Optimization (Karnaugh maps)

a	b	c	d	e
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	1
0	1	1	1	0
1	0	0	0	1
1	0	0	1	1
1	0	1	0	1
1	0	1	1	1
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	0

Optimization (Karnaugh maps)

a	b	c	d	e
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	1
0	1	1	1	0
1	0	0	0	1
1	0	0	1	1
1	0	1	0	1
1	0	1	1	1
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	0

Optimization (Karnaugh maps)

a	b	c	d	e
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	1
0	1	1	1	0
1	0	0	0	1
1	0	0	1	1
1	0	1	0	1
1	0	1	1	1
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	0

$a' \cdot b \cdot c \cdot d'$

Optimization (Karnaugh maps)

a	b	c	d	e
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	1
0	1	1	1	0
1	0	0	0	1
1	0	0	1	1
1	0	1	0	1
1	0	1	1	1
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	0

$a' \cdot b \cdot c \cdot d'$

$a \cdot b' \cdot c' \cdot d'$

Optimization (Karnaugh maps)

a	b	c	d	e	
0	0	0	0	0	
0	0	0	1	0	
0	0	1	0	0	
0	0	1	1	0	
0	1	0	0	0	
0	1	0	1	0	
0	1	1	0	1	a’ · b · c · d’
0	1	1	1	0	
1	0	0	0	1	a · b’ · c’ · d’
1	0	0	1	1	a · b’ · c’ · d
1	0	1	0	1	
1	0	1	1	1	
1	1	0	0	1	
1	1	0	1	1	
1	1	1	0	1	
1	1	1	1	0	

Optimization (Karnaugh maps)

a	b	c	d	e	
0	0	0	0	0	
0	0	0	1	0	
0	0	1	0	0	
0	0	1	1	0	
0	1	0	0	0	
0	1	0	1	0	
0	1	1	0	1	a’ · b · c · d’
0	1	1	1	0	
1	0	0	0	1	a · b’ · c’ · d’
1	0	0	1	1	a · b’ · c’ · d
1	0	1	0	1	a · b’ · c · d’
1	0	1	1	1	
1	1	0	0	1	
1	1	0	1	1	
1	1	1	0	1	
1	1	1	1	0	

Optimization (Karnaugh maps)

a	b	c	d	e	
0	0	0	0	0	
0	0	0	1	0	
0	0	1	0	0	
0	0	1	1	0	
0	1	0	0	0	
0	1	0	1	0	
0	1	1	0	1	a’ · b · c · d’
0	1	1	1	0	
1	0	0	0	1	a · b’ · c’ · d’
1	0	0	1	1	a · b’ · c’ · d
1	0	1	0	1	a · b’ · c · d’
1	0	1	1	1	a · b’ · c · d
1	1	0	0	1	a · b · c’ · d’
1	1	0	1	1	a · b · c’ · d
1	1	1	0	1	a · b · c · d’
1	1	1	1	0	

Optimization (Karnaugh maps)

a	b	c	d	e	
0	0	0	0	0	
0	0	0	1	0	
0	0	1	0	0	
0	0	1	1	0	
0	1	0	0	0	
0	1	0	1	0	
0	1	1	0	1	a’ · b · c · d’
0	1	1	1	0	
1	0	0	0	1	a · b’ · c’ · d’
1	0	0	1	1	a · b’ · c’ · d
1	0	1	0	1	a · b’ · c · d’
1	0	1	1	1	a · b’ · c · d
1	1	0	0	1	a · b · c’ · d’
1	1	0	1	1	a · b · c’ · d
1	1	1	0	1	a · b · c · d’
1	1	1	1	0	

$\}$ or

Optimization (Karnaugh maps)

a	b	c	d	e
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	1
0	1	1	1	0
1	0	0	0	1
1	0	0	1	1
1	0	1	0	1
1	0	1	1	1
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	0

ab

cd

	00	01	11	10
00				
01				
11				
10				

Optimization (Karnaugh maps)

a	b	c	d	e
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	1
0	1	1	1	0
1	0	0	0	1
1	0	0	1	1
1	0	1	0	1
1	0	1	1	1
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	0

ab

cd

	00	01	11	10
00	0			
01				
11				
10				

Optimization (Karnaugh maps)

a	b	c	d	e
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	1
0	1	1	1	0
1	0	0	0	1
1	0	0	1	1
1	0	1	0	1
1	0	1	1	1
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	0

ab

cd

	00	01	11	10
00	0			1
01				
11				
10				

Optimization (Karnaugh maps)

a	b	c	d	e
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	1
0	1	1	1	0
1	0	0	0	1
1	0	0	1	1
1	0	1	0	1
1	0	1	1	1
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	0

ab

cd

	00	01	11	10
00	0			1
01				
11			0	
10				

Optimization (Karnaugh maps)

a	b	c	d	e
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	1
0	1	1	1	0
1	0	0	0	1
1	0	0	1	1
1	0	1	0	1
1	0	1	1	1
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	0

ab

cd

	00	01	11	10
00	0			1
01				
11			0	
10		1		

Optimization (Karnaugh maps)

a	b	c	d	e
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	1
0	1	1	1	0
1	0	0	0	1
1	0	0	1	1
1	0	1	0	1
1	0	1	1	1
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	0

ab

cd

	00	01	11	10
00	0	0	1	1
01	0	0	1	1
11	0	0	0	1
10	0	1	1	1

Optimization (Karnaugh maps)

a	b	c	d	e
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	1
0	1	1	1	0
1	0	0	0	1
1	0	0	1	1
1	0	1	0	1
1	0	1	1	1
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	0

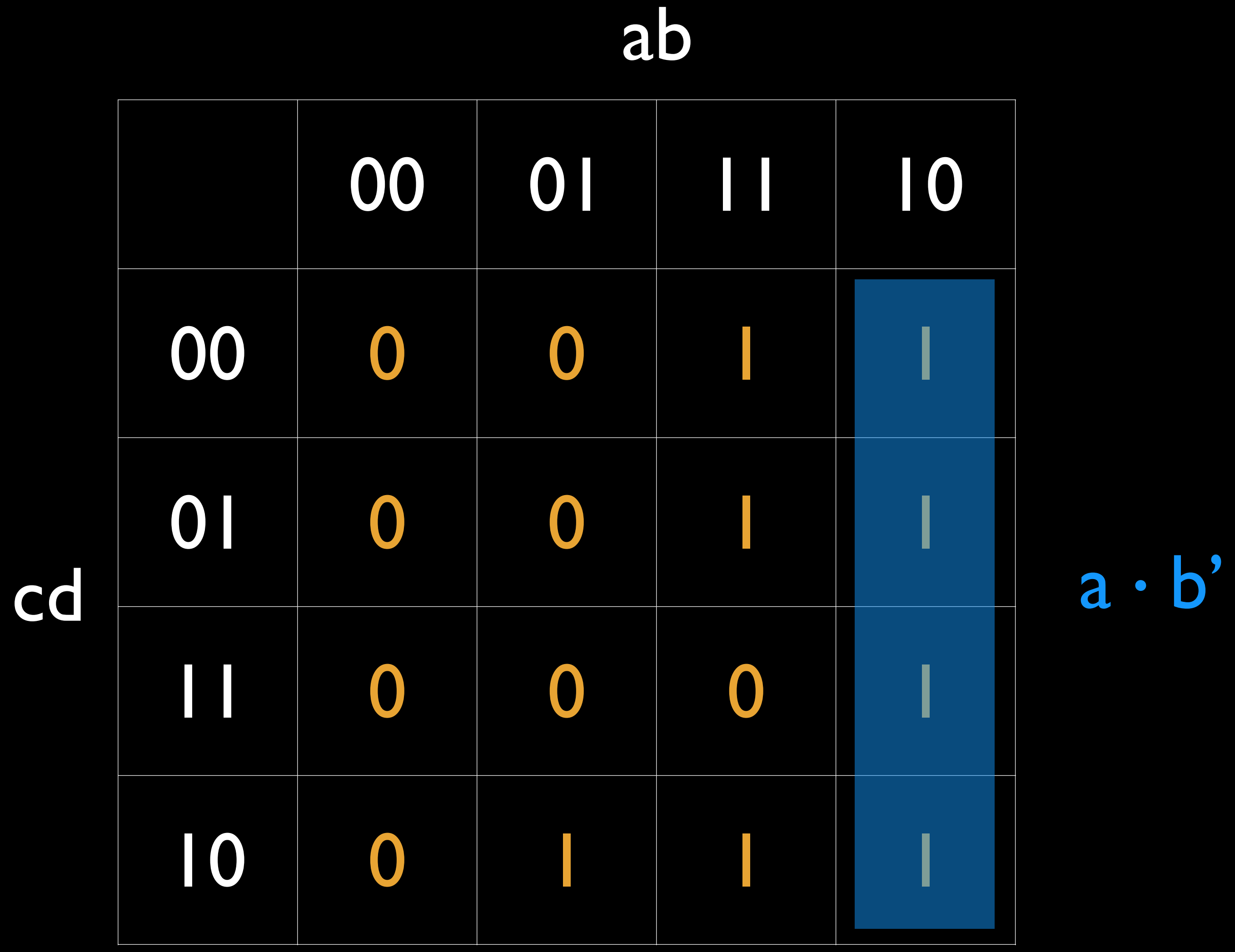
ab

cd

	00	01	11	10
00	0	0	1	1
01	0	0	1	1
11	0	0	0	1
10	0	1	1	1

Optimization (Karnaugh maps)

a	b	c	d	e
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	1
0	1	1	1	0
1	0	0	0	1
1	0	0	1	1
1	0	1	0	1
1	0	1	1	1
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	0



ab

cd	00	01	11	10
00	0	0	1	1
01	0	0	1	1
11	0	0	0	1
10	0	1	1	1

$a \cdot b'$

Optimization (Karnaugh maps)

a	b	c	d	e
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	1
0	1	1	1	0
1	0	0	0	1
1	0	0	1	1
1	0	1	0	1
1	0	1	1	1
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	0

