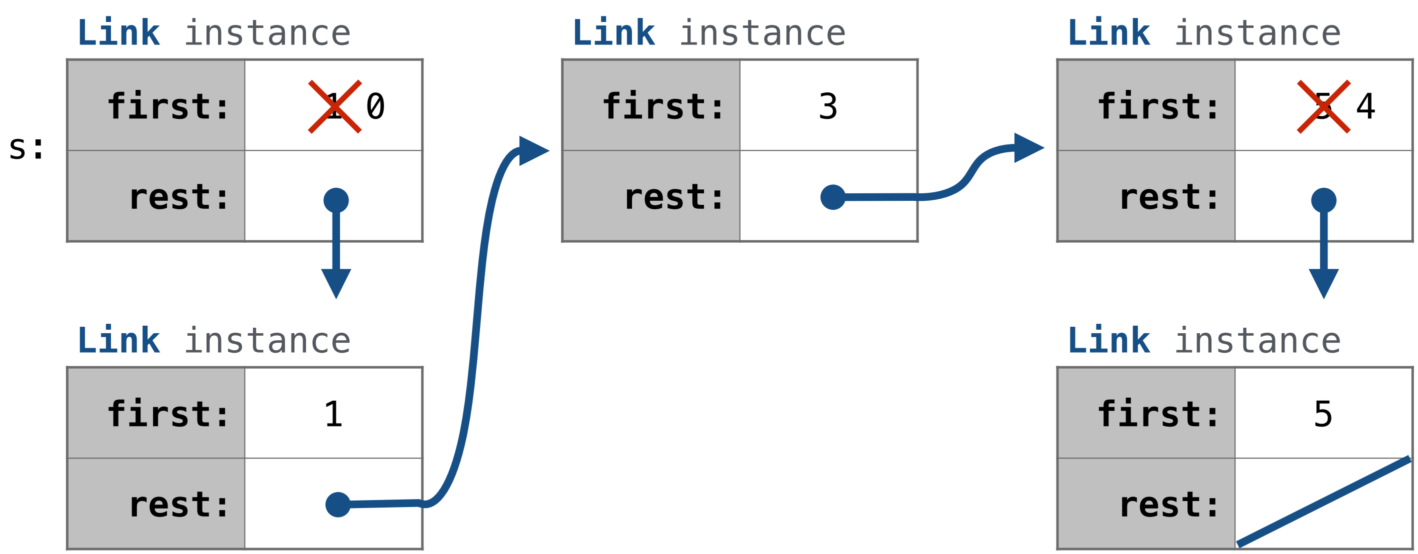
        add(s, 0)       add(s, 3)       add(s, 4)
```

Adding to an Ordered List

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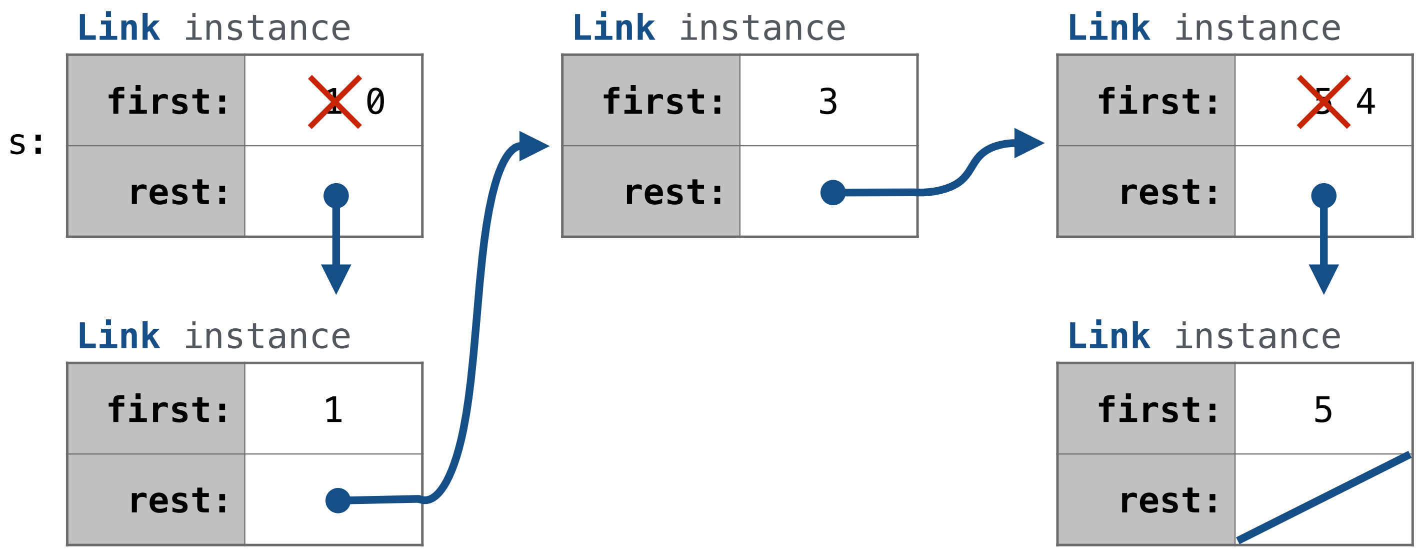
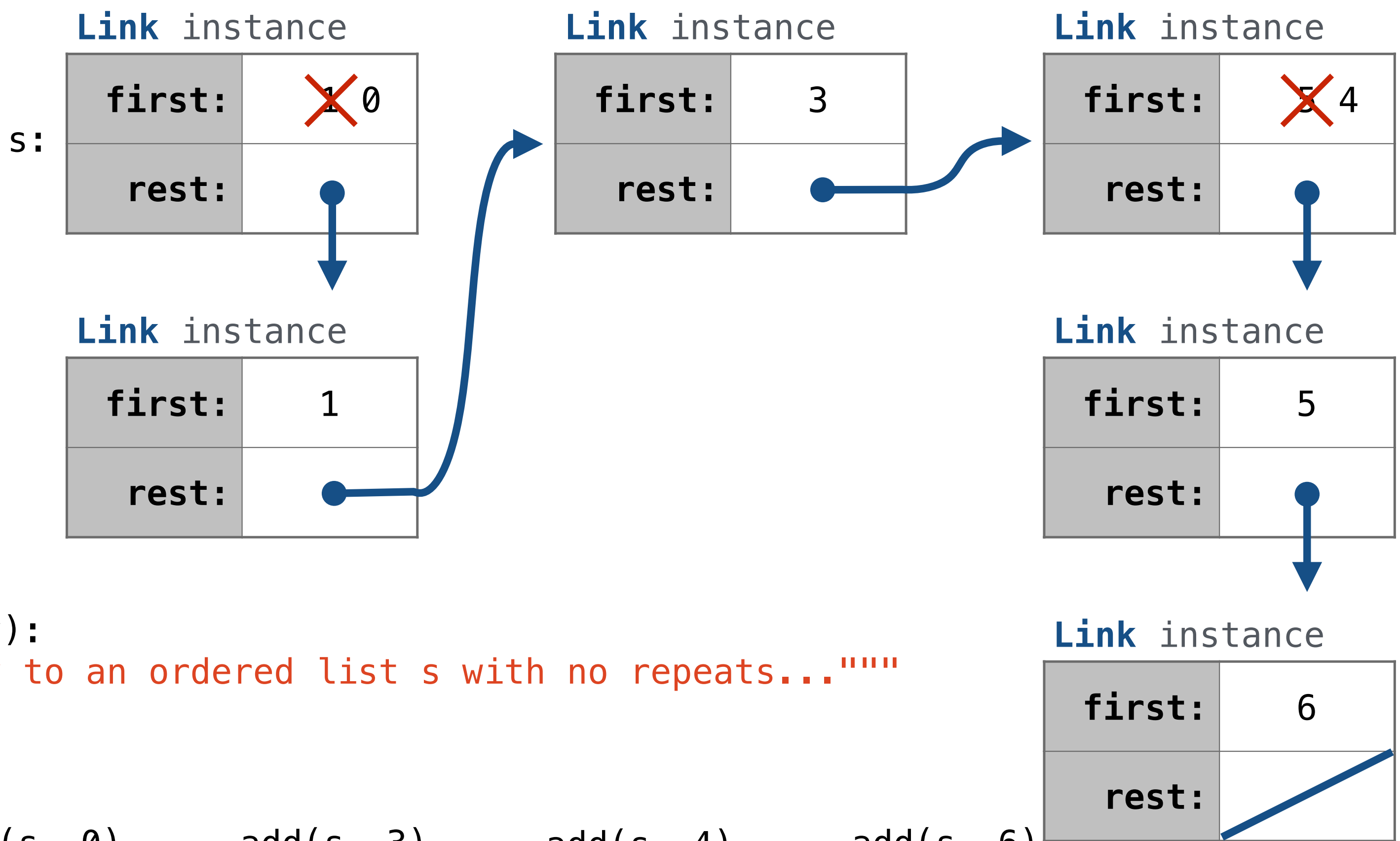



