Mutable Functions

Announcements

Mutable Functions

A Function with Behavior That Varies Over Time

Let's model a bank account that has a balance of $100

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```
>>> withdraw(25)
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```

Argument:
amount to withdraw

A Function with Behavior That Varies Over Time

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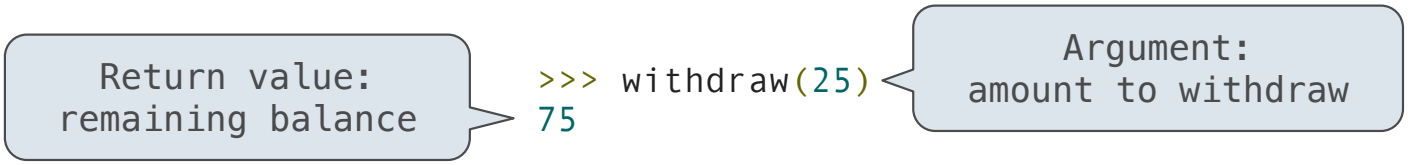

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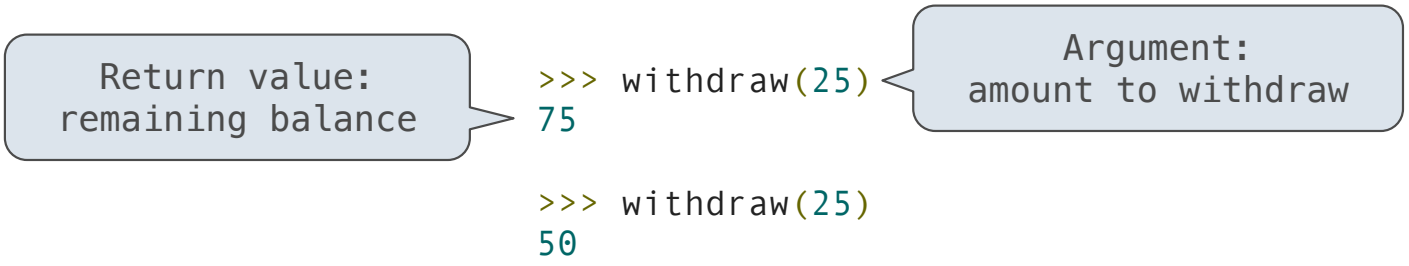

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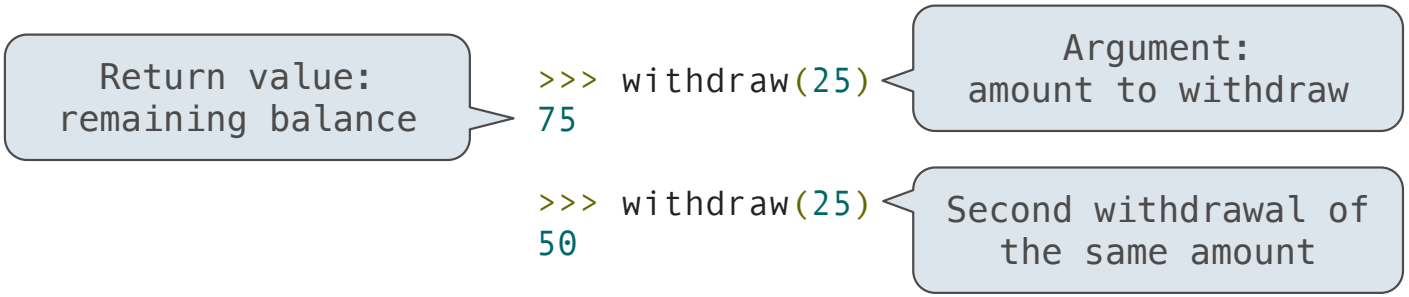

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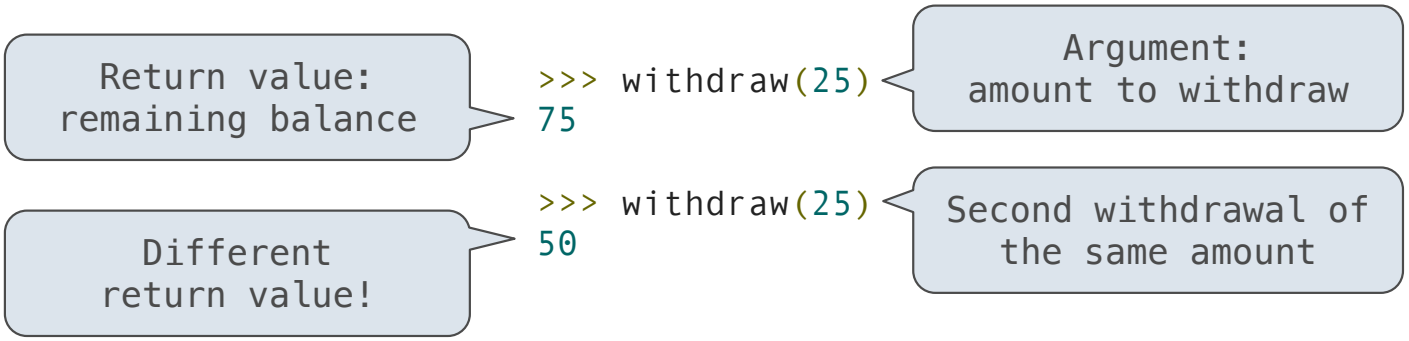

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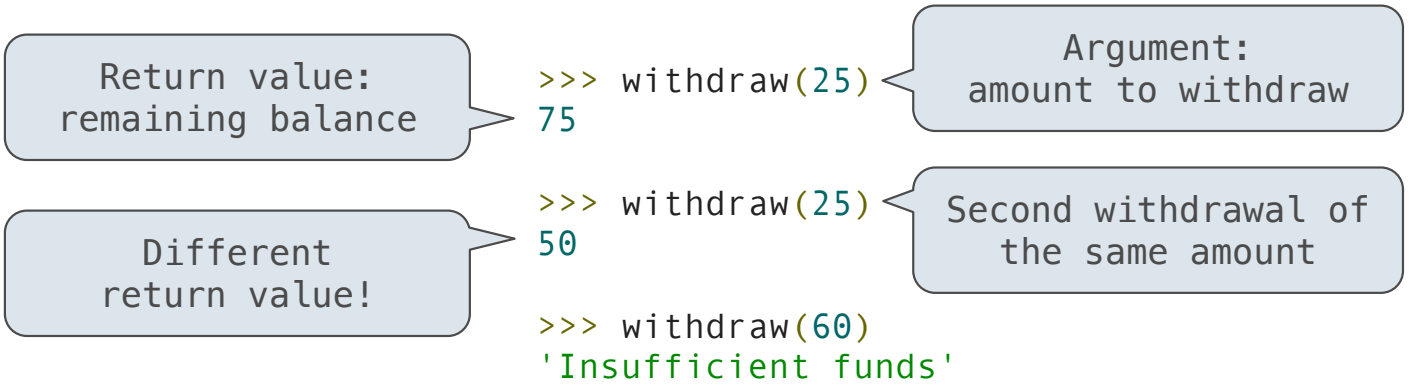