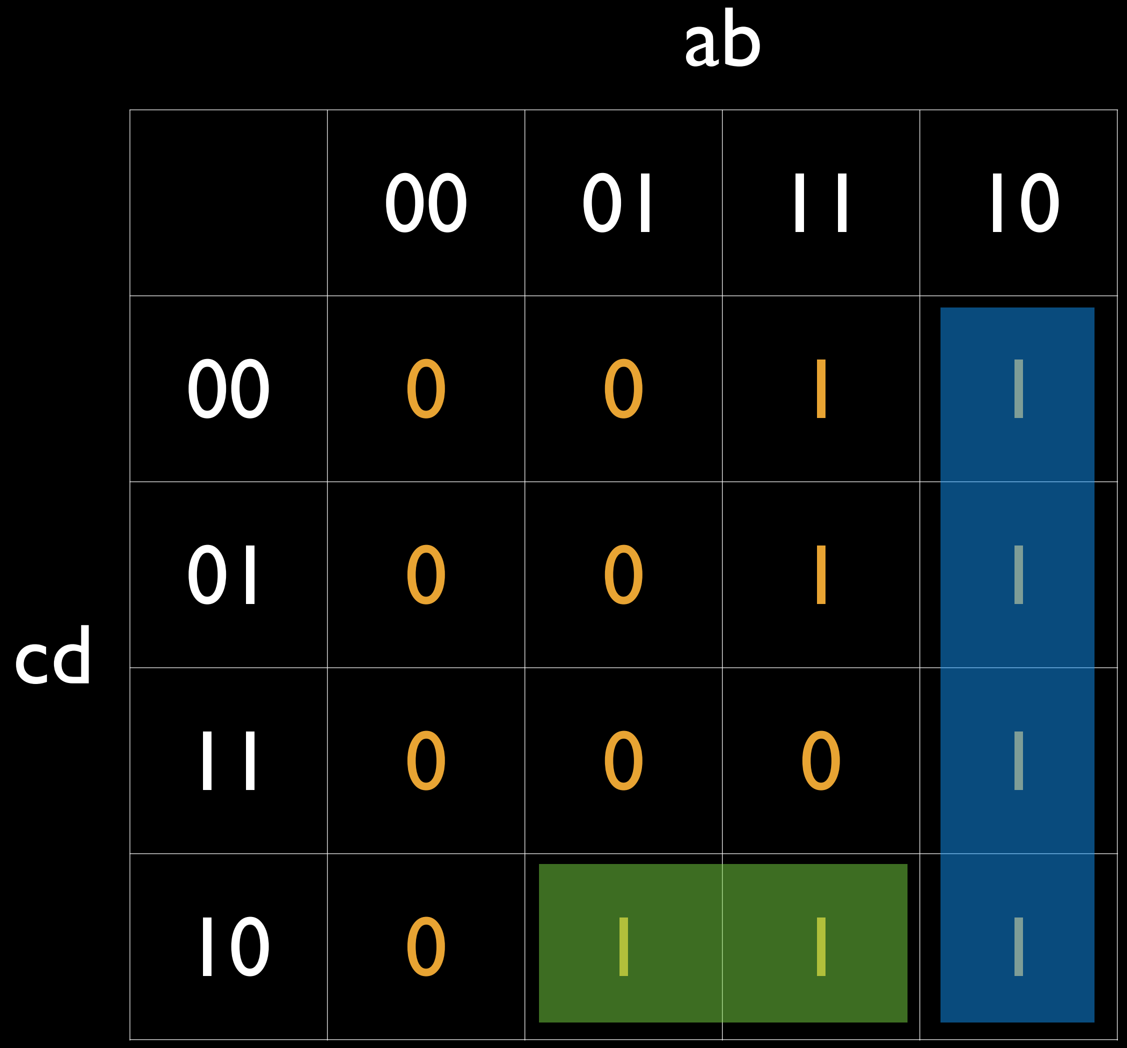
ab

cd

	00	01	11	10
00	0	0	1	1
01	0	0	1	1
11	0	0	0	1
10	0	1	1	1

Optimization (Karnaugh maps)

a	b	c	d	e
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	1
0	1	1	1	0
1	0	0	0	1
1	0	0	1	1
1	0	1	0	1
1	0	1	1	1
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	0



ab

cd

	00	01	11	10
00	0	0	1	1
01	0	0	1	1
11	0	0	0	1
10	0	1	1	1

b · c · d’

Optimization (Karnaugh maps)

a	b	c	d	e
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	1
0	1	1	1	0
1	0	0	0	1
1	0	0	1	1
1	0	1	0	1
1	0	1	1	1
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	0

ab

cd

	00	01	11	10
00	0	0	1	1
01	0	0	1	1
11	0	0	0	1
10	0	1	1	1

Optimization (Karnaugh maps)

a	b	c	d	e
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	1
0	1	1	1	0
1	0	0	0	1
1	0	0	1	1
1	0	1	0	1
1	0	1	1	1
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	0

ab

cd

	00	01	11	10
00	0	0	1	1
01	0	0	1	1
11	0	0	0	1
10	0	1	1	1

a · c’

Optimization (Karnaugh maps)

a	b	c	d	e
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	1
0	1	1	1	0
1	0	0	0	1
1	0	0	1	1
1	0	1	0	1
1	0	1	1	1
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	0

cd \ ab	00	01	11	10
00	0	0	1	1
01	0	0	1	1
11	0	0	0	1
10	0	1	1	1

$$e = a \cdot b' + b \cdot c \cdot d' + a \cdot c'$$

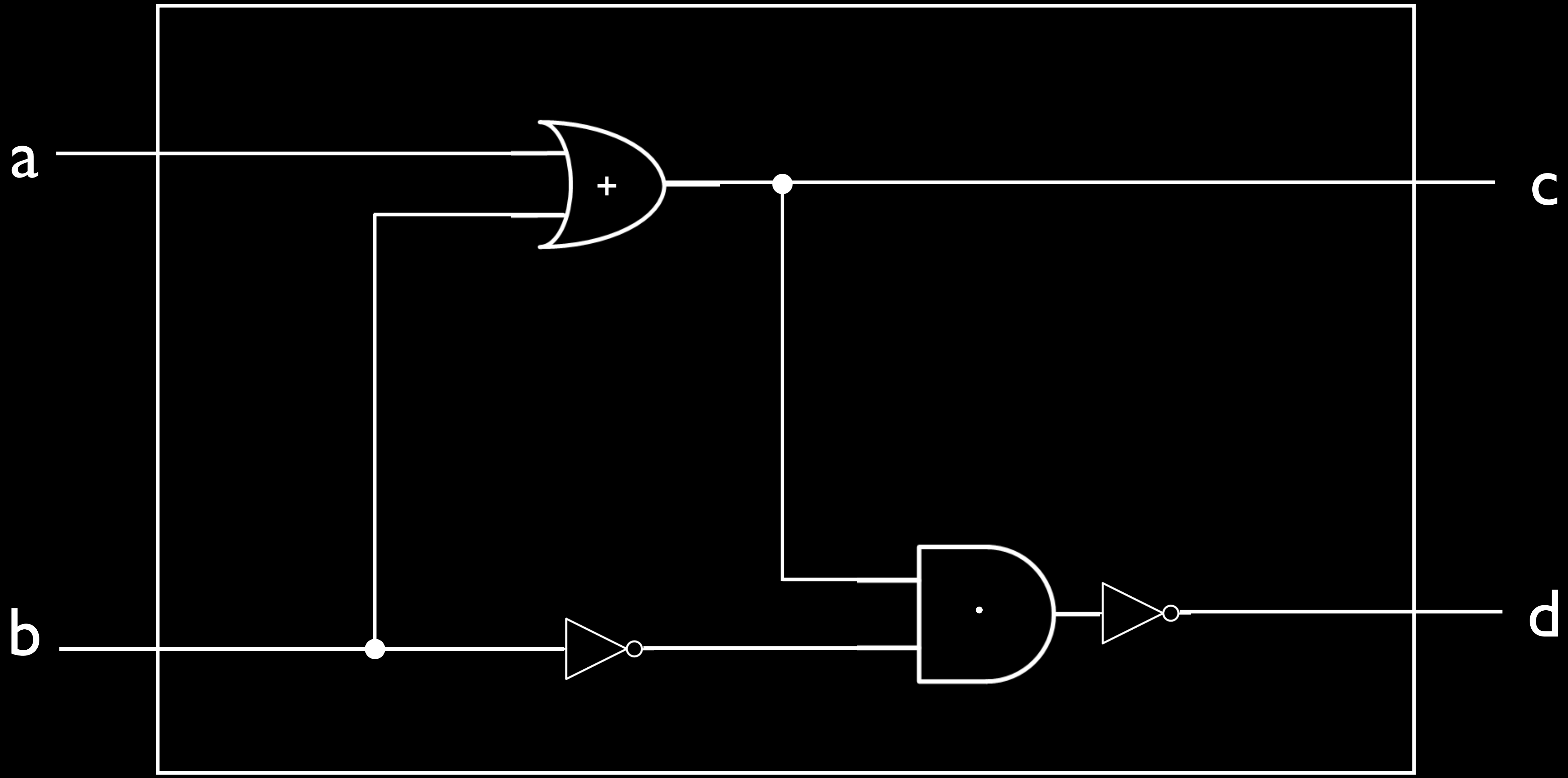
Optimization (Karnaugh maps)

a	b	c	d	e	
0	0	0	0	0	
0	0	0	1	0	
0	0	1	0	0	
0	0	1	1	0	
0	1	0	0	0	
0	1	0	1	0	
0	1	1	0	1	a’ · b · c · d’
0	1	1	1	0	
1	0	0	0	1	a · b’ · c’ · d’
1	0	0	1	1	a · b’ · c’ · d
1	0	1	0	1	a · b’ · c · d’
1	0	1	1	1	a · b’ · c · d
1	1	0	0	1	a · b · c’ · d’
1	1	0	1	1	a · b · c’ · d
1	1	1	0	1	a · b · c · d’
1	1	1	1	0	

Circuits

A circuit is a collection of logical gates that transforms a set of binary inputs into a set of binary outputs.