```
def add(s, v):
    """Add v to an ordered list s with no repeats..."""
```

add(s, 0)   add(s, 3)   add(s, 4)   add(s, 6)

Adding to an Ordered List

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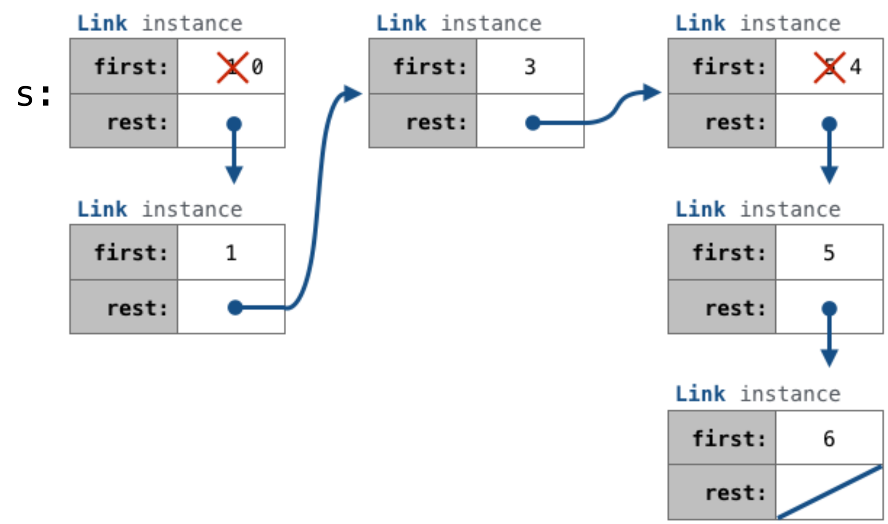
Adding to a Set Represented as an Ordered List



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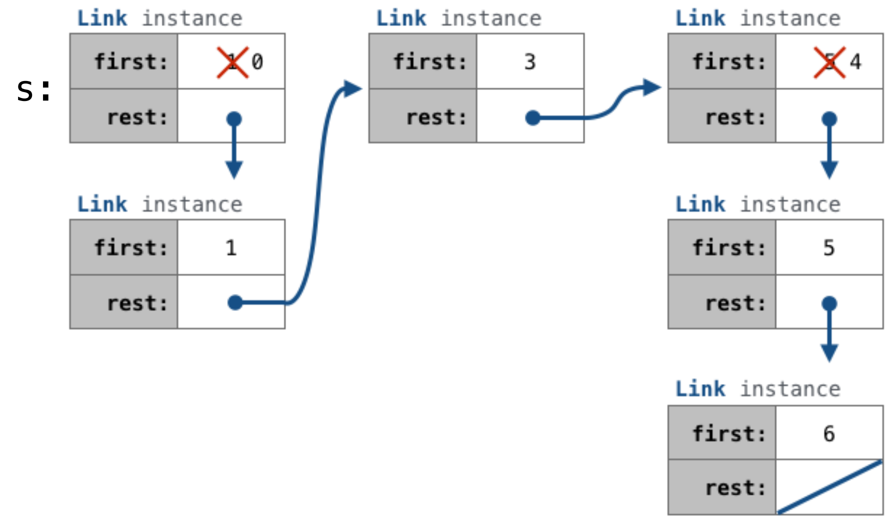

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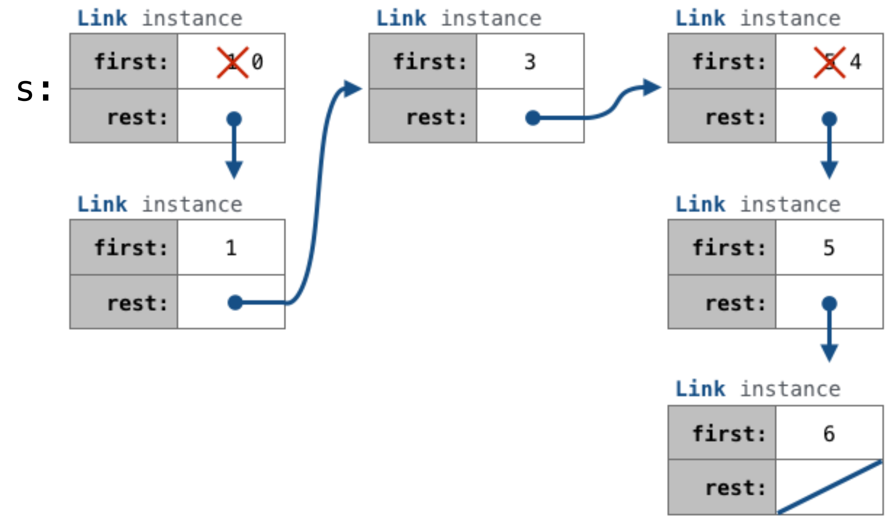

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```
def add(s, v):
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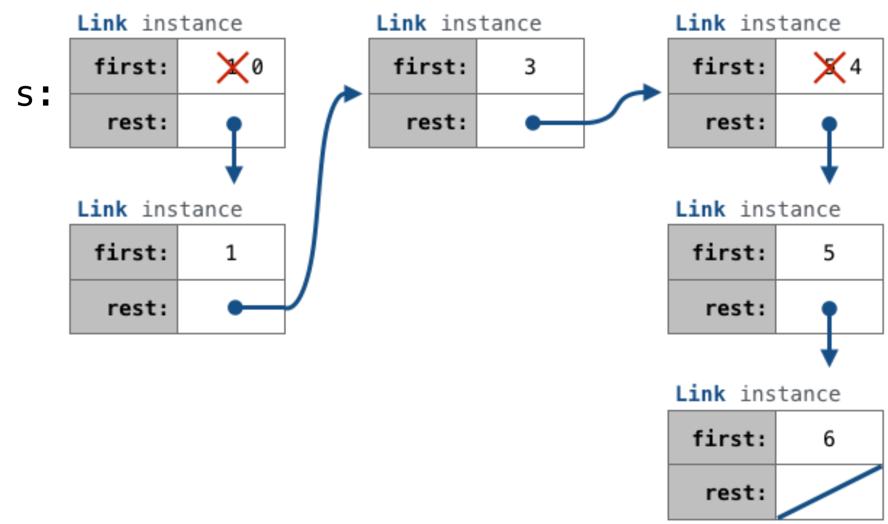
    >>> s = Link(1, Link(3, Link(5)))
```



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    Link(0, Link(1, Link(3, Link(5))))
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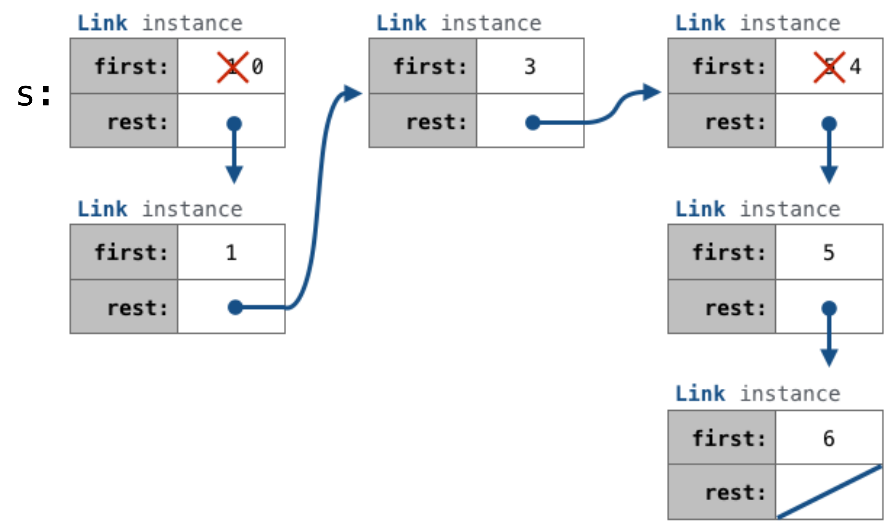

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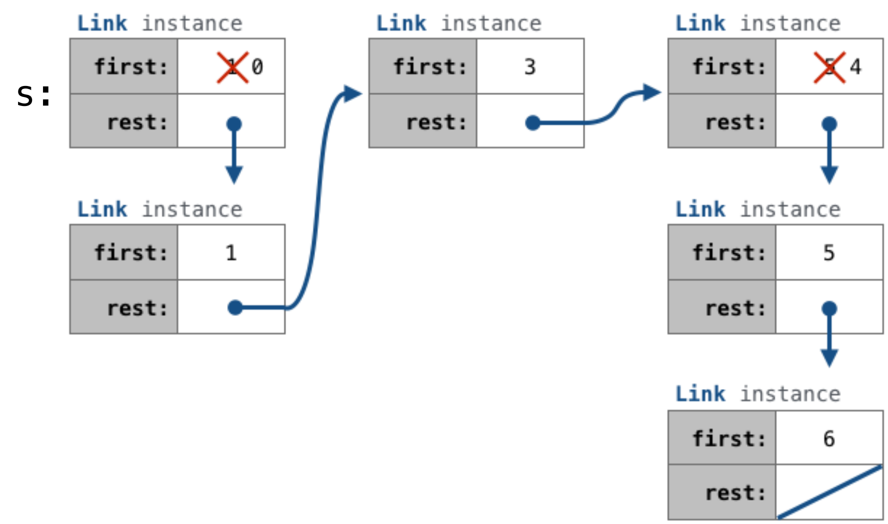

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    >>> add(s, 4)
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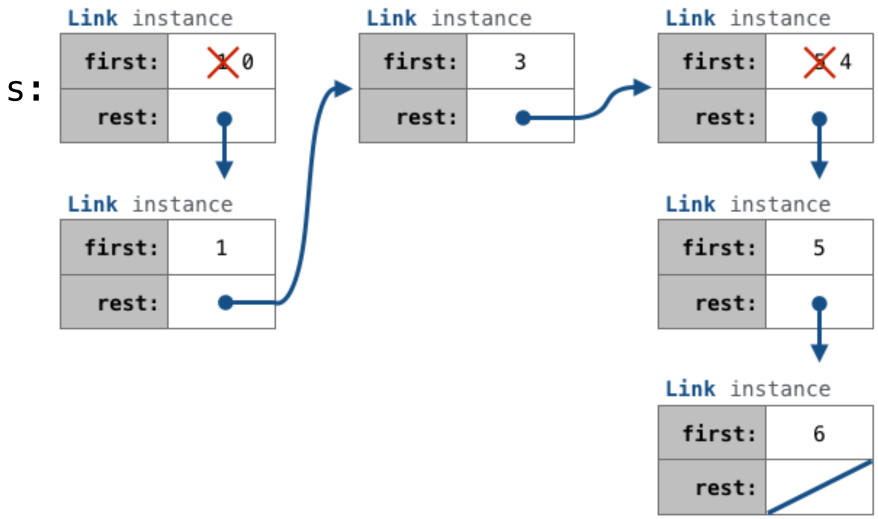
```



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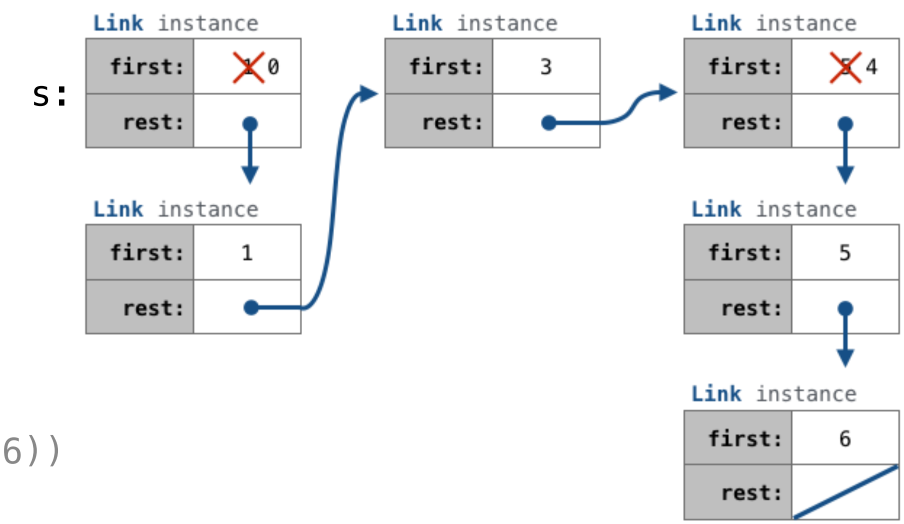
    >>> add(s, 4)
    Link(0, Link(1, Link(3, Link(4, Link(5)))))
    >>> add(s, 6)
    Link(0, Link(1, Link(3, Link(4, Link(5, Link(6))
    """
```