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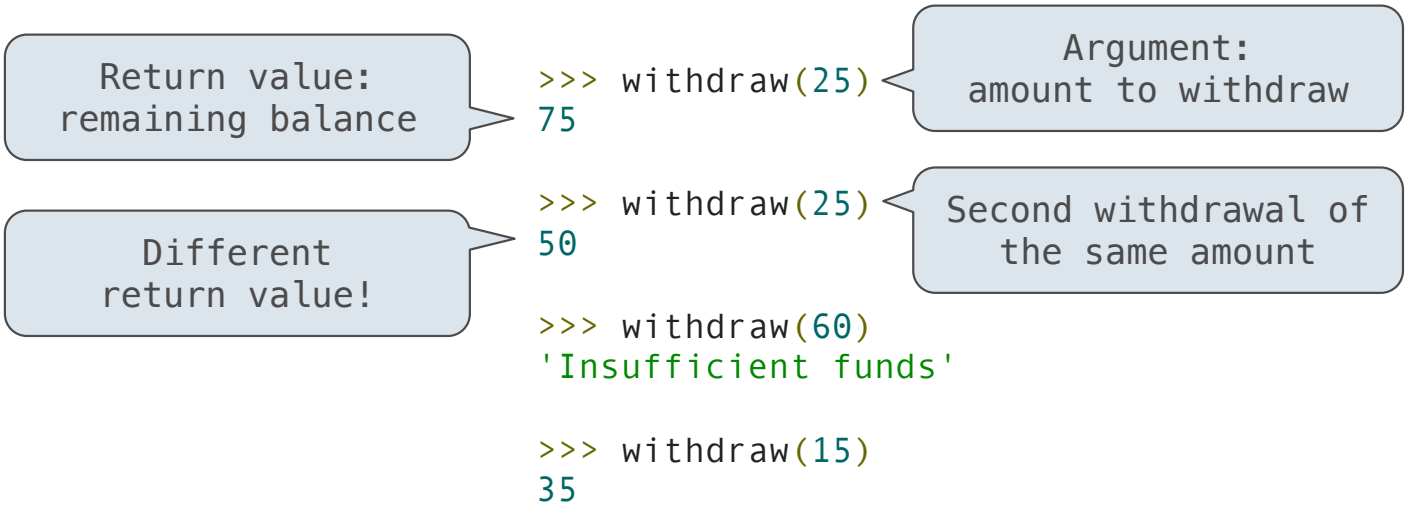


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>>> withdraw(15)
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```

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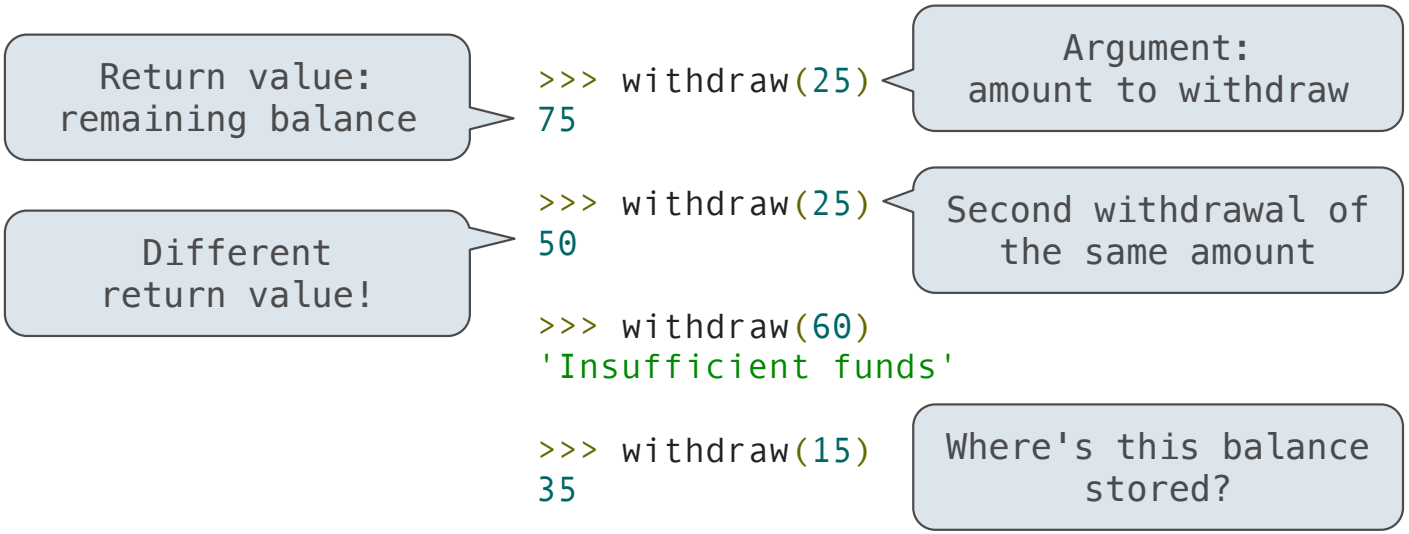

A Function with Behavior That Varies Over Time

Let's model a bank account that has a balance of $100

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>>> withdraw = make_withdraw(100)

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Return value: remaining balance

Argument: amount to withdraw

Different return value!

Second withdrawal of the same amount

Where's this balance stored?

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Within the parent frame of the function!