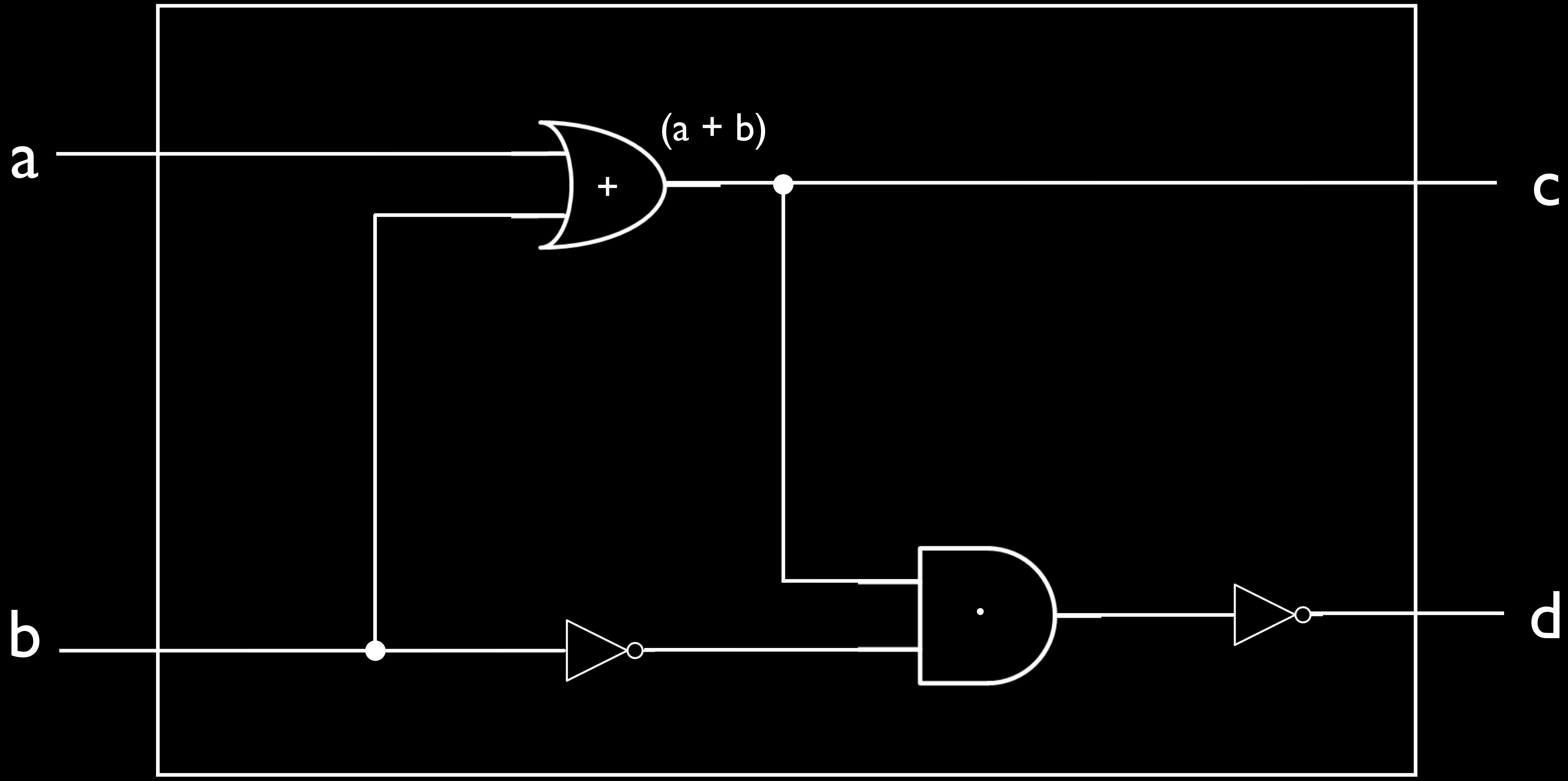
Circuits



input

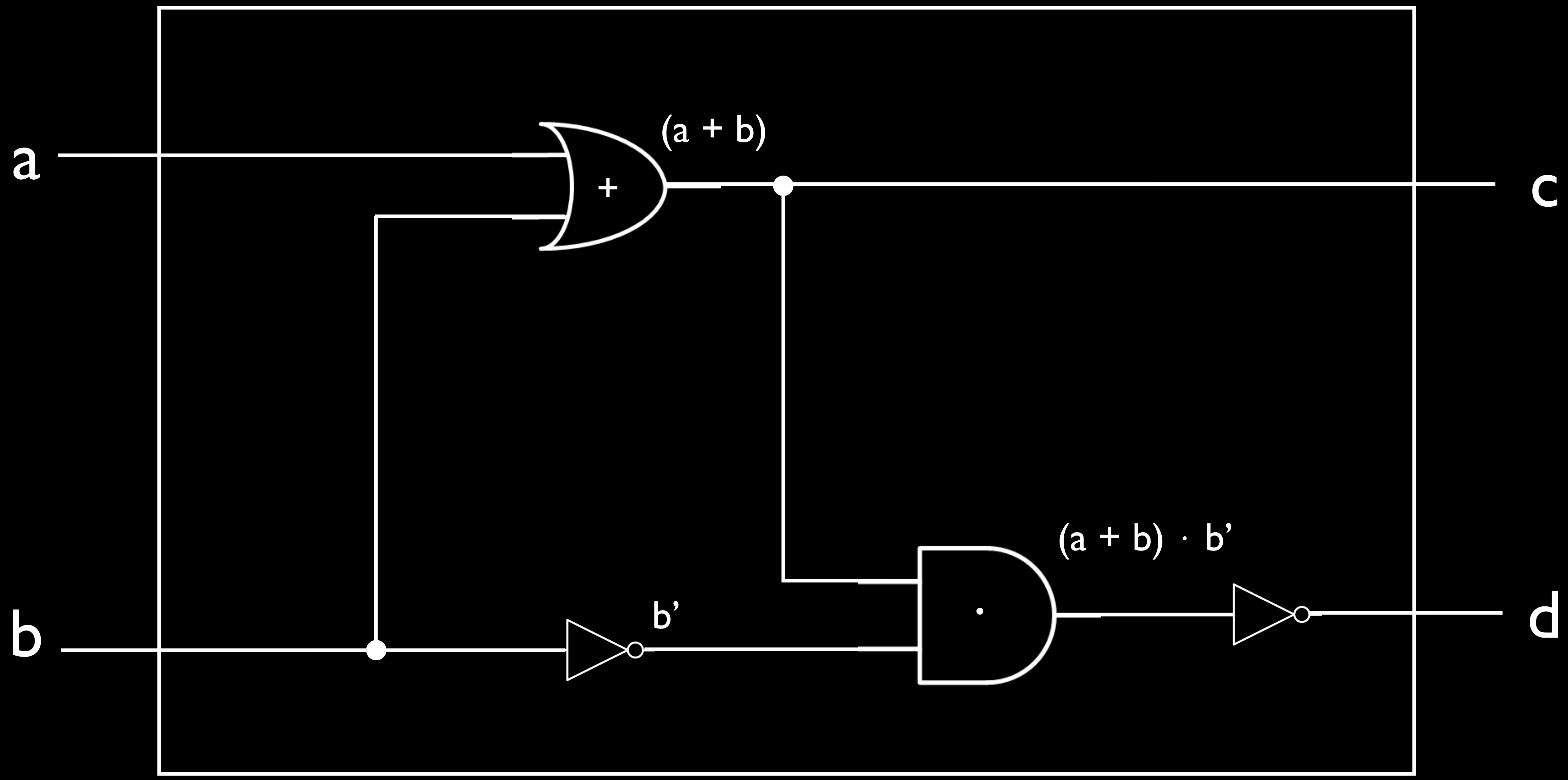
output

Circuits

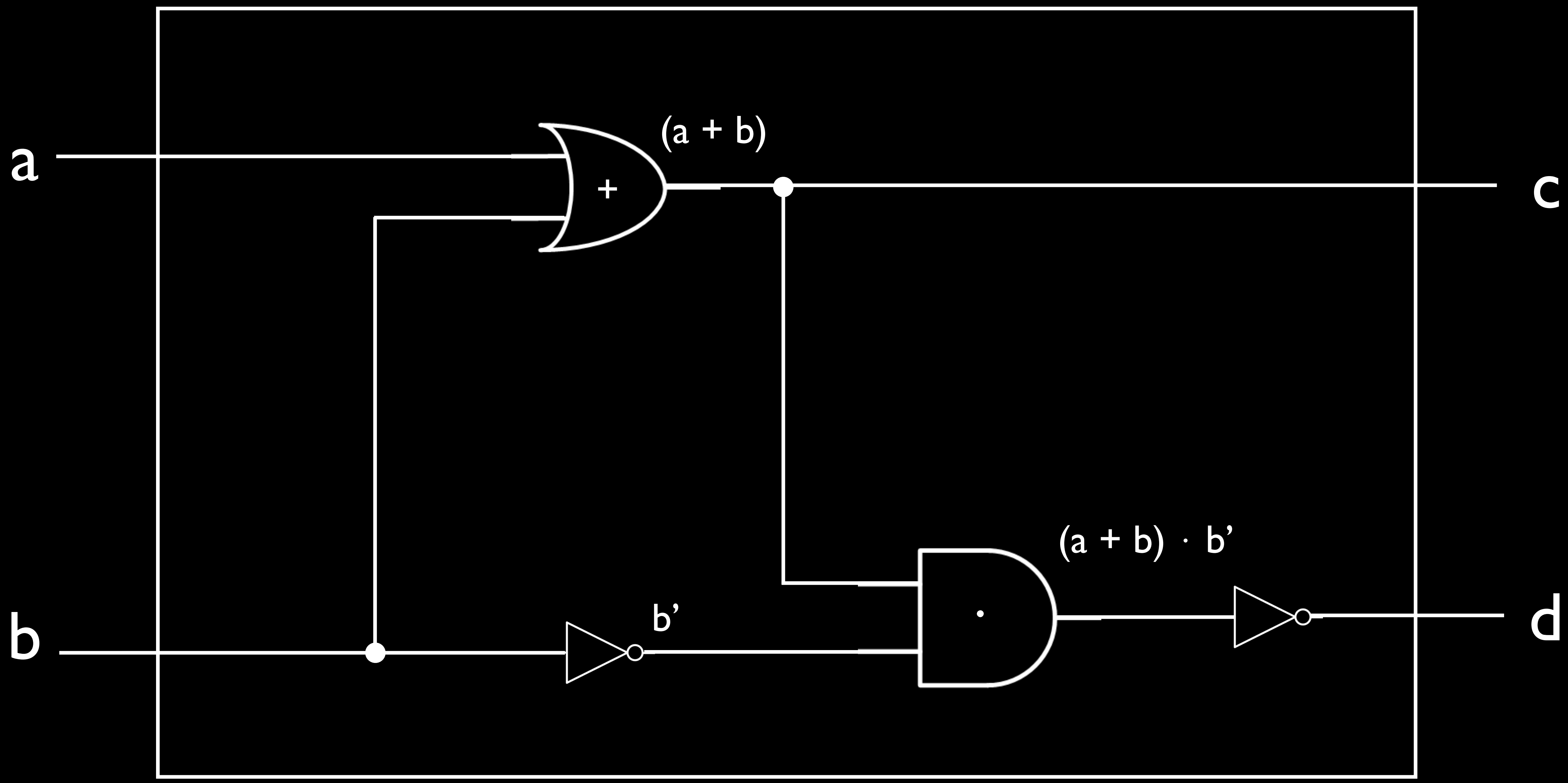


$$c = (a + b)$$

Circuits



Circuits



$$d = ((a + b) \cdot b')'$$

Designing Circuits

input		output	
a	b	c	d
0	0		
0	1		
1	0		
1	1		

step 1. build truth-table for all possible input/output values

Designing Circuits

input output

a	b	c	d
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

arbitrary, for now

step 1. build truth-table for all possible input/output values

Designing Circuits

input output

a	b	c	d
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

arbitrary, for now

step 2. build sub-expressions with *and/not* for each output column

Designing Circuits

input		output	
a	b	c	d
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

$d = a \cdot b$

step 2. build sub-expressions with *and/not* for each output column