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    assert s is not List.empty
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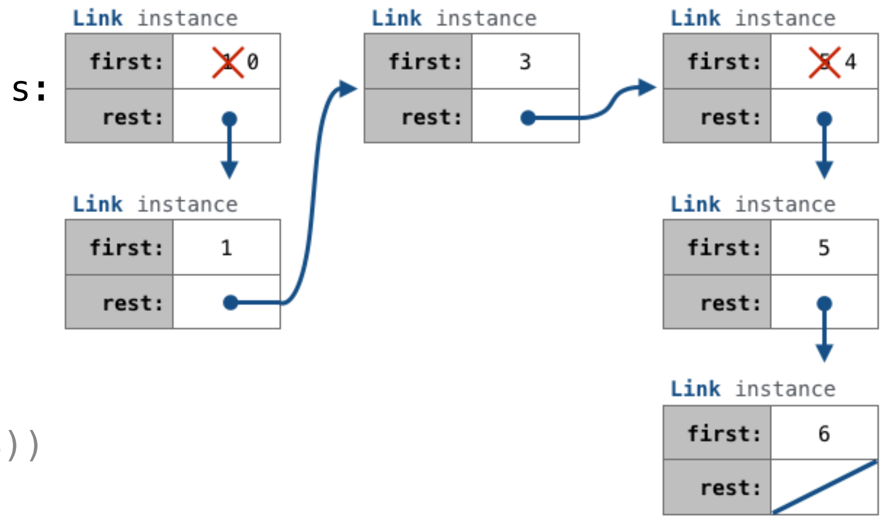

Adding to a Set Represented as an Ordered List

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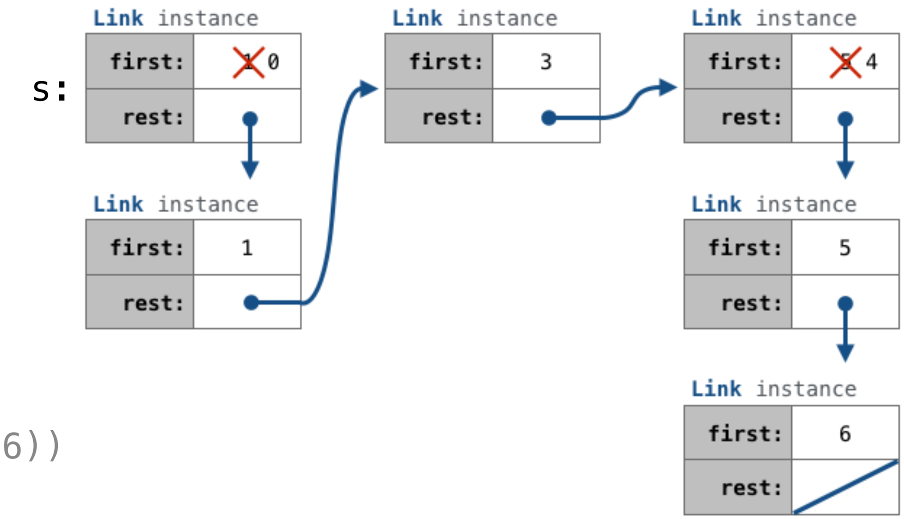

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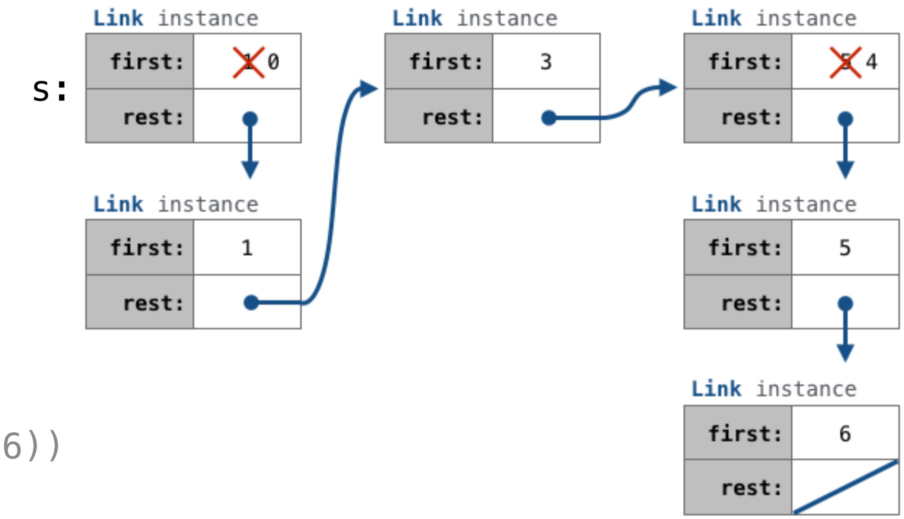

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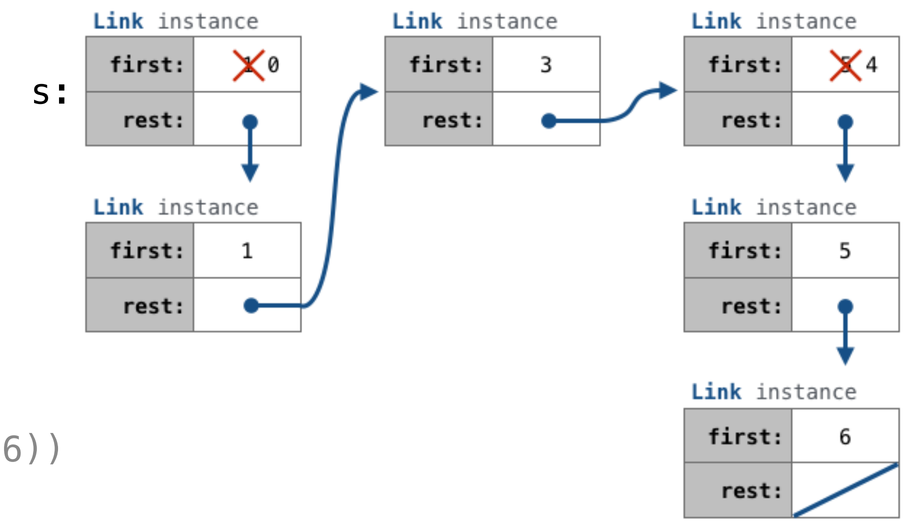

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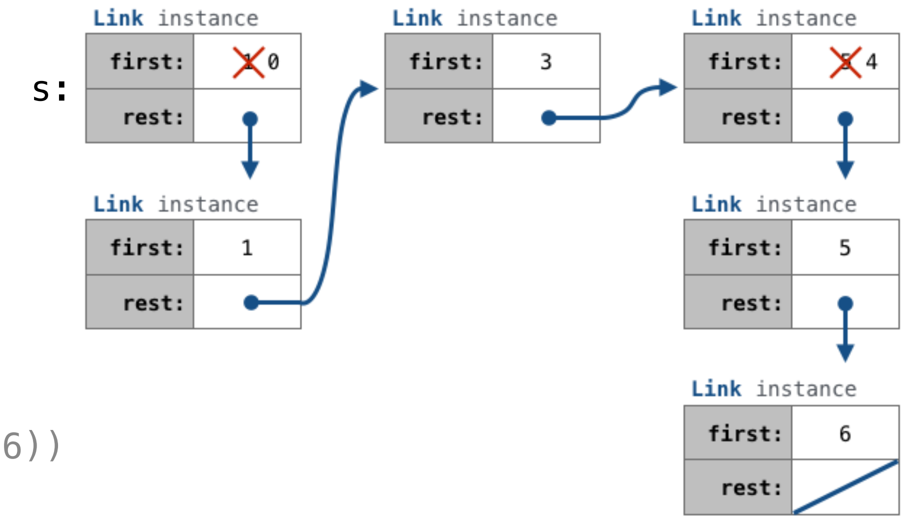