Argument: amount to withdraw

Return value: remaining balance

Second withdrawal of the same amount

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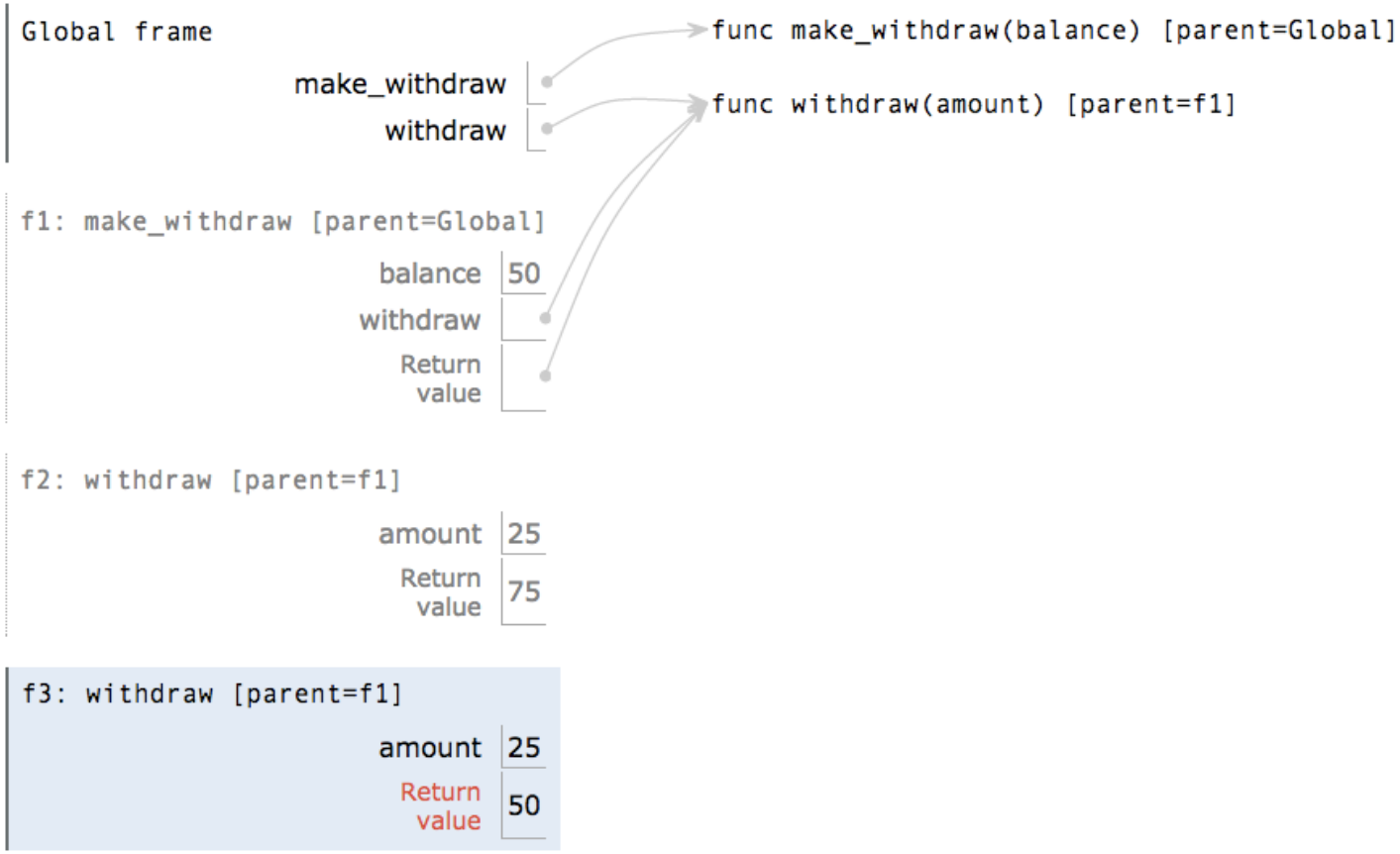
Where's this balance stored?

Persistent Local State Using Environments



pythontutor.com/composingprograms.html#code=def%20make_withdraw%28balance%29%3A%0A%20%20%20%20def%20withdraw%28amount%29%3A%0A%20%20%20%20%20%20%20%20nonlocal%20balance%0A%20%20%20%20%20%20%20%20if%20amount%20%3E%20balance%3A%0A%20%20%20%20%20%20%20%20%20%20%20%20return%20'Insufficient%20funds'%0A%20%20%20%20%20%20%20%20balance%20%3D%20balance%20-%20amount%0A%20%20%20%20%20%20%20%20return%20balance%0A%20%20%20%20return%20withdraw%0A%0Awithdraw%20%3D%20make_withdraw%28100%29%0Awithdraw%2825%29%0Awithdraw%2825%29%0Awithdraw%2860%29%0Awithdraw%2815%29&mode=display&origin=composingprograms.js&cumulative=true&py=3&rawInputLstJSON=[]&curInstr=0

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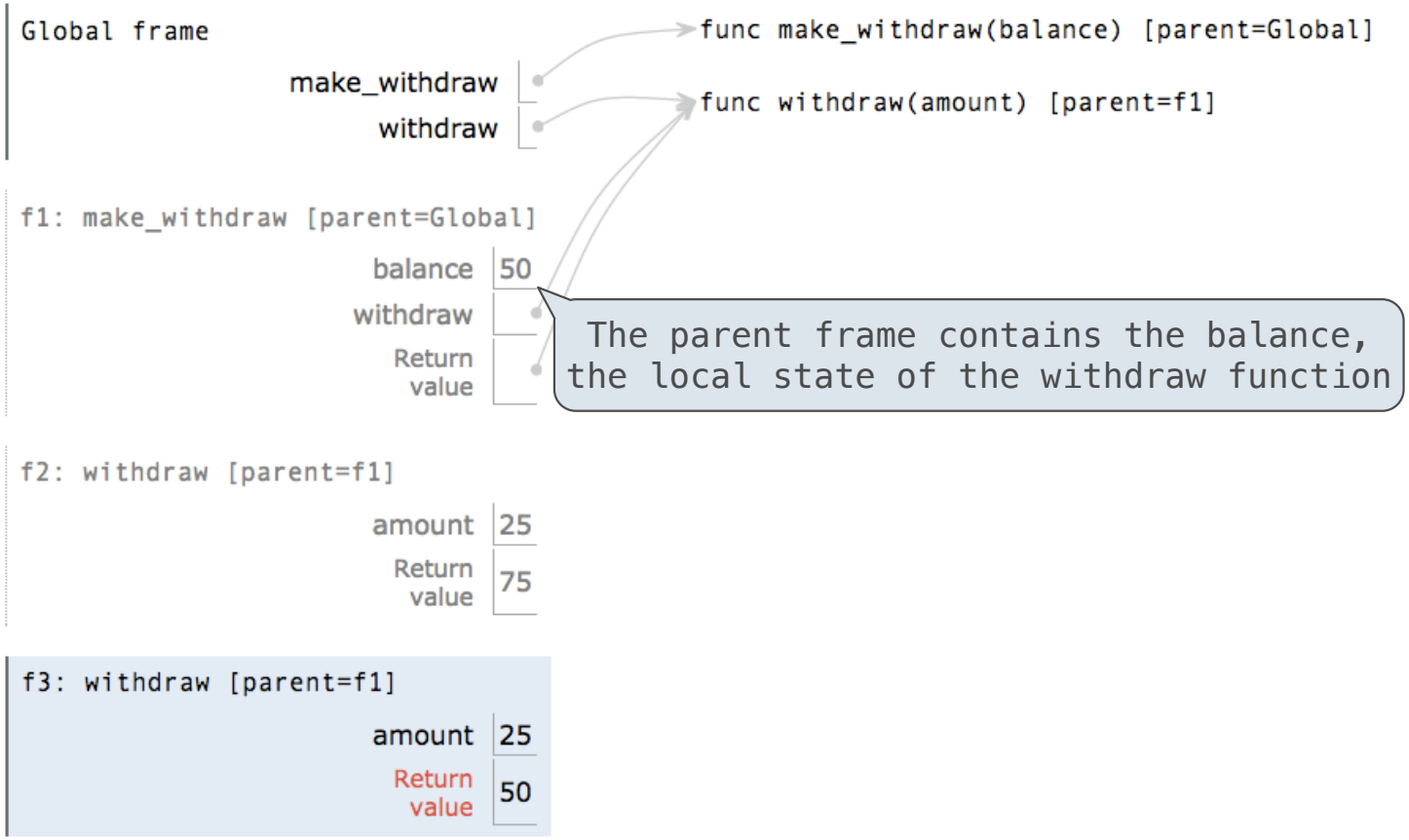