Designing Circuits

input		output	
a	b	c	d
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

$c = a' \cdot b$

step 2. build sub-expressions with *and*/*not* for each output column

Designing Circuits

input		output	
a	b	c	d
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

$c = a' \cdot b$

$c = a \cdot b'$

step 2. build sub-expressions with *and/not* for each output column

Designing Circuits

input		output	
a	b	c	d
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

step 3. combine, two at a time, sub-expressions with an *or*

Designing Circuits

input		output	
a	b	c	d
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

$$c = (a' \cdot b) + (a \cdot b')$$

step 3. combine, two at a time, sub-expressions with an *or*

Designing Circuits

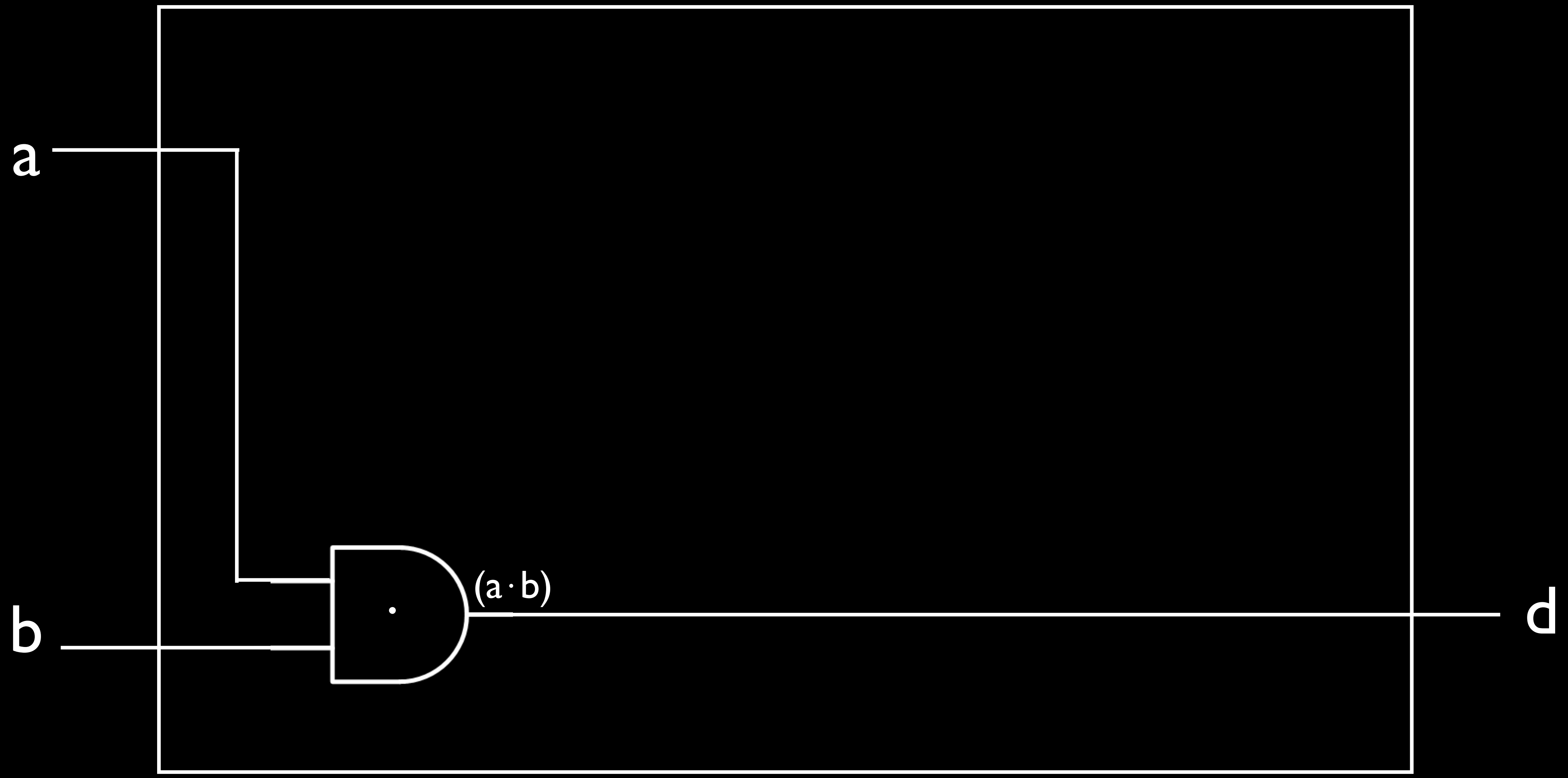
input		output	
a	b	c	d
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