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    if s.first > v:
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        ________________________
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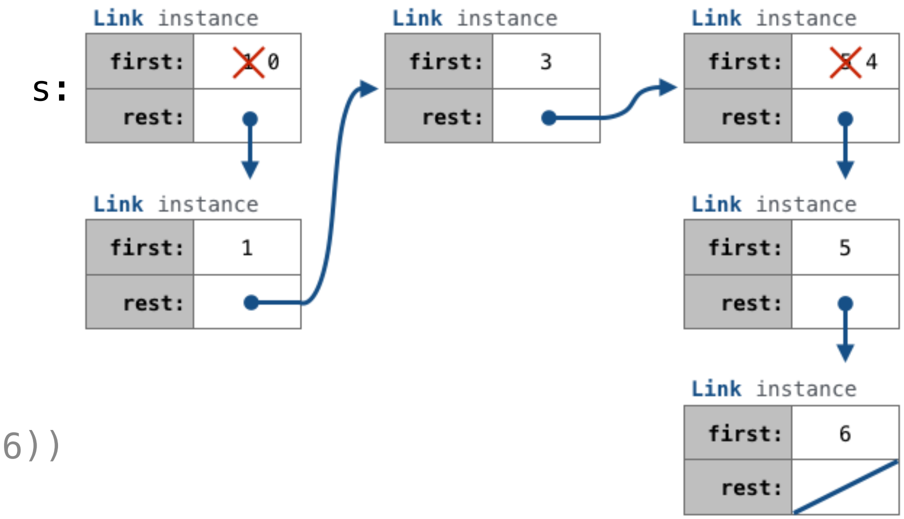
```



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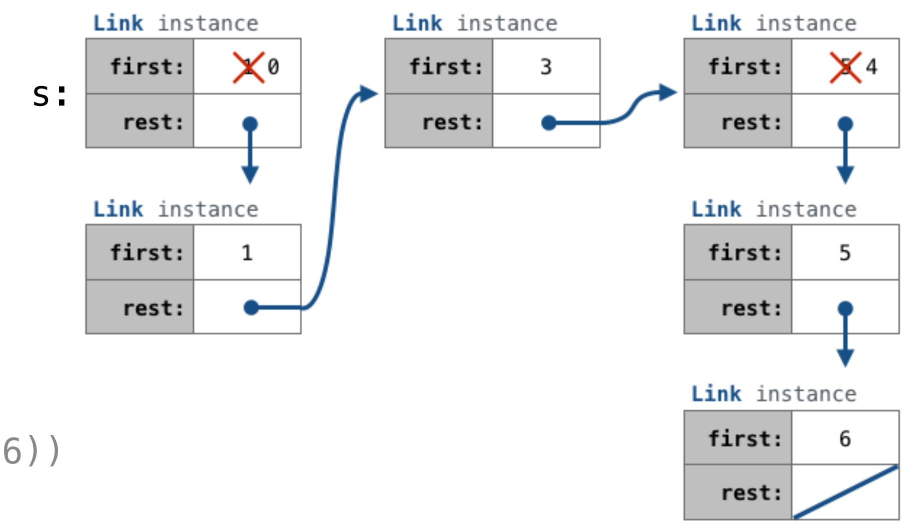

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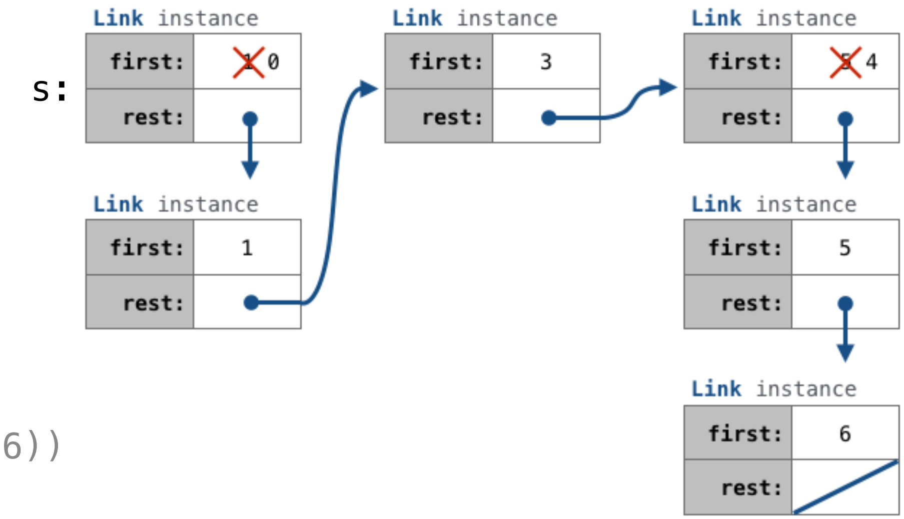

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    """

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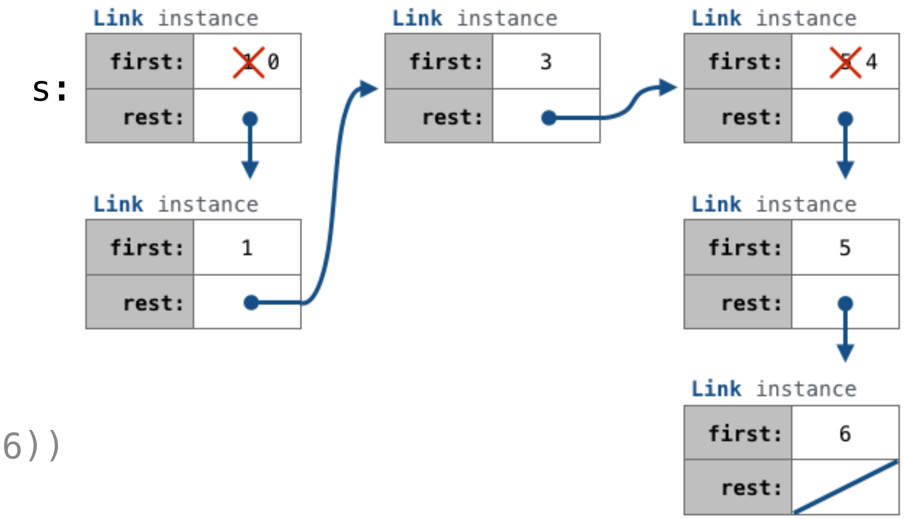
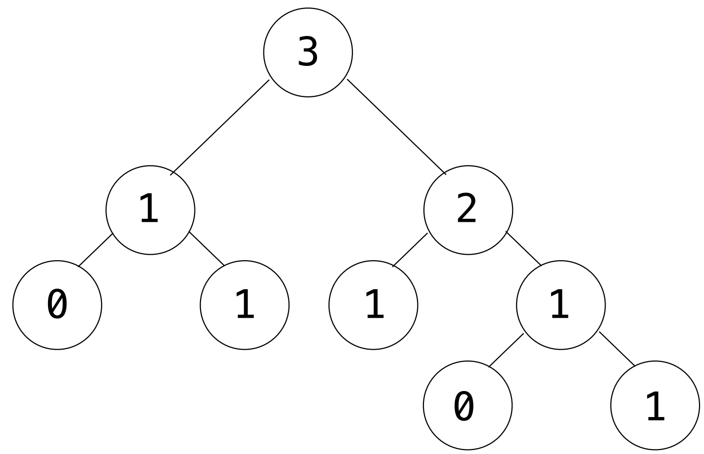
```



Tree Class

Tree Abstraction (Review)



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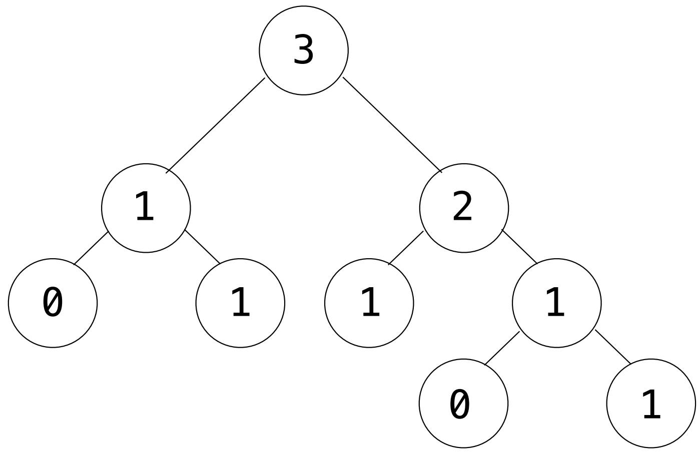
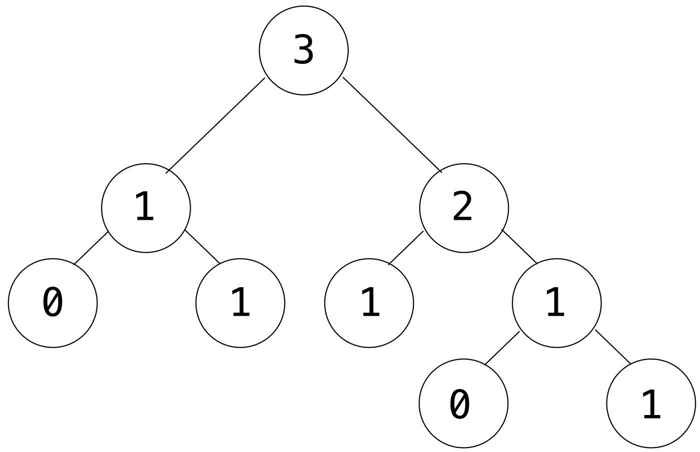


Recursive description (wooden trees):

Relative description (family trees):

Tree Abstraction (Review)



Recursive description (wooden trees):

A **tree** has a **root label** and a list of **branches**

Relative description (family trees):

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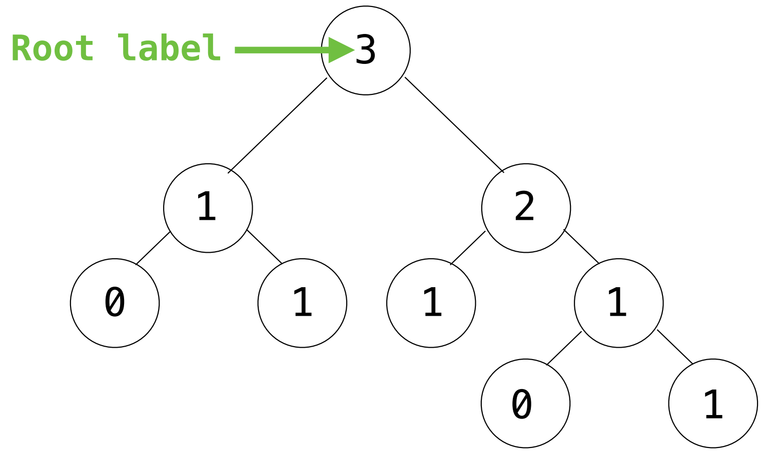