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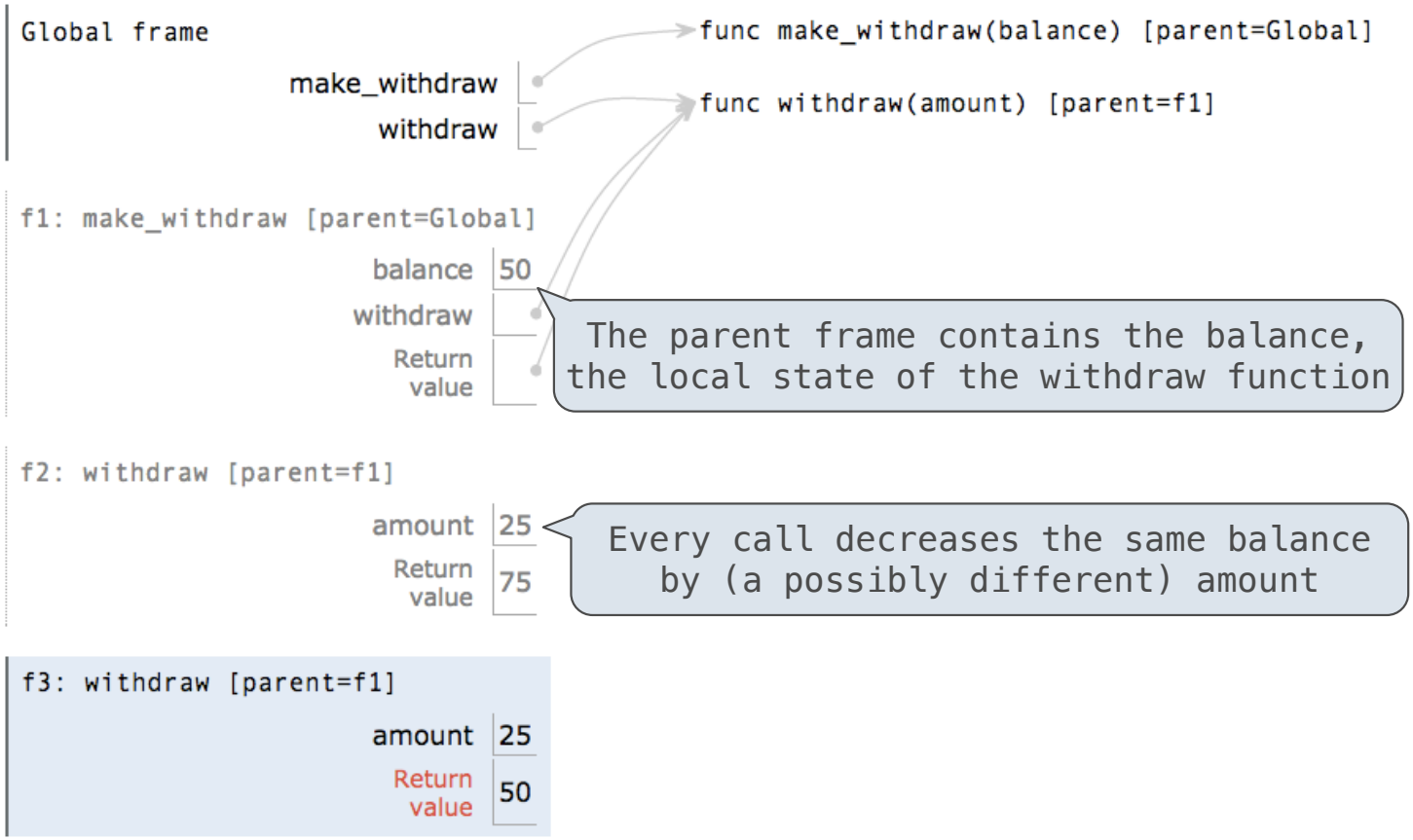

pythontutor.com/composingprograms.html#code=def%20make_withdraw%28balance%29%3A%0A%20%20%20%20def%20withdraw%28amount%29%3A%0A%20%20%20%20%20%20%20%20nonlocal%20balance%0A%20%20%20%20%20%20%20%20if%20amount%20>%20balance%3A%0A%20%20%20%20%20%20%20%20%20%20%20%20return%20'Insufficient%20funds'%0A%20%20%20%20%20%20%20%20balance%20%3D%20balance%20-%20amount%0A%20%20%20%20%20%20%20%20return%20balance%0A%20%20%20%20return%20withdraw%0A%0Awithdraw%20%3D%20make_withdraw%28100%29%0Awithdraw%2825%29%0Awithdraw%2825%29%0Awithdraw%2860%29%0Awithdraw%2815%29&mode=display&origin=composingprograms.js&cumulative=true&py=3&rawInputLstJSON=[]&curInstr=0

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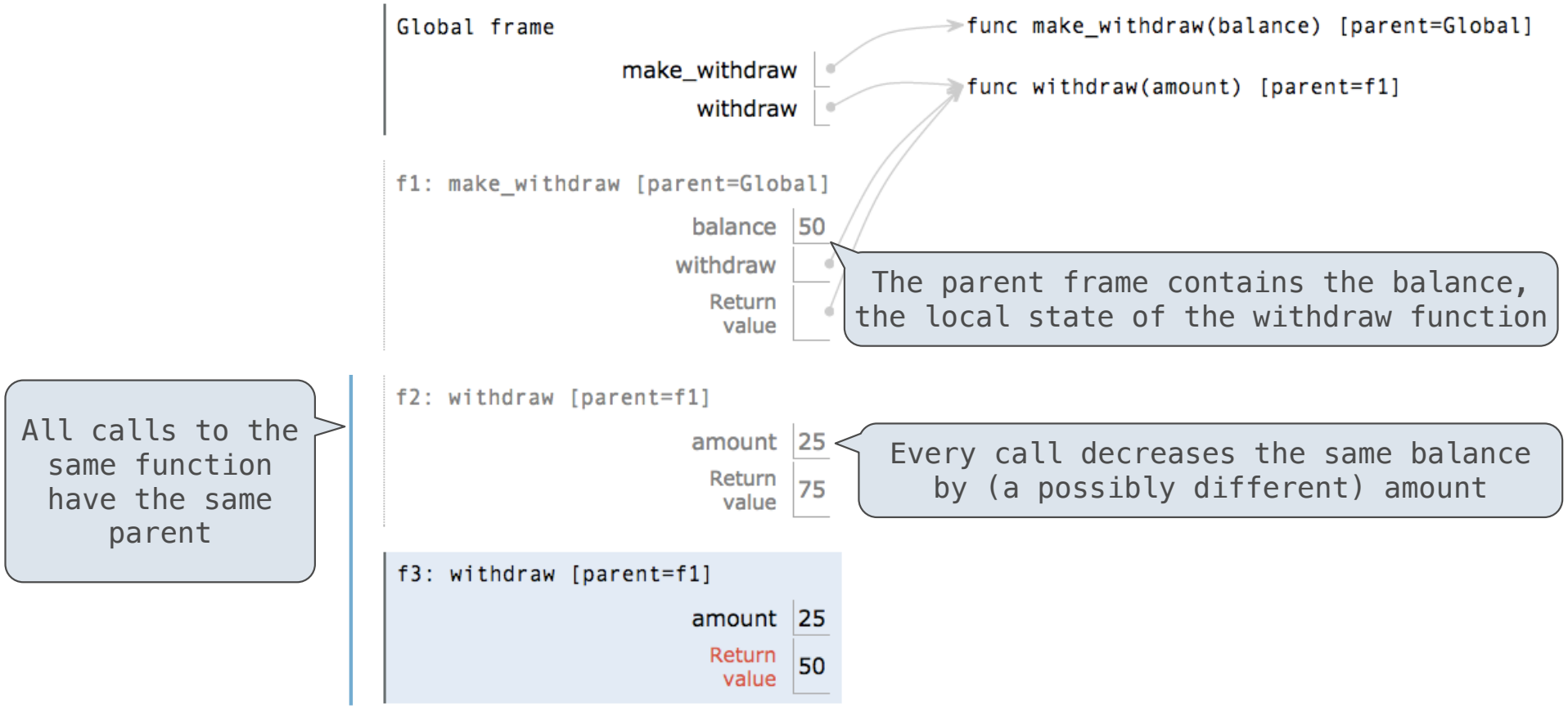