$$c = (a' \cdot b) + (a \cdot b')$$

$$d = a \cdot b$$

step 3. combine, two at a time, sub-expressions with an *or*

Designing Circuits

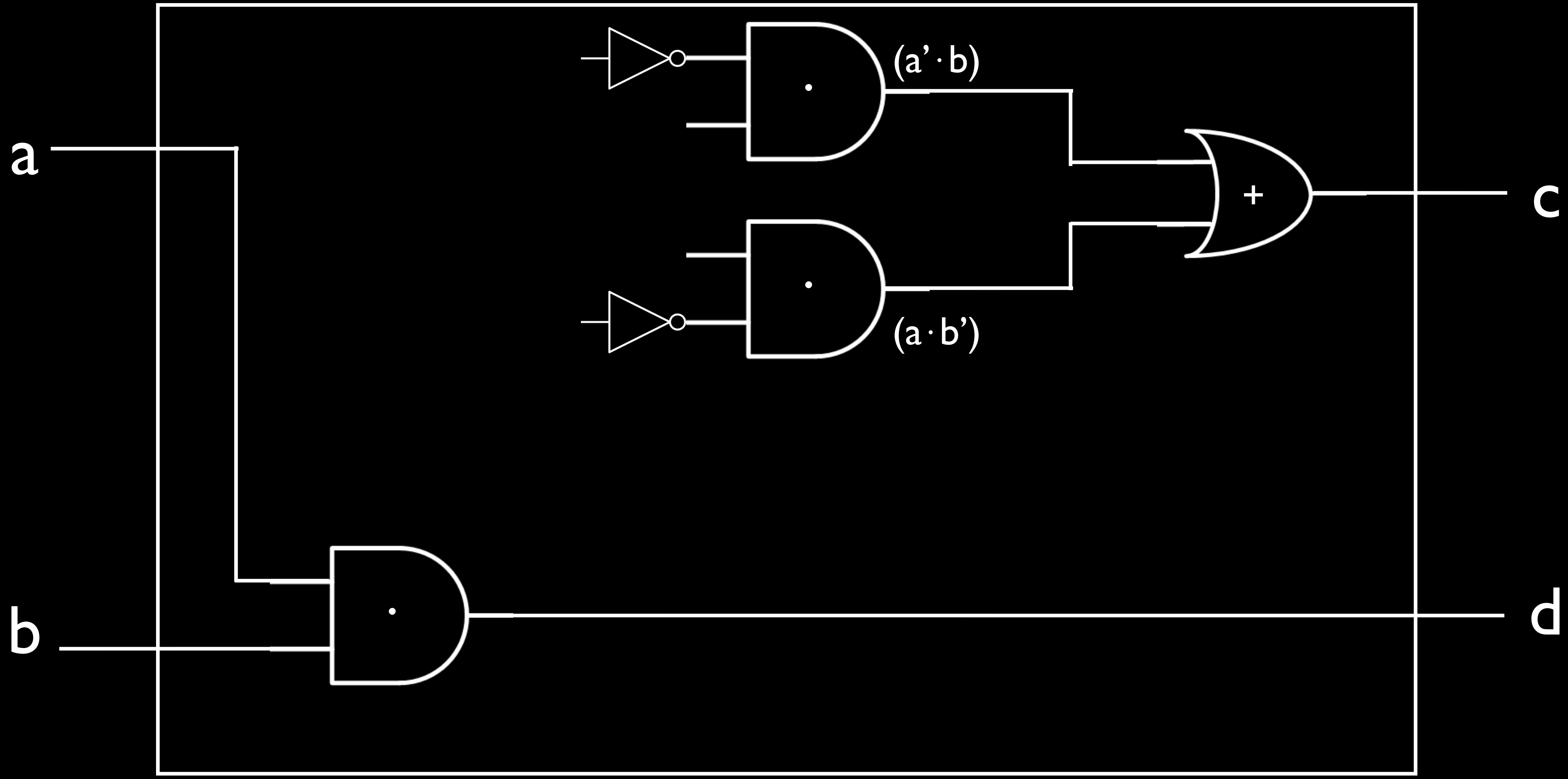


$d = a \cdot b$

step 4. draw circuit diagram

Designing Circuits

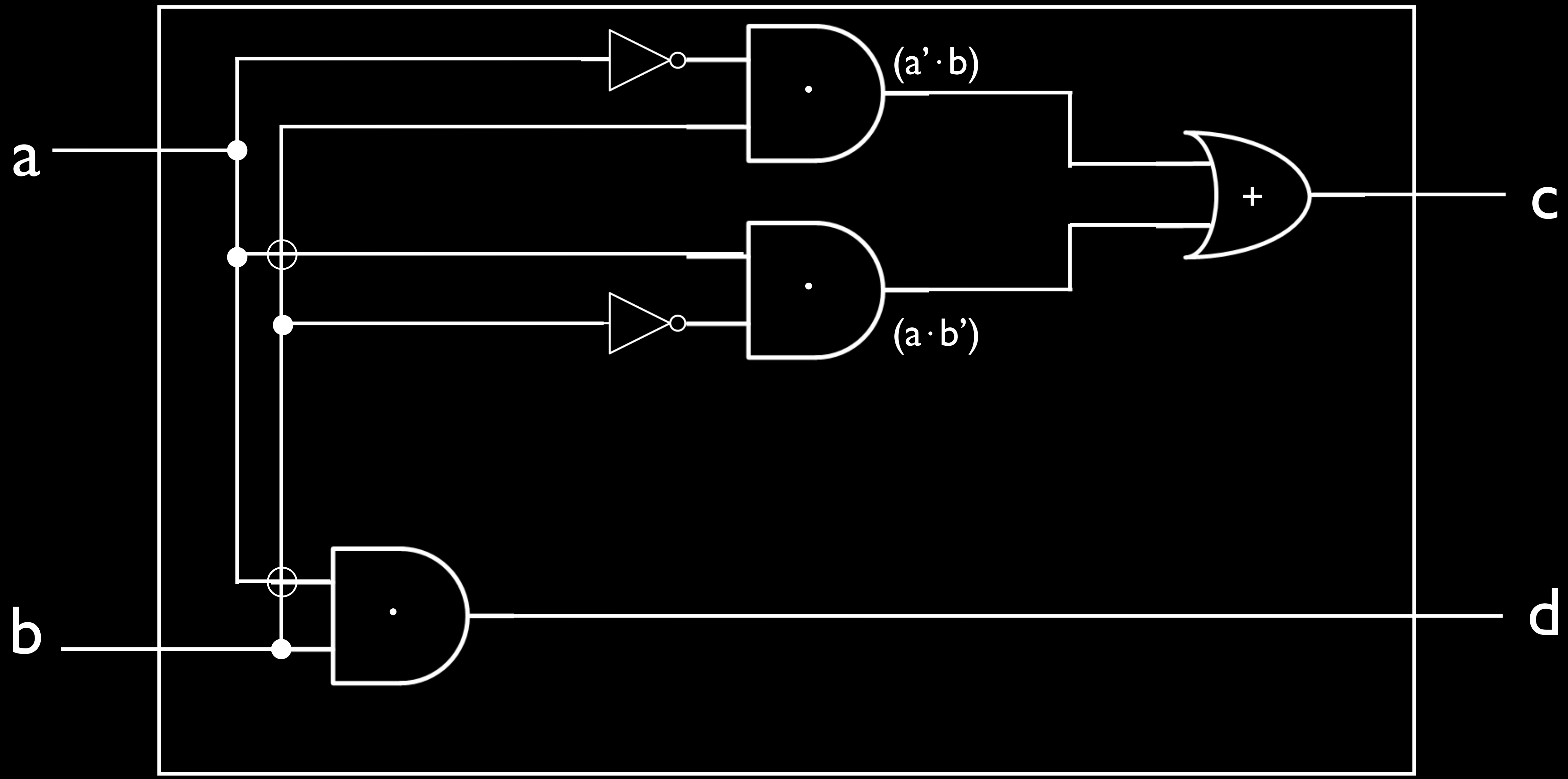
c = (a’ · b) + (a · b’)



step 4. draw circuit diagram

Designing Circuits

$c = (a' \cdot b) + (a \cdot b')$



step 4. draw circuit diagram

Designing Circuits

step 1. build truth-table for all possible input/output values

step 2. build sub-expressions with *and/not* for each output column

step 3. combine, two at a time, sub-expressions with an *or*

step 4. draw circuit diagram

1-Bit Compare for Equality (CE)

```
If two bits $a$, $b$ are equal then
    return 1
else
    return 0
```

1-Bit Compare for Equality (CE)

input		output
a	b	c
0	0	
0	1	
1	0	
1	1	

1-Bit Compare for Equality (CE)

input		output
a	b	c
0	0	1
0	1	0
1	0	0
1	1	1

1-Bit Compare for Equality (CE)

input output

a	b	c	sub-expression
0	0	1	
0	1	0	
1	0	0	
1	1	1	

1-Bit Compare for Equality (CE)

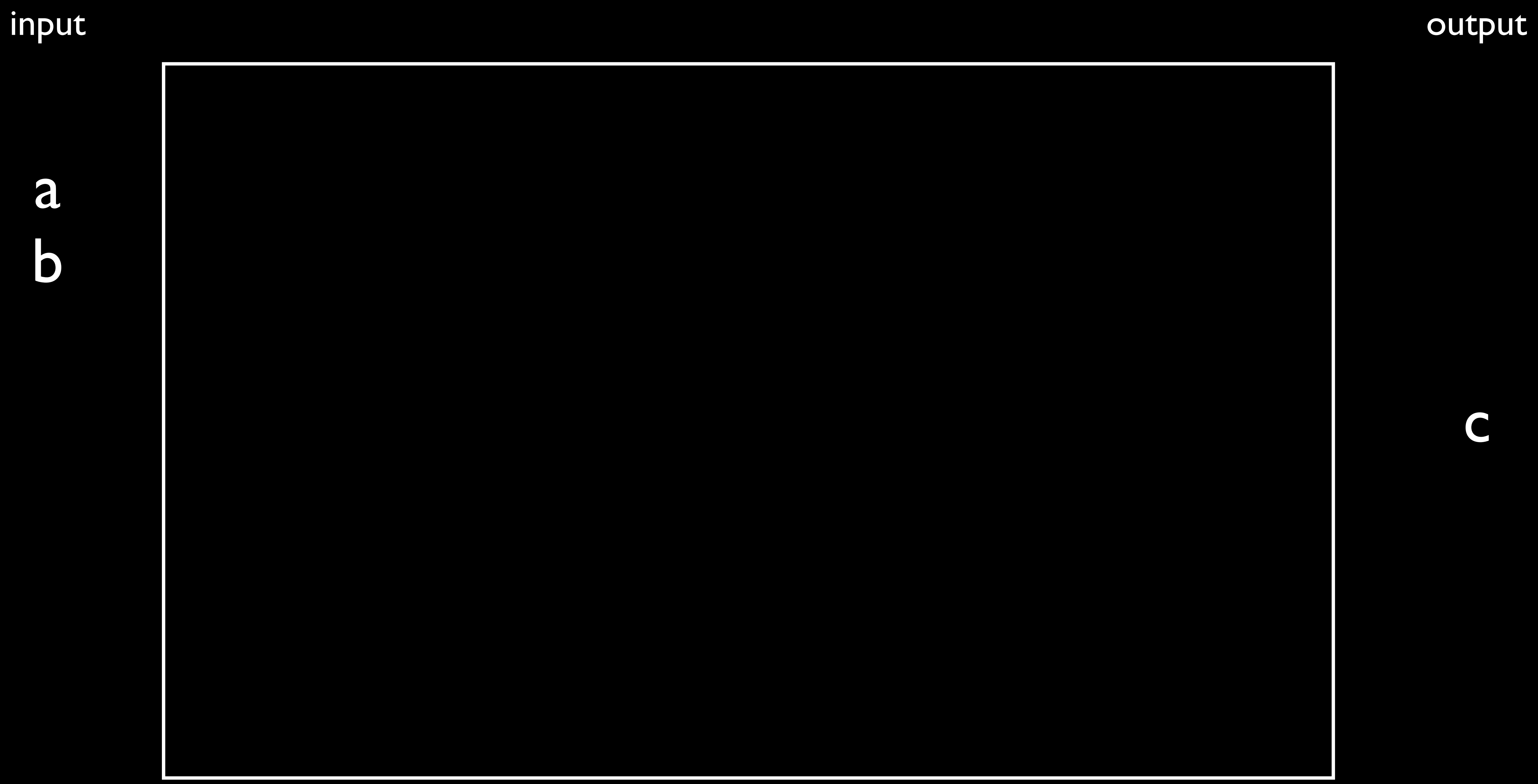
input		output	
a	b	c	sub-expression
0	0	1	$a' \cdot b'$
0	1	0	
1	0	0	
1	1	1	$a \cdot b$