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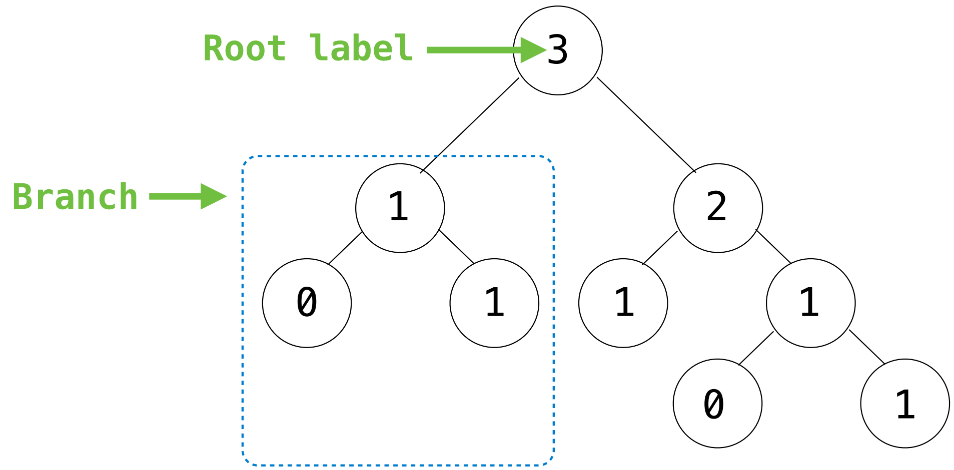

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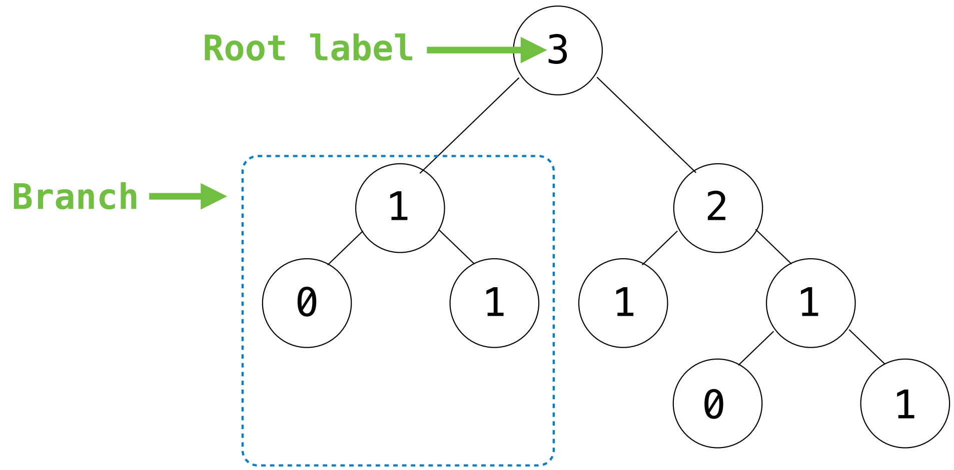

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Each **branch** is a **tree**

Relative description (family trees):

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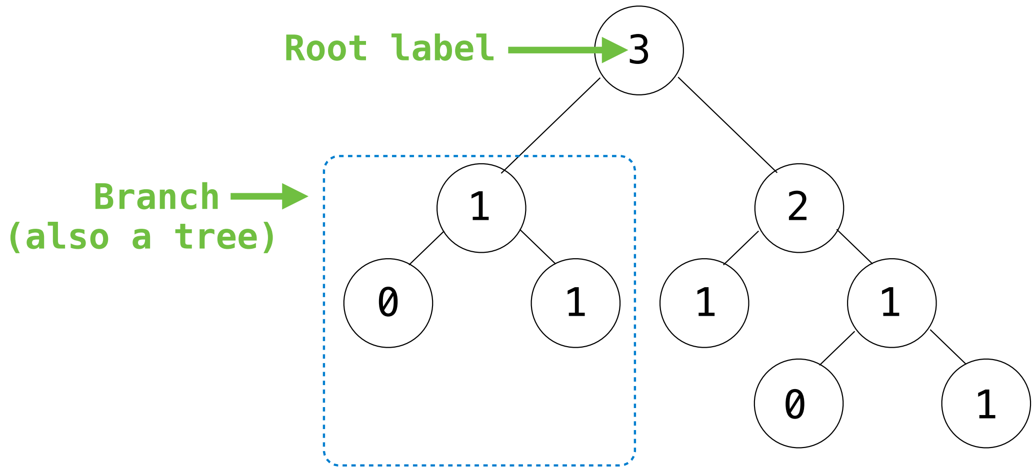

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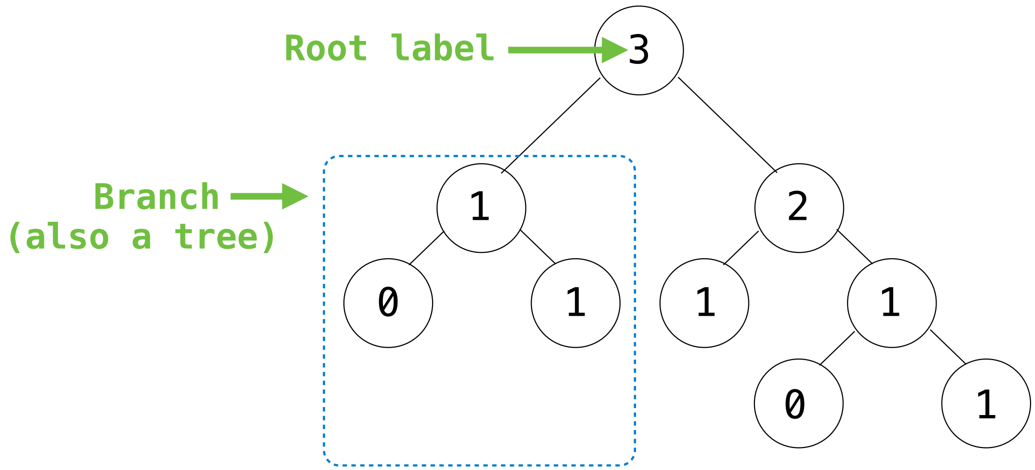

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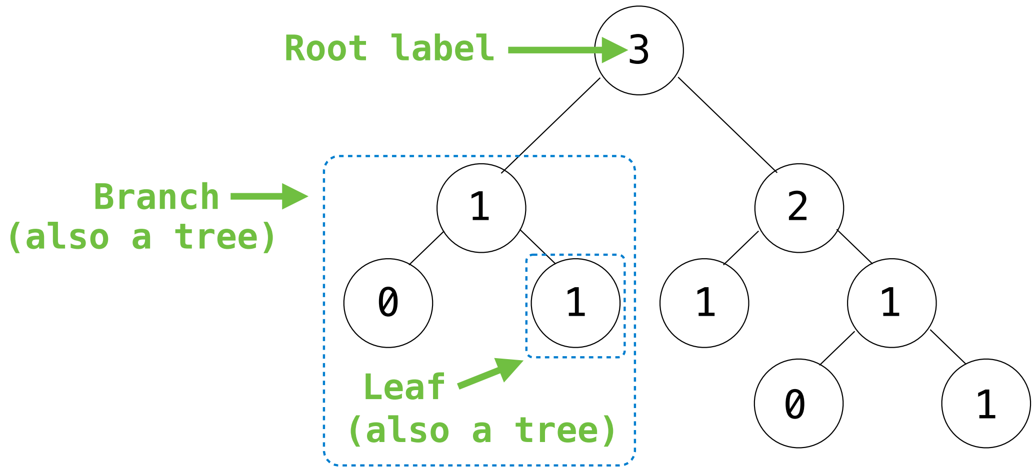

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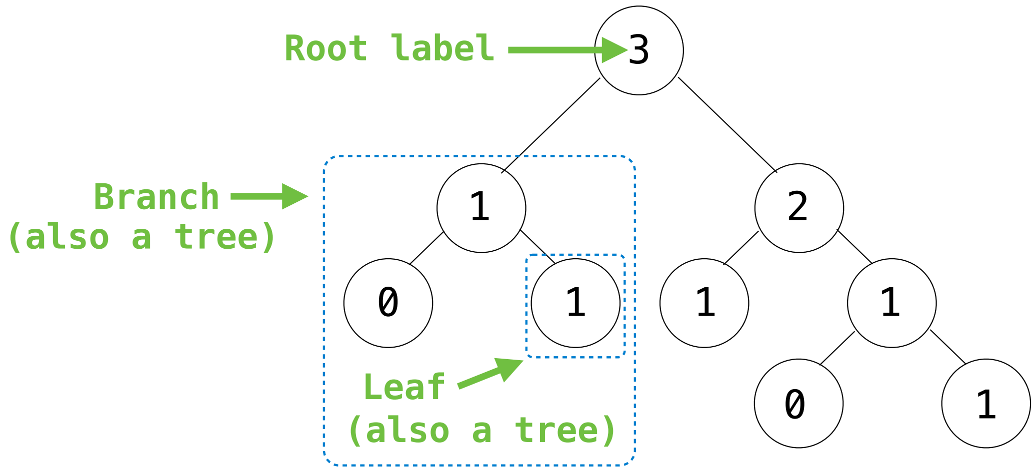