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Reminder: Local Assignment

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def percent_difference(x, y):
        difference = abs(x-y)
        return 100 * difference / x
diff = percent_difference(40, 50)
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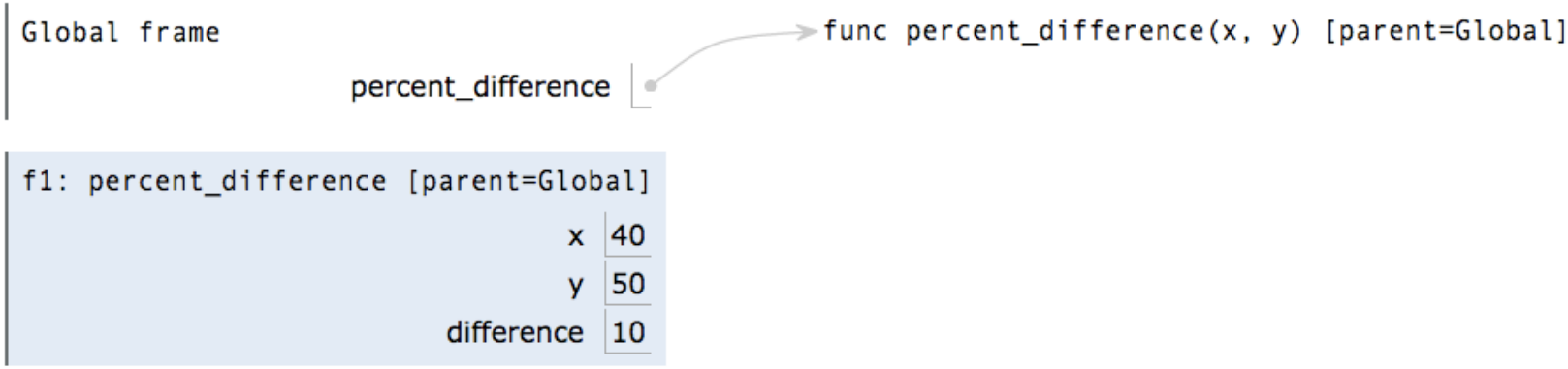

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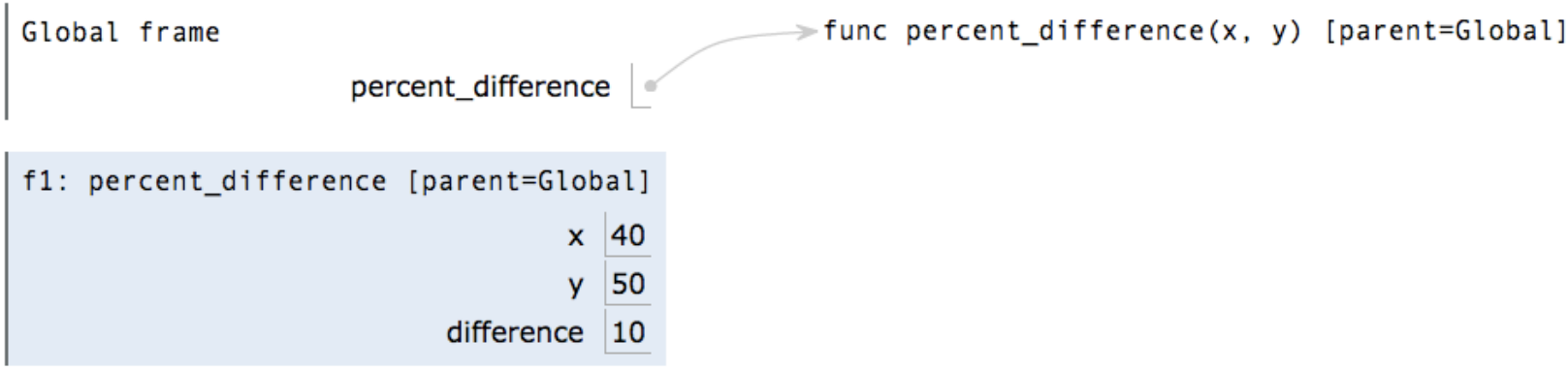
Assignment binds name(s) to value(s) in the first frame of the current environment



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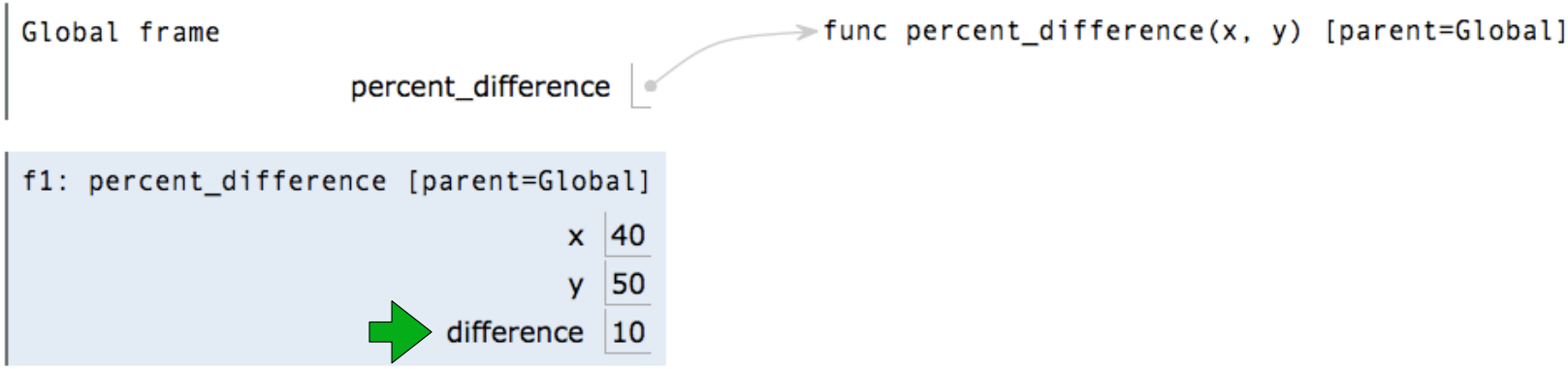