1-Bit Compare for Equality (CE)

input		output	
a	b	c	sub-expression
0	0	1	$a' \cdot b'$
0	1	0	
1	0	0	
1	1	1	$a \cdot b$

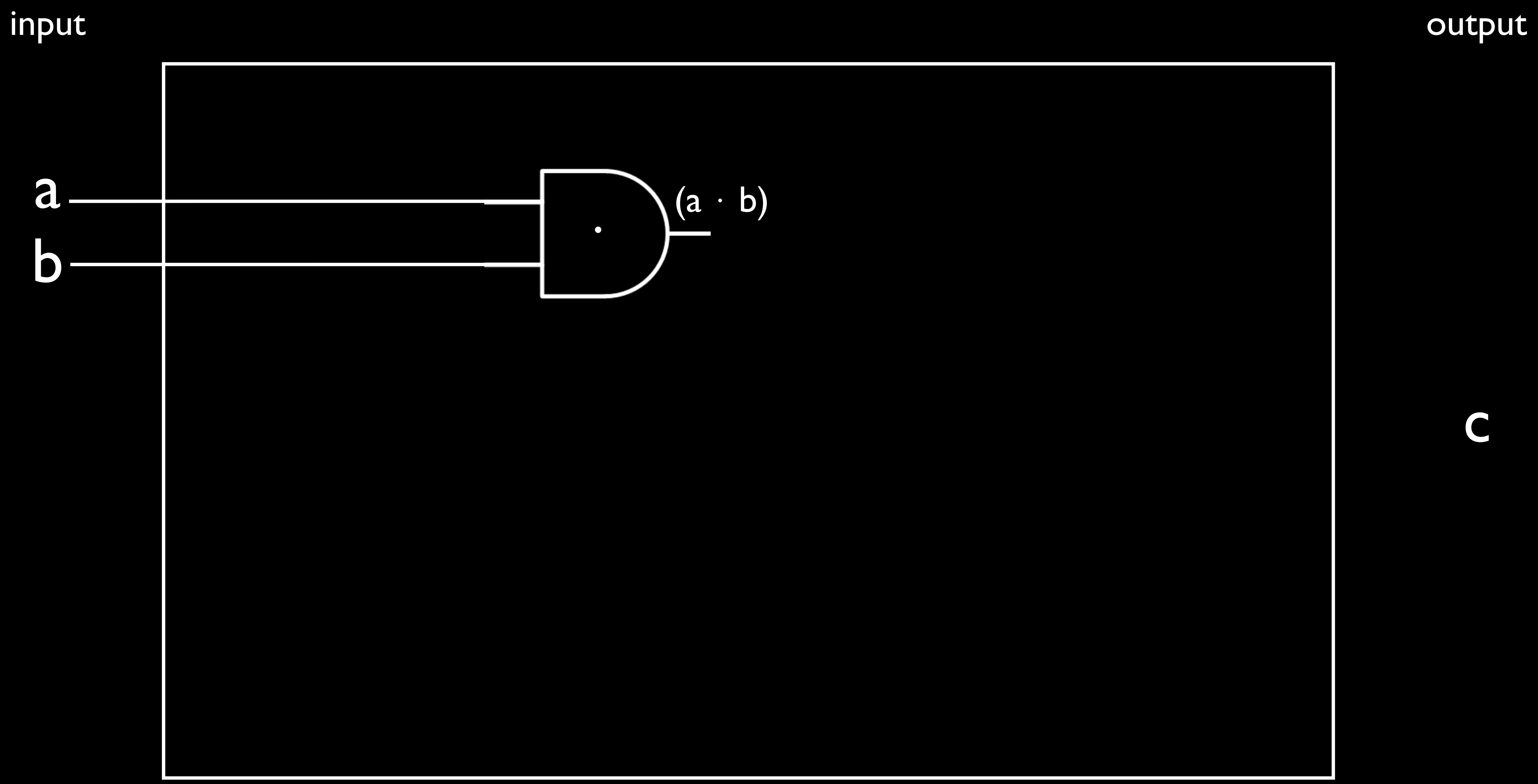
$$c = (a' \cdot b') + (a \cdot b)$$

1-Bit Compare for Equality (CE)



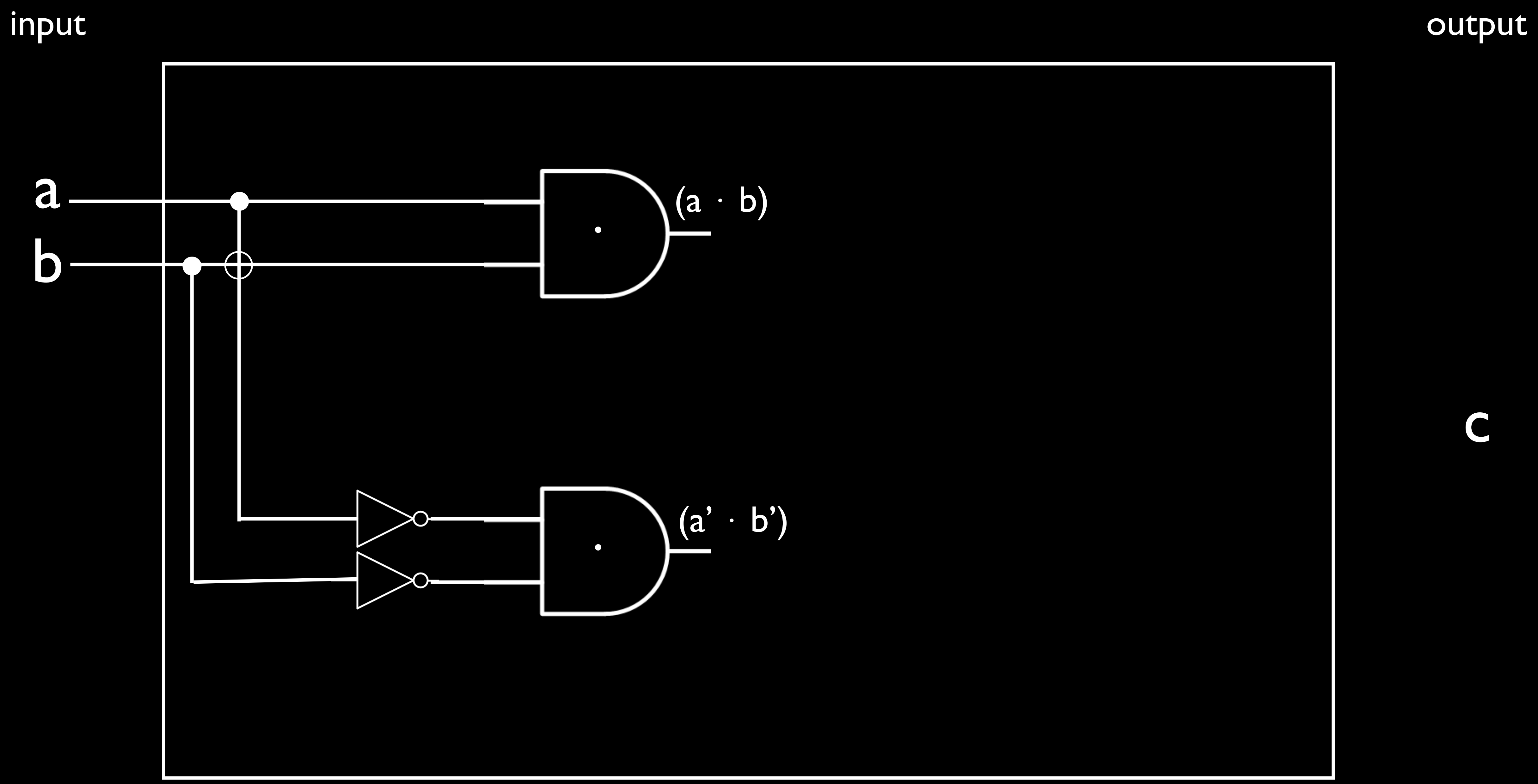
$$c = (a' \cdot b') + (a \cdot b)$$

1-Bit Compare for Equality (CE)