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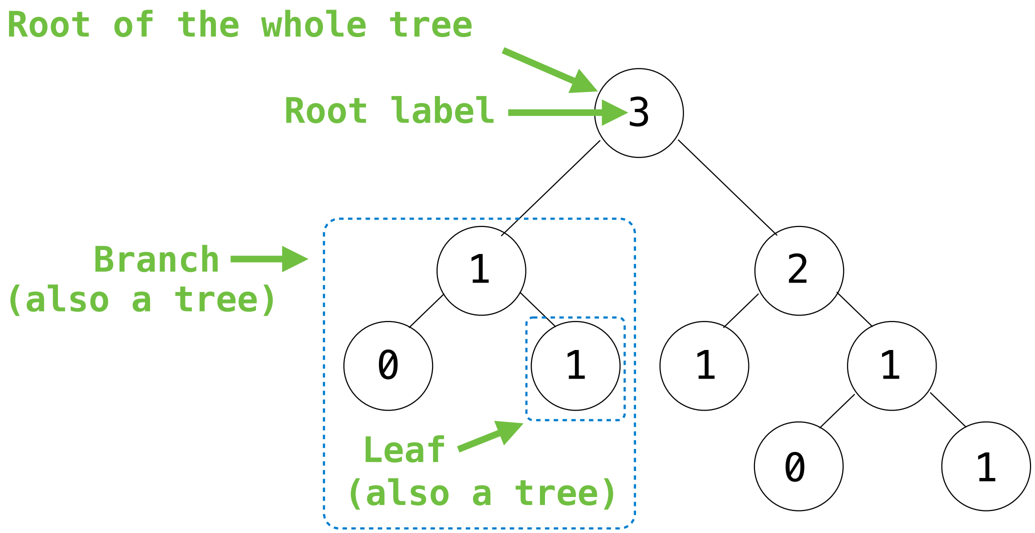

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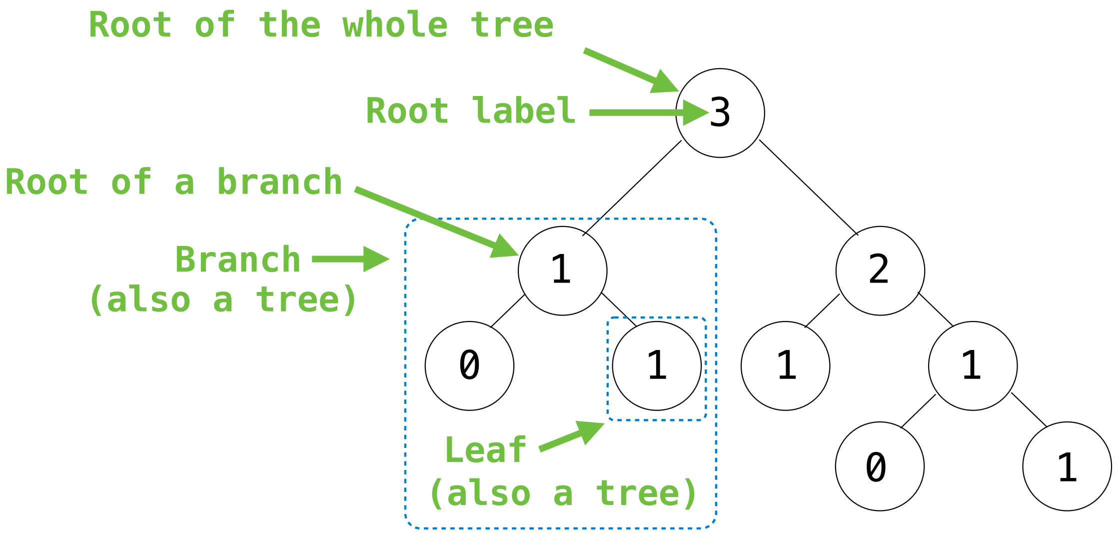

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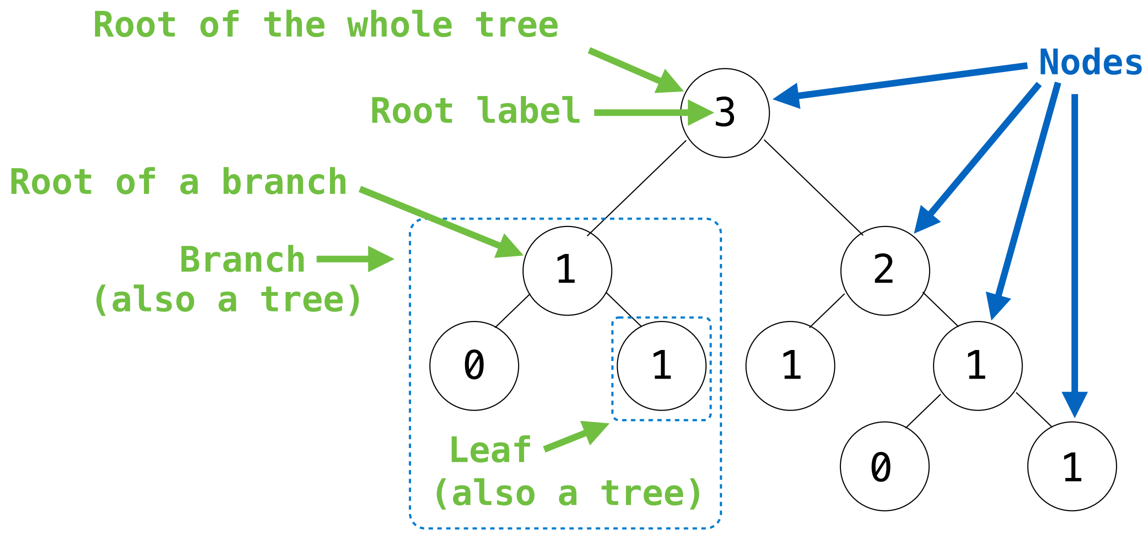

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Relative description (family trees):

Each location in a tree is called a **node**

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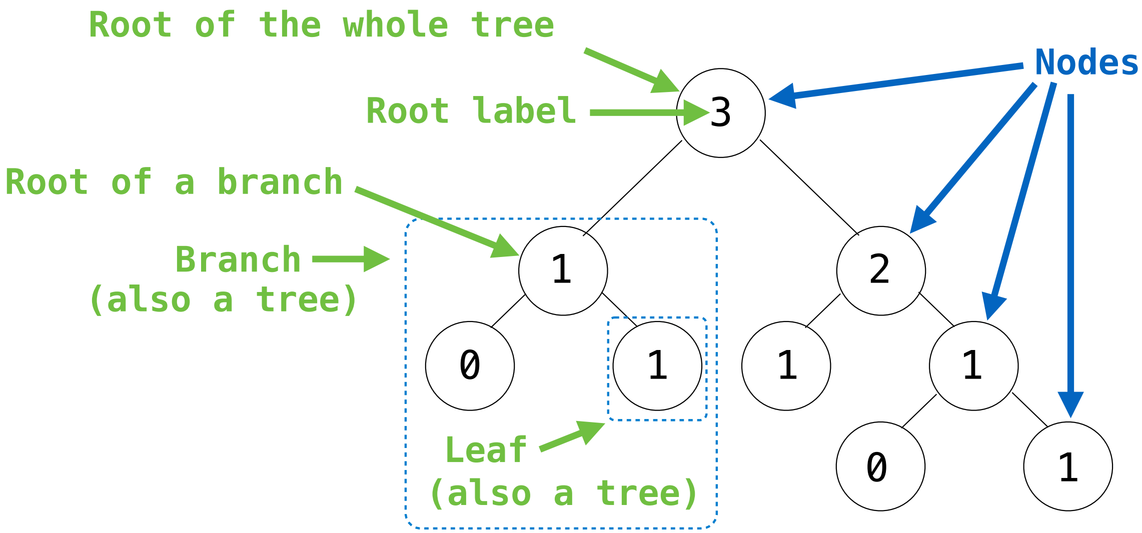

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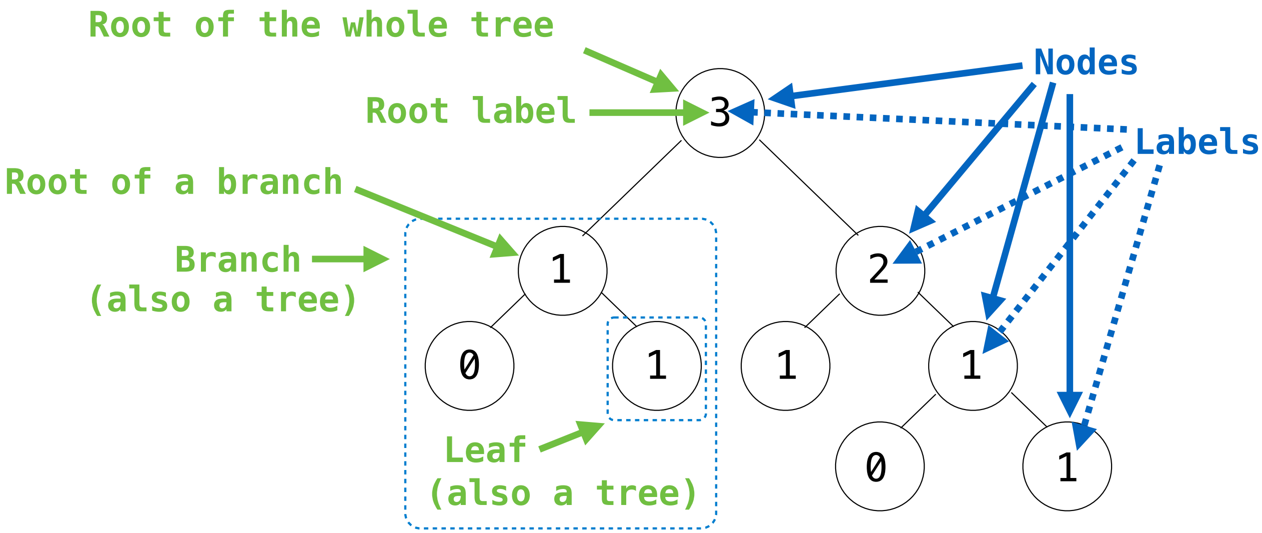
Each location in a tree is called a **node**

Each **node** has a **label** that can be any value

Tree Abstraction (Review)