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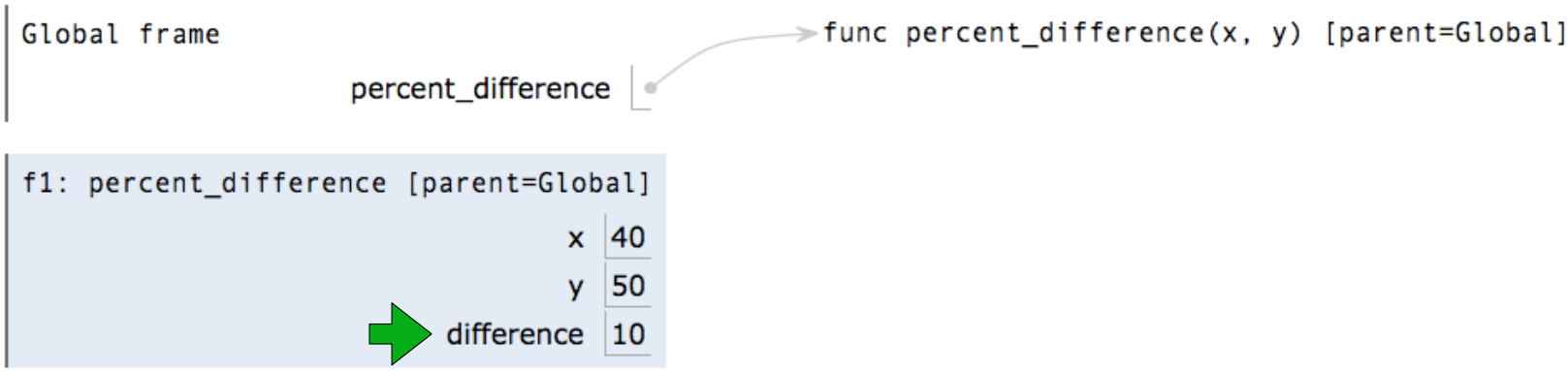

Execution rule for assignment statements:

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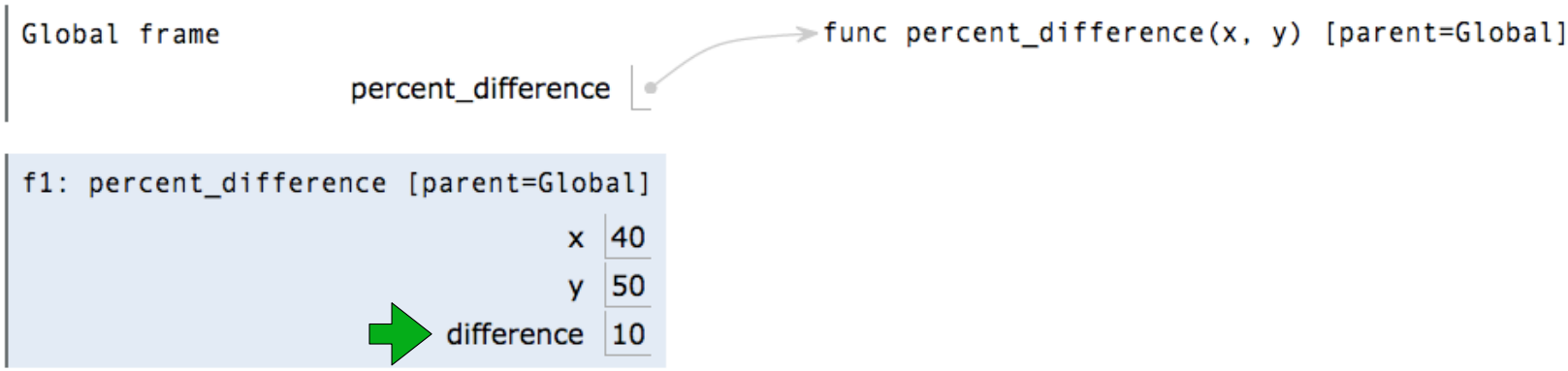

Execution rule for assignment statements:

1. Evaluate all expressions right of =, from left to right

2. Bind the names on the left to the resulting values in the **current frame**

Non-Local Assignment & Persistent Local State

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Non-Local Assignment & Persistent Local State

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        if amount > balance:
```

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(Demo)

Non-Local Assignment

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http://www.python.org/dev/peps/pep-3104/

The Many Meanings of Assignment Statements

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Python Particulars

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def make_withdraw(balance):
    def withdraw(amount):
        if amount > balance:
            return 'Insufficient funds'
        balance = balance - amount
        return balance
    return withdraw

wd = make_withdraw(20)
wd(5)
```

pythontutor.com/composingprograms.html#code=def%20make_withdraw%28balance%29%3A%0A%20%20%20%20def%20withdraw%28amount%29%3A%0A%20%20%20%20%20%20%20%20if%20amount%20%3E%20balance%3A%0A%20%20%20%20%20%20%20%20%20%20%20%20return%20%27Insufficient%20funds%27%0A%20%20%20%20%20%20%20%20balance%20%3D%20balance%20-%20amount%0A%20%20%20%20%20%20%20%20return%20balance%0A%20%20%20%20return%20withdraw%0A%0Awithdraw%20%3D%20make_withdraw%2820%29%0Awithdraw%285%29&mode=display&origin=composingprograms.js&cumulative=true&py=3&rawInputLstJSON=[]&curInstr=0

Python Particulars

Python pre-computes which frame contains each name before executing the body of a function.

Within the body of a function, all instances of a name must refer to the same frame.

```
def make_withdraw(balance):
    def withdraw(amount):
        if amount > balance:
            return 'Insufficient funds'
        balance = balance - amount
        return balance
    return withdraw

wd = make_withdraw(20)
wd(5)
```

Local assignment

pythontutor.com/composingprograms.html#code=def%20make_withdraw%28balance%29%3A%0A%20%20%20%20def%20withdraw%28amount%29%3A%0A%20%20%20%20%20%20%20%20if%20amount%20%3E%20balance%3A%0A%20%20%20%20%20%20%20%20%20%20%20%20return%20%27Insufficient%20funds%27%0A%20%20%20%20%20%20%20%20balance%20%3D%20balance%20-%20amount%0A%20%20%20%20%20%20%20%20return%20balance%0A%20%20%20%20return%20withdraw%0A%0Awithdraw%20%3D%20make_withdraw%2820%29%0Awithdraw%285%29&mode=display&origin=composingprograms.js&cumulative=true&py=3&rawInputLstJSON=[]&curInstr=0

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Local assignment

UnboundLocalError: local variable 'balance' referenced before assignment

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Mutable Values & Persistent Local State

Mutable values can be changed *without* a nonlocal statement.

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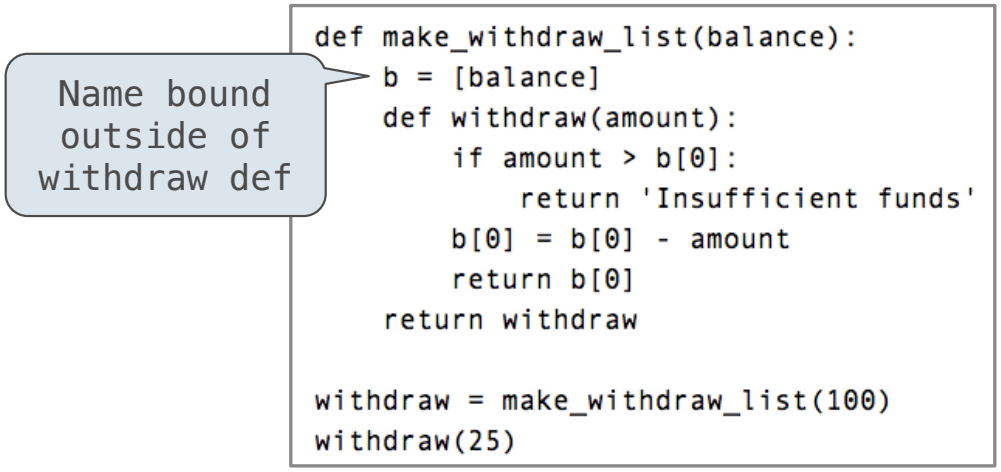
```
def make_withdraw_list(balance):
    b = [balance]
    def withdraw(amount):
        if amount > b[0]:
            return 'Insufficient funds'
        b[0] = b[0] - amount
        return b[0]
    return withdraw

withdraw = make_withdraw_list(100)
withdraw(25)
```

Mutable Values & Persistent Local State

Mutable values can be changed *without* a nonlocal statement.



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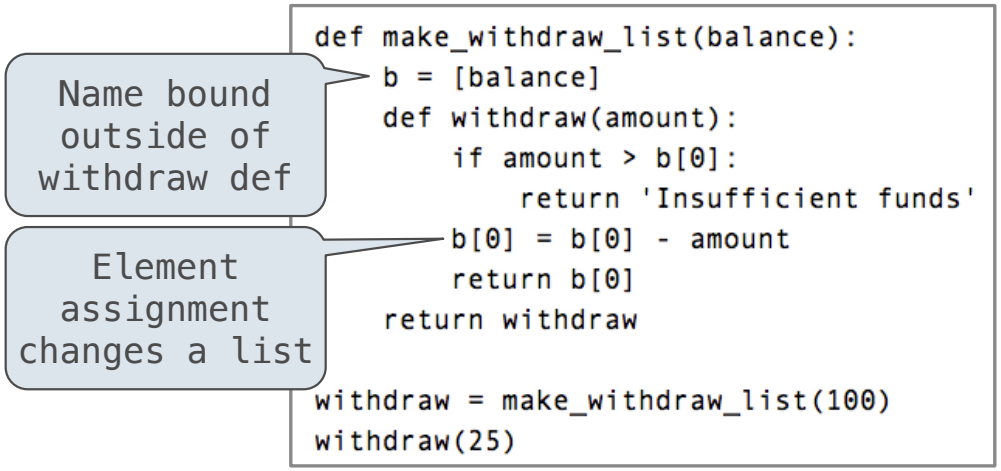

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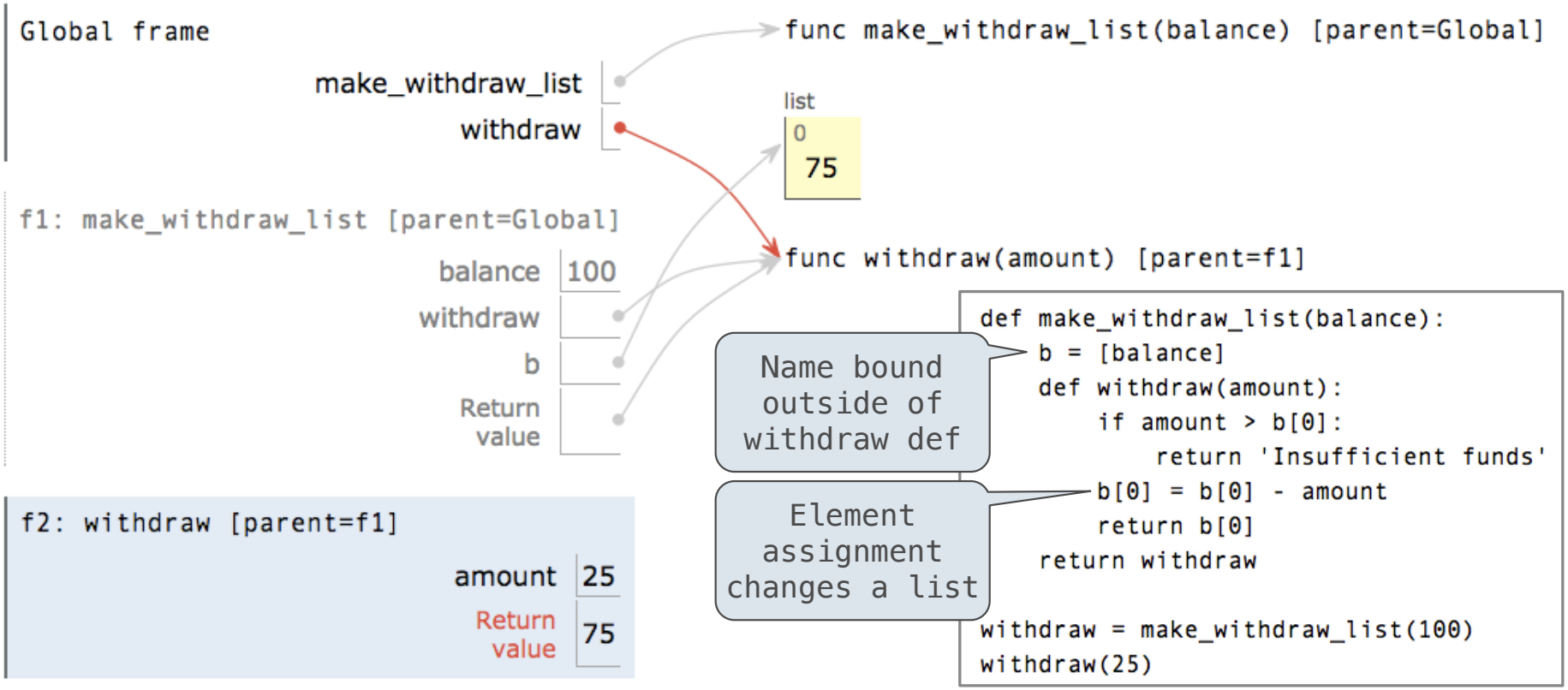