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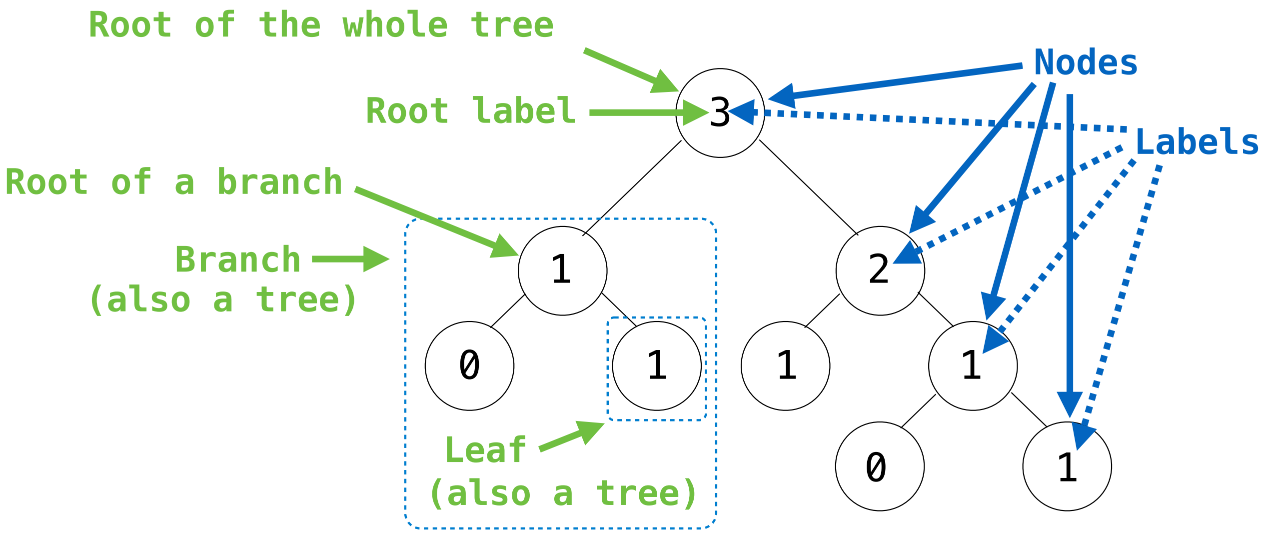

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One node can be the **parent/child** of another

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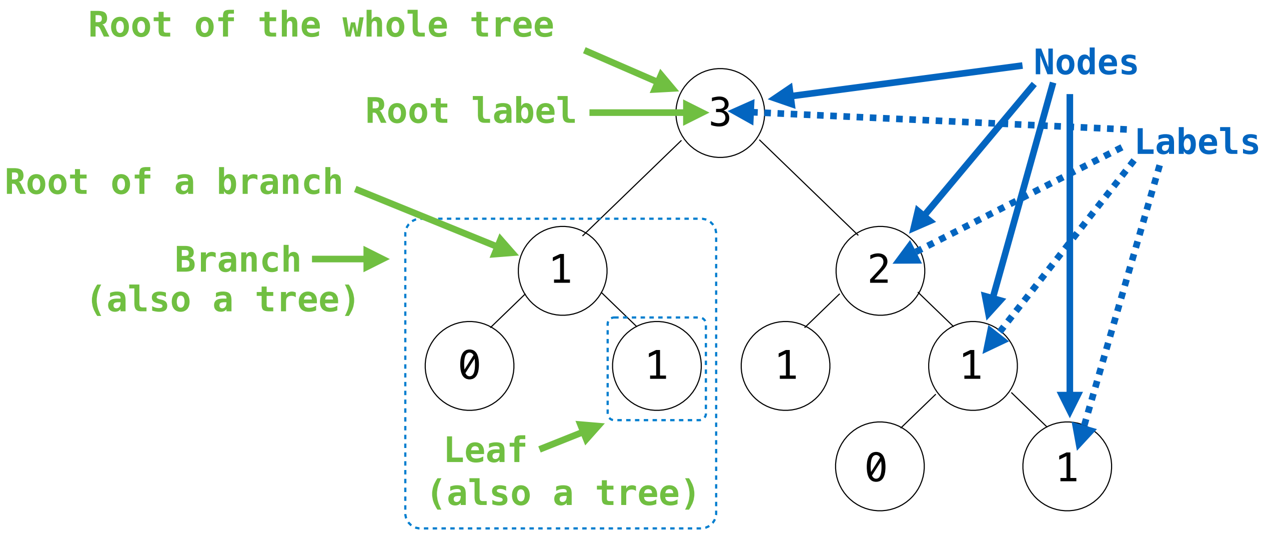

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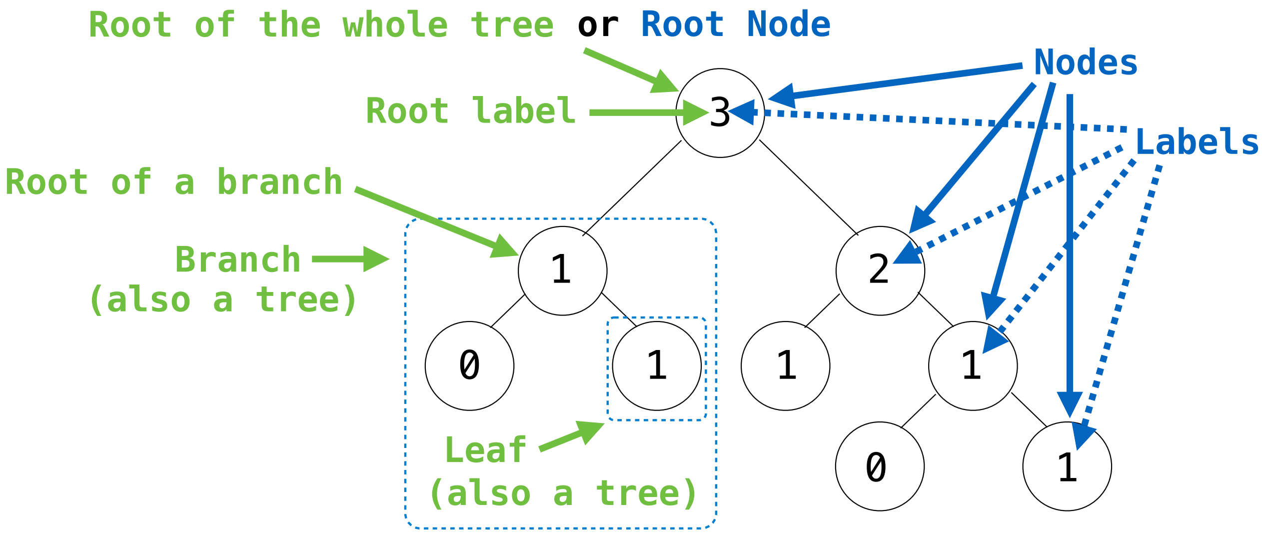

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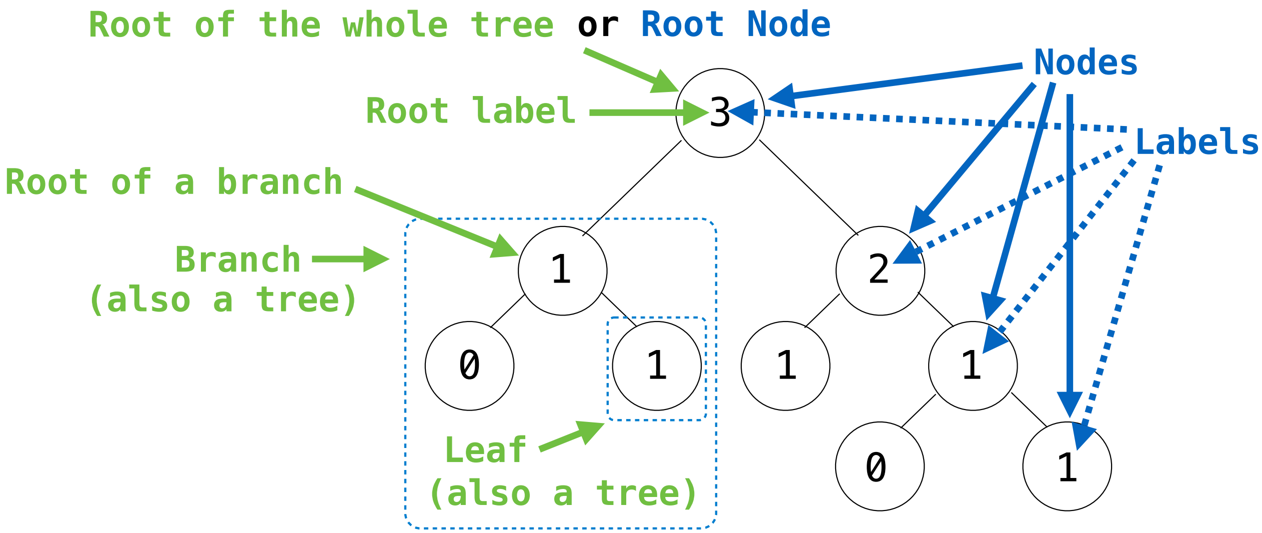

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People often refer to labels by their locations: "each parent is the sum of its children"

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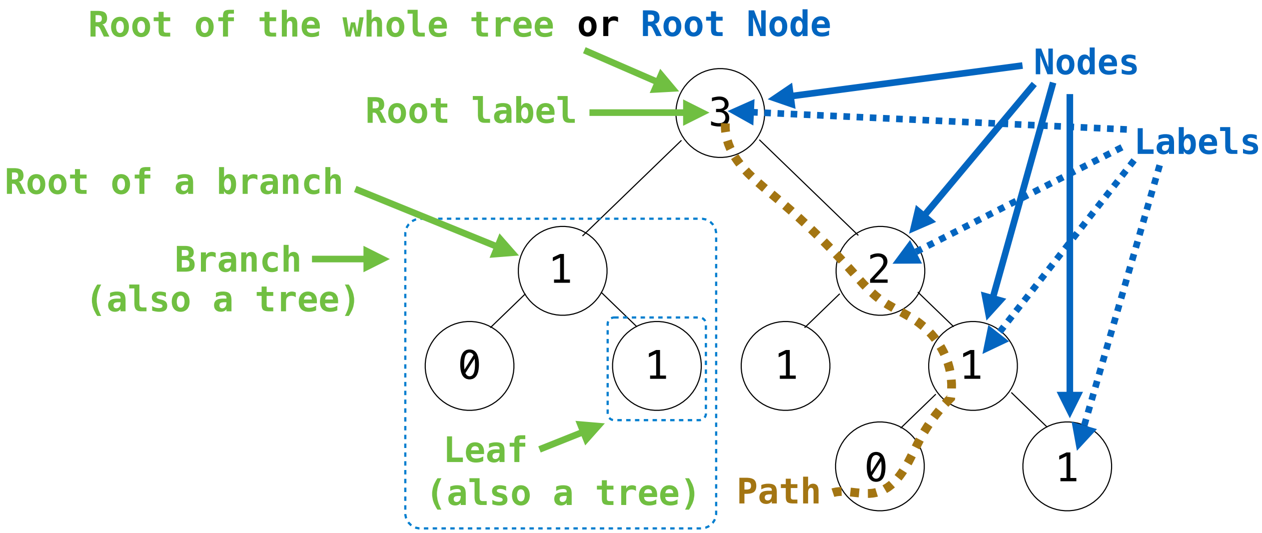

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People often refer to labels by their locations: "each parent is the sum of its children"

Tree Class

`A Tree has a label and a list of branches; each branch is a Tree`

Tree Class

A Tree has a label and a list of branches; each branch is a Tree

```
class Tree:
```

Tree Class

A Tree has a label and a list of branches; each branch is a Tree

```
class Tree:
    def __init__(self, label, branches=[]):
```

Tree Class

A Tree has a label and a list of branches; each branch is a Tree

```
class Tree:
    def __init__(self, label, branches=[]):
        self.label = label
```

Tree Class

A Tree has a label and a list of branches; each branch is a Tree

```
class Tree:
    def __init__(self, label, branches=[]):
        self.label = label
        for branch in branches:
            assert isinstance(branch, Tree)
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Tree Class

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class Tree:
    def __init__(self, label, branches=[]):
        self.label = label
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        self.branches = list(branches)
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    def __init__(self, label, branches=[]):
        self.label = label
        for branch in branches:
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        self.branches = list(branches)
```

```
def tree(label, branches=[]):
    for branch in branches:
        assert is_tree(branch)
    return [label] + list(branches)
def label(tree):
    return tree[0]
def branches(tree):
    return tree[1:]
```

Tree Class

A Tree has a label and a list of branches; each branch is a Tree

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def label(tree):
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```

```
def fib_tree(n):
    if n == 0 or n == 1:
        return Tree(n)
    else:
        left = fib_tree(n-2)
        right = fib_tree(n-1)
        fib_n = left.label + right.label
        return Tree(fib_n, [left, right])
```

Tree Class

A Tree has a label and a list of branches; each branch is a Tree

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class Tree:
    def __init__(self, label, branches=[]):
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    else:
        left = fib_tree(n-2)
        right = fib_tree(n-1)
        fib_n = left.label + right.label
        return Tree(fib_n, [left, right])
```

```
def fib_tree(n):
    if n == 0 or n == 1:
        return tree(n)
    else:
        left = fib_tree(n-2)
        right = fib_tree(n-1)
        fib_n = label(left) + label(right)
        return tree(fib_n, [left, right])
```

Tree Class

A Tree has a label and a list of branches; each branch is a Tree

```
class Tree:
    def __init__(self, label, branches=[]):
        self.label = label
        for branch in branches:
            assert isinstance(branch, Tree)
        self.branches = list(branches)
```

```
def tree(label, branches=[]):
    for branch in branches:
        assert is_tree(branch)
    return [label] + list(branches)
def label(tree):
    return tree[0]
def branches(tree):
    return tree[1:]
```

```
def fib_tree(n):
    if n == 0 or n == 1:
        return Tree(n)
    else:
        left = fib_tree(n-2)
        right = fib_tree(n-1)
        fib_n = left.label + right.label
        return Tree(fib_n, [left, right])
```

```
def fib_tree(n):
    if n == 0 or n == 1:
        return tree(n)
    else:
        left = fib_tree(n-2)
        right = fib_tree(n-1)
        fib_n = label(left) + label(right)
        return tree(fib_n, [left, right])
```

(Demo)

Tree Mutation

Example: Pruning Trees

Removing subtrees from a tree is called *pruning*

Prune branches before recursive processing

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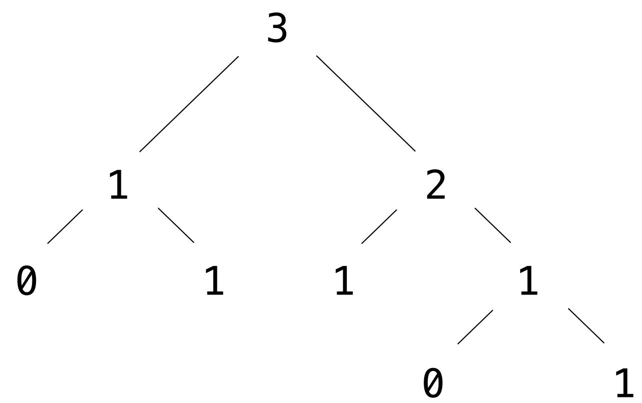
Prune branches before recursive processing



Example: Pruning Trees

Removing subtrees from a tree is called *pruning*

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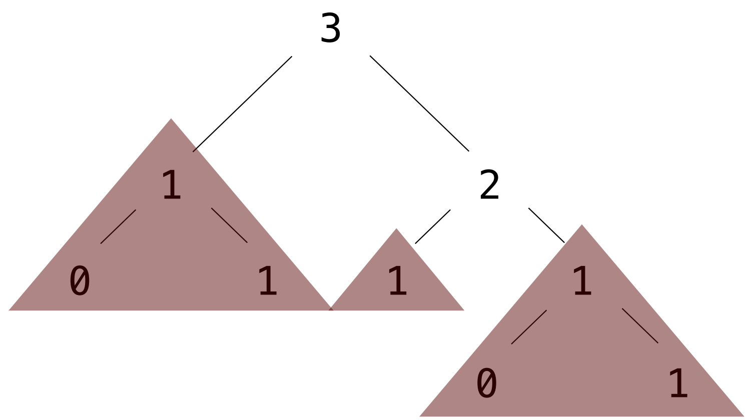

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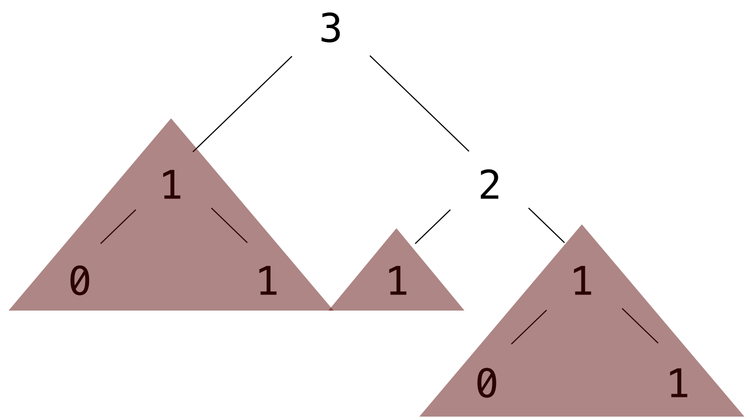
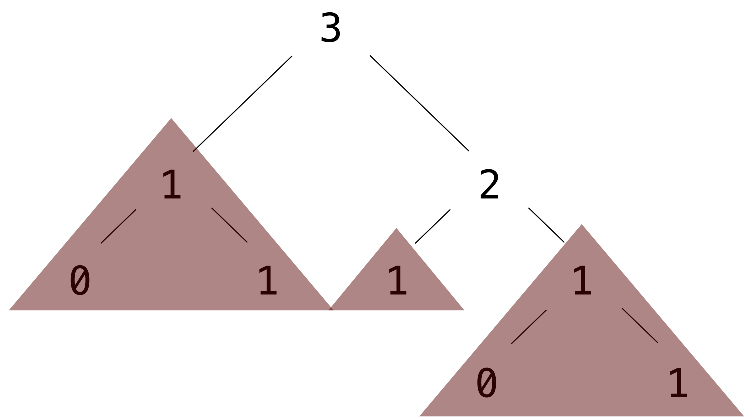



```
def prune(t, n):
    """Prune all sub-trees whose label is n."""
    t.branches = [_______________ for b in t.branches if _______________________]
    for b in t.branches:
        prune(_________________________________, _________________________________)
```

Example: Pruning Trees

Removing subtrees from a tree is called *pruning*

Prune branches before recursive processing

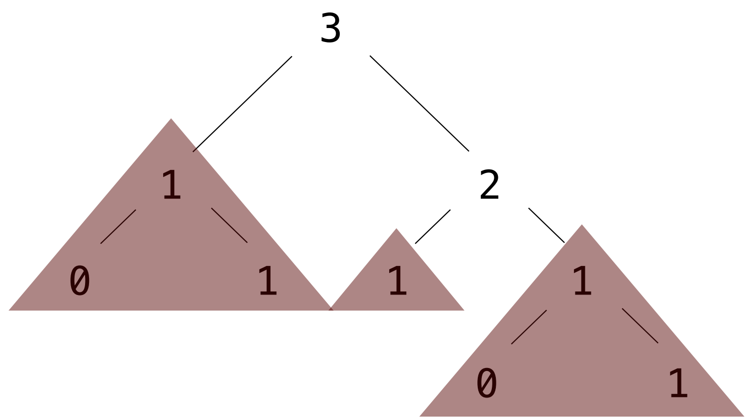



```
def prune(t, n):
    """Prune all sub-trees whose label is n."""
    t.branches = [_____b_____ for b in t.branches if ____b.label != n____]
    for b in t.branches:
        prune(_______________________, _______________________)
```

Example: Pruning Trees

Removing subtrees from a tree is called *pruning*

Prune branches before recursive processing




```
def prune(t, n):
    """Prune all sub-trees whose label is n."""
    t.branches = [____b____ for b in t.branches if ____b.label != n____]
    for b in t.branches:
        prune(____b____, ____n____)
```