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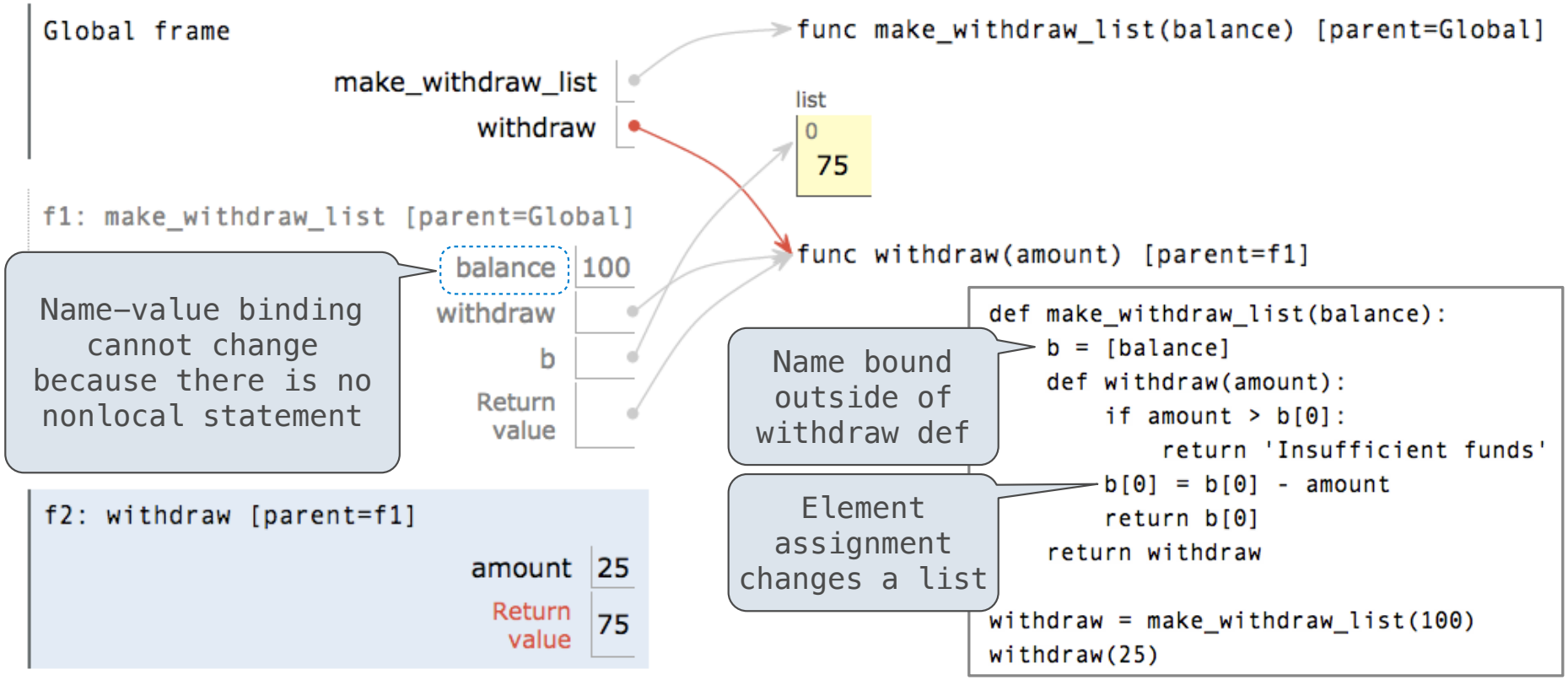

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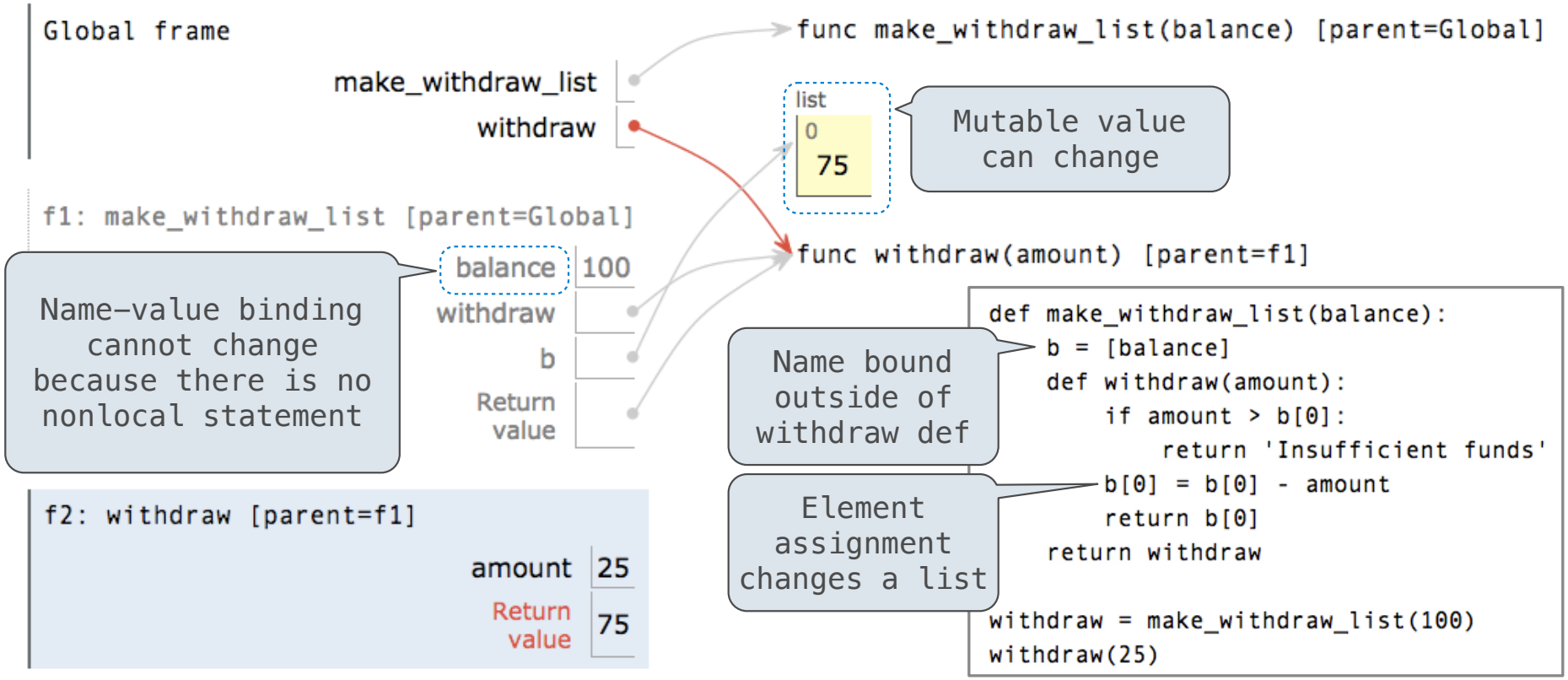

Multiple Mutable Functions

(Demo)

Referential Transparency, Lost

pythontutor.com/composingprograms.html#code=def%20f%28x%29%3A%0A%20%20%20%20x%20%3D%204%0A%20%20%20%20def%20g%28y%29%3A%0A%20%20%20%20%20%20%20%20def%20h%28z%29%3A%0A%20%20%20%20%20%20%20%20%20%20%20%20nonlocal%20x%0A%20%20%20%20%20%20%20%20%20%20%20%20x%20%3D%20x%20%2B%201%0A%20%20%20%20%20%20%20%20%20%20%20%20return%20x%20%2B%20y%20%2B%20z%0A%20%20%20%20%20%20%20%20return%20h%0A%20%20%20%20return%20g%0Aa%20%3D%20f%281%29%0Ab%20%3D%20a%282%29%0Atotal%20%3D%20b%283%29%20%2B%20b%284%29&mode=display&origin=composingprograms.js&cumulative=true&py=3&rawInputLstJSON=[]&curInstr=0

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mul(add(2, mul(4, 6)), add(3, 5))
```


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Review Problem