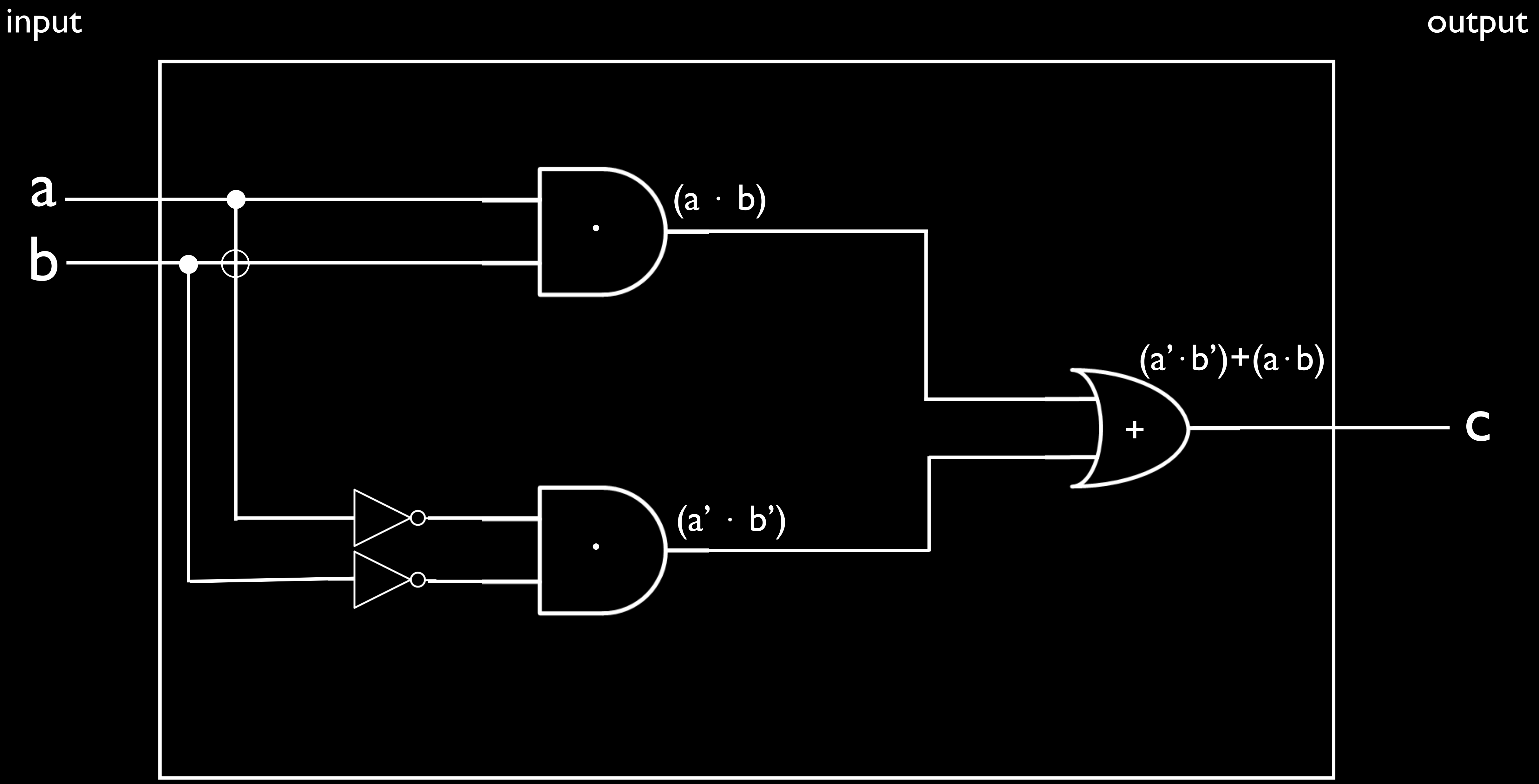
$$c = (a' \cdot b') + (a \cdot b)$$

1-Bit Compare for Equality (CE)



$$c = (a' \cdot b') + (a \cdot b)$$

1-Bit Compare for Equality (CE)



$$c = (a' \cdot b') + (a \cdot b)$$

4-Bit Compare for Equality (CE)

```
If two 4-bit numbers are equal then
    return 1
else
    return 0
```

4-Bit Compare for Equality (CE)

```
If two 4-bit numbers are equal then
    return 1
else
    return 0
```

$$a_3\ a_2\ a_1\ a_0 == b_3\ b_2\ b_1\ b_0$$

4-Bit Compare for Equality (CE)

input

output

a_3	a_2	a_1	a_0	b_3	b_2	b_1	b_0	c

how many rows?

4-Bit Compare for Equality (CE)

256

4-Bit Compare for Equality (CE)

two 4-bit numbers are equal if:

$a_3 == b_3$ and

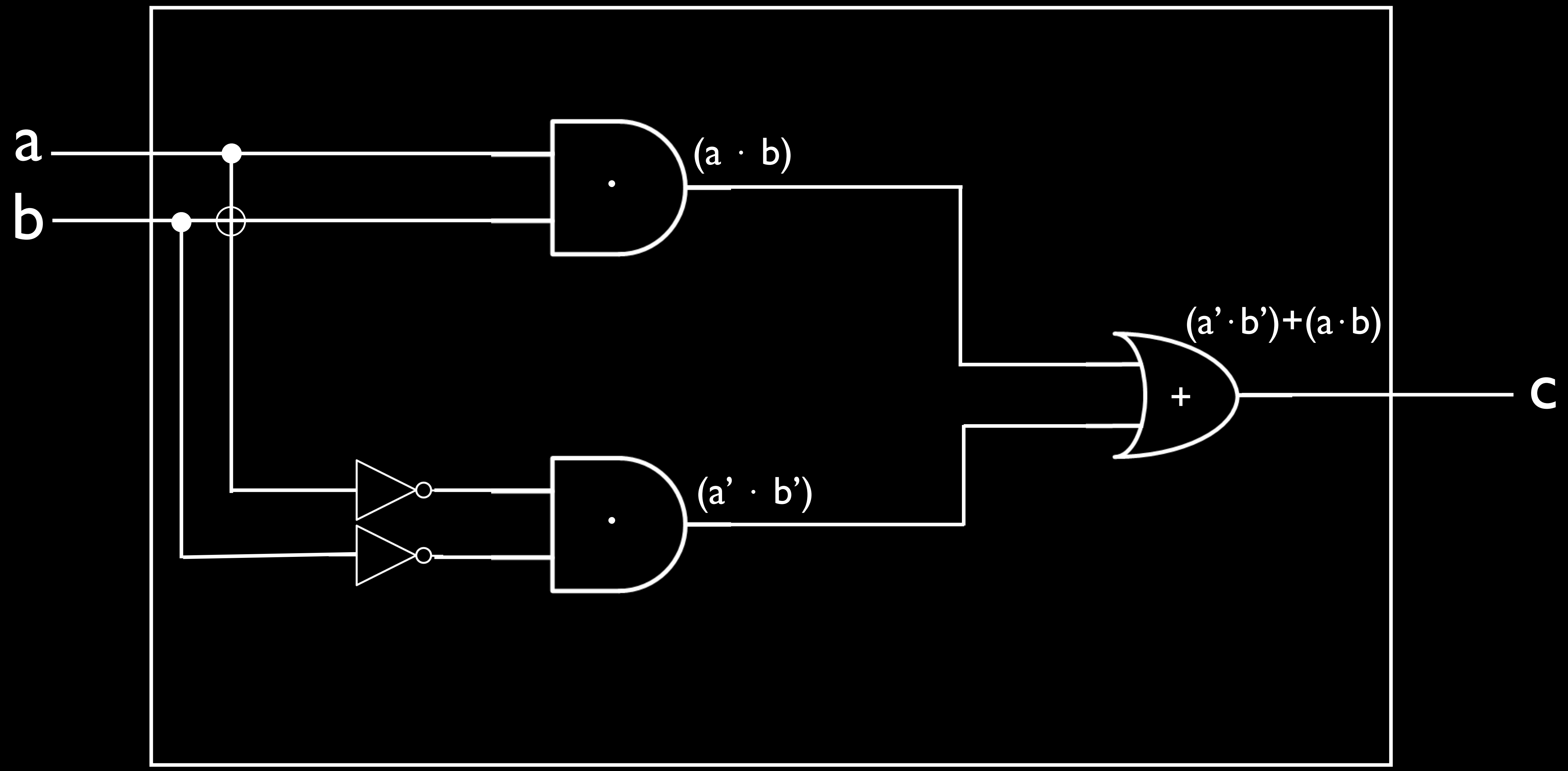
$a_2 == b_2$ and

$a_1 == b_1$ and

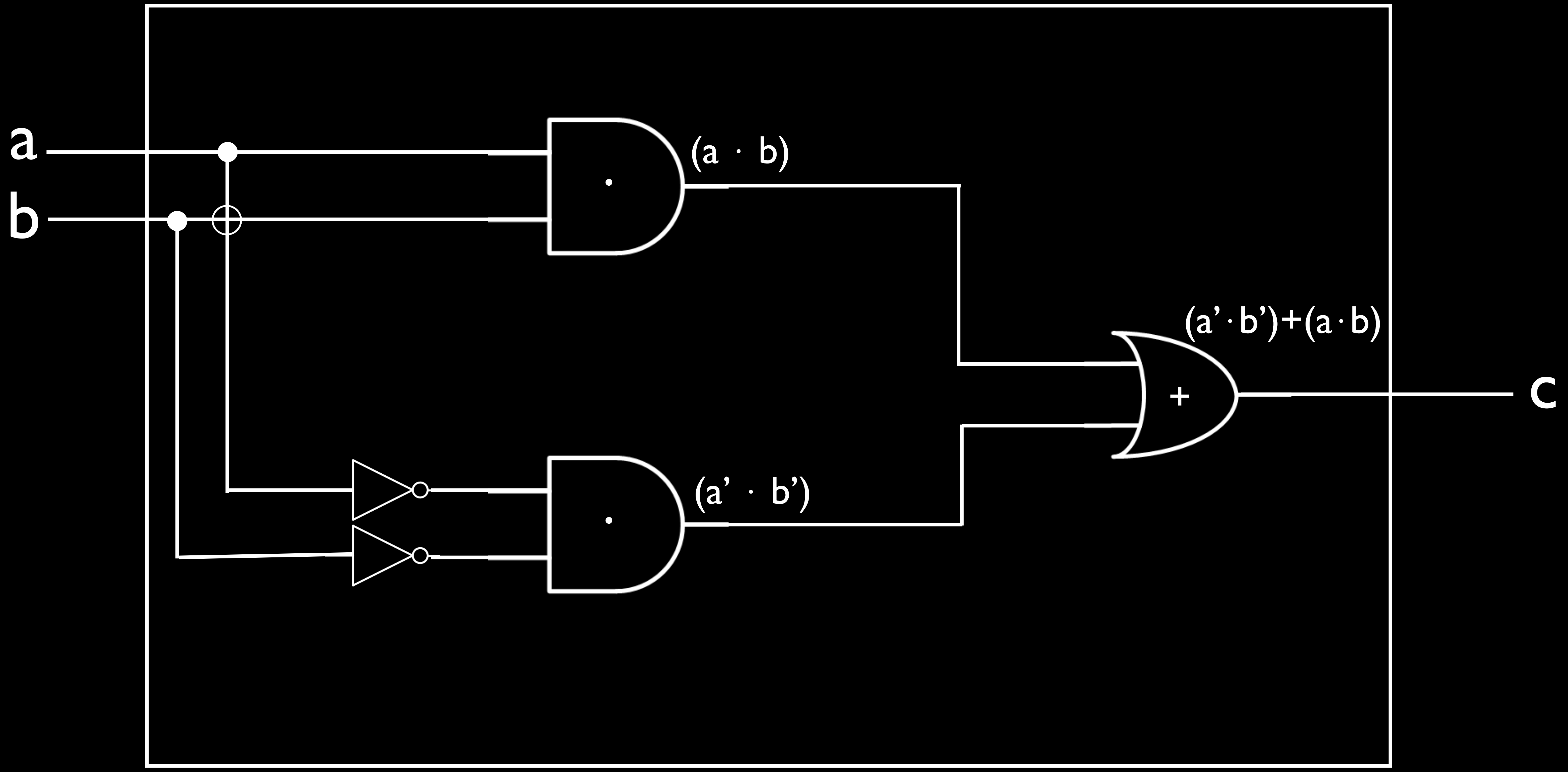
$a_0 == b_0$

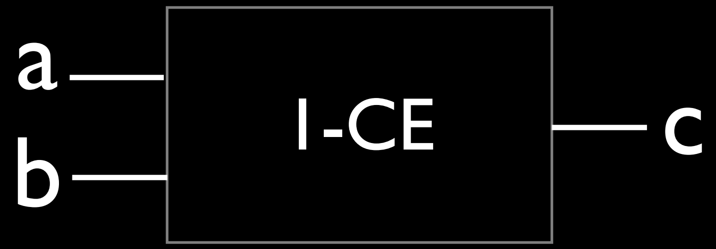
$$a_3\ a_2\ a_1\ a_0 == b_3\ b_2\ b_1\ b_0$$

4-Bit Compare for Equality (CE)

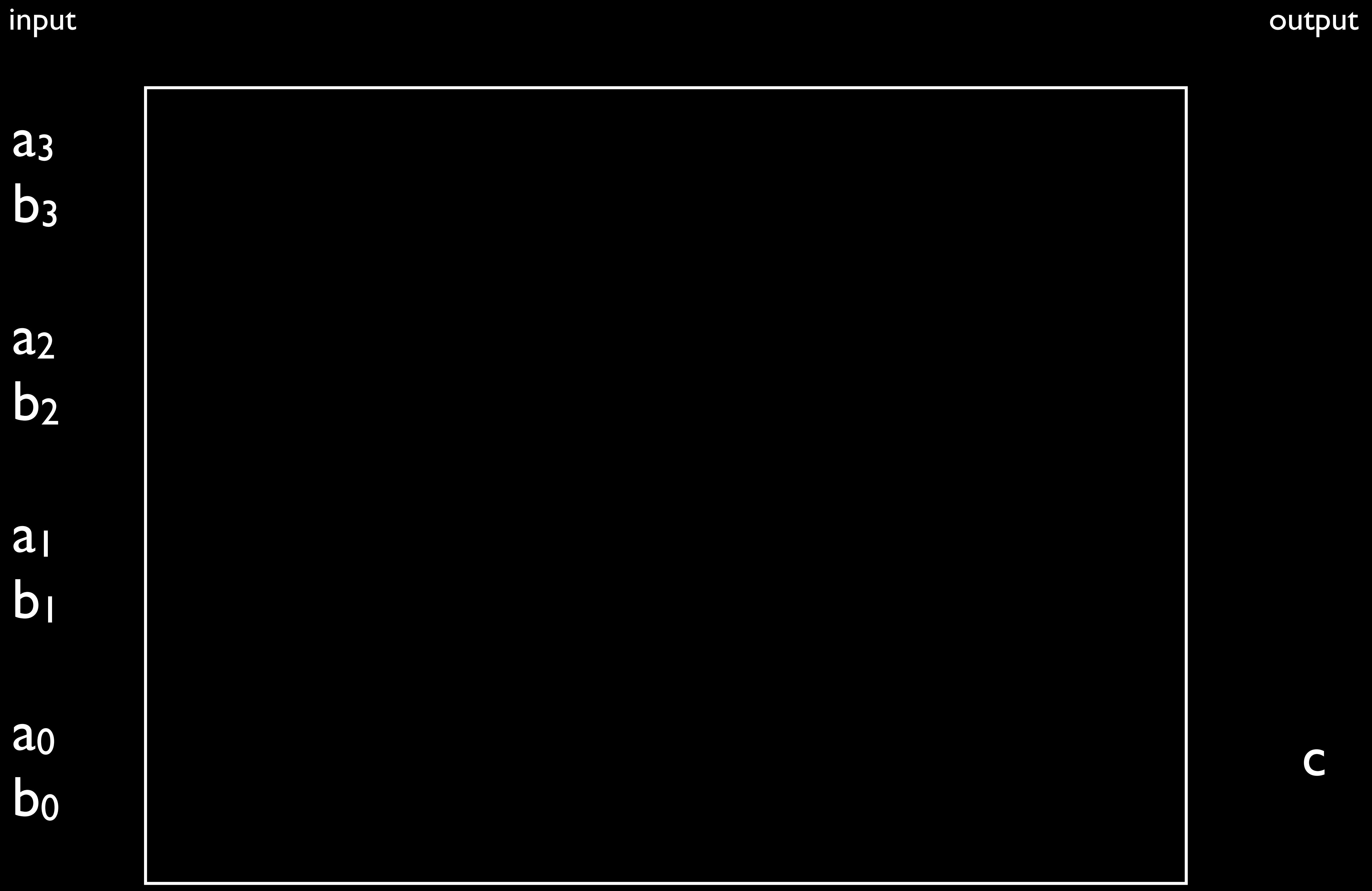


4-Bit Compare for Equality (CE)





4-Bit Compare for Equality (CE)



input

a_3
b_3

a_2
b_2

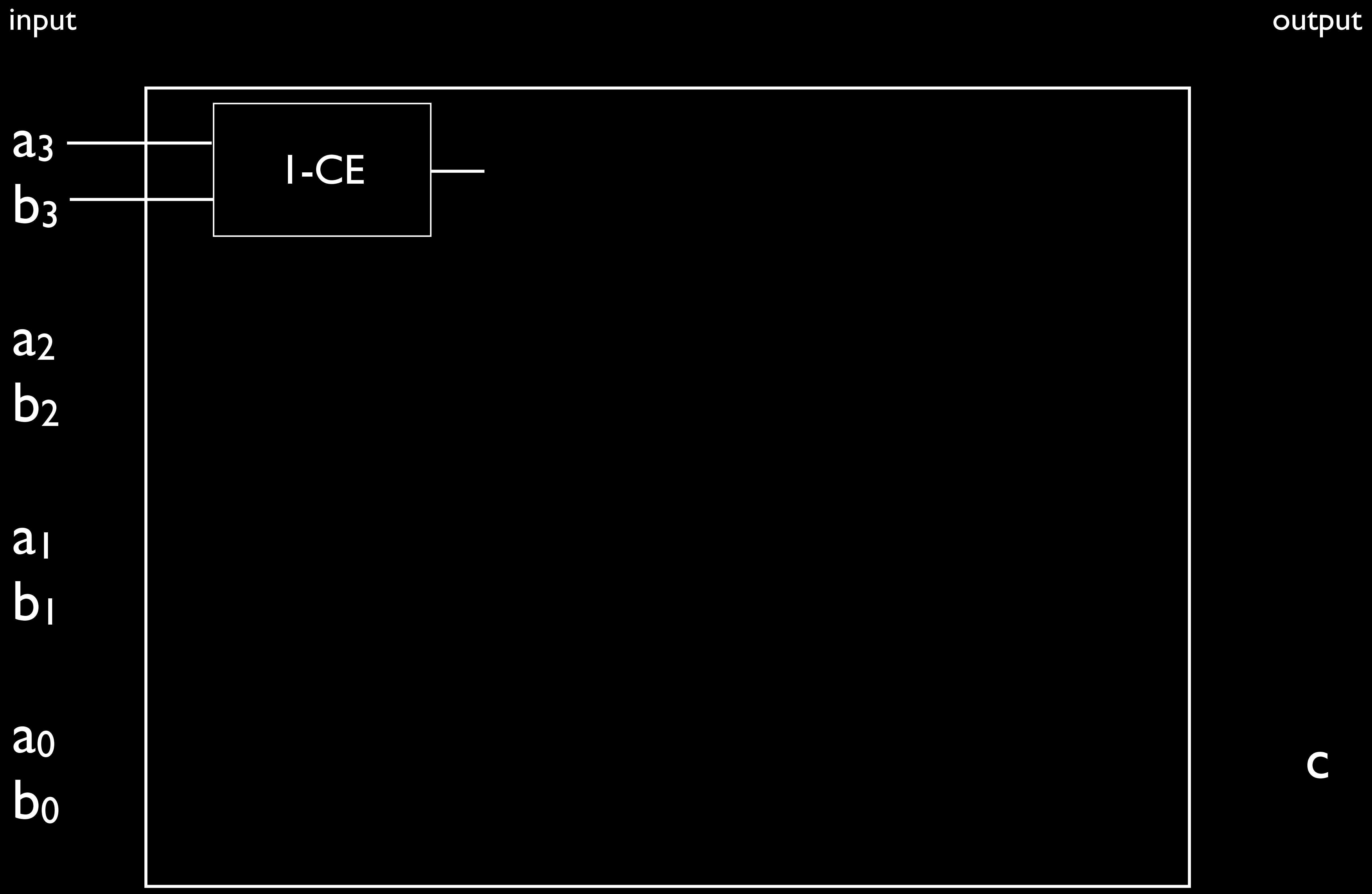
a_1
b_1

a_0
b_0

output

c

4-Bit Compare for Equality (CE)



input

output

a_3

b_3

1-CE

a_2

b_2

a_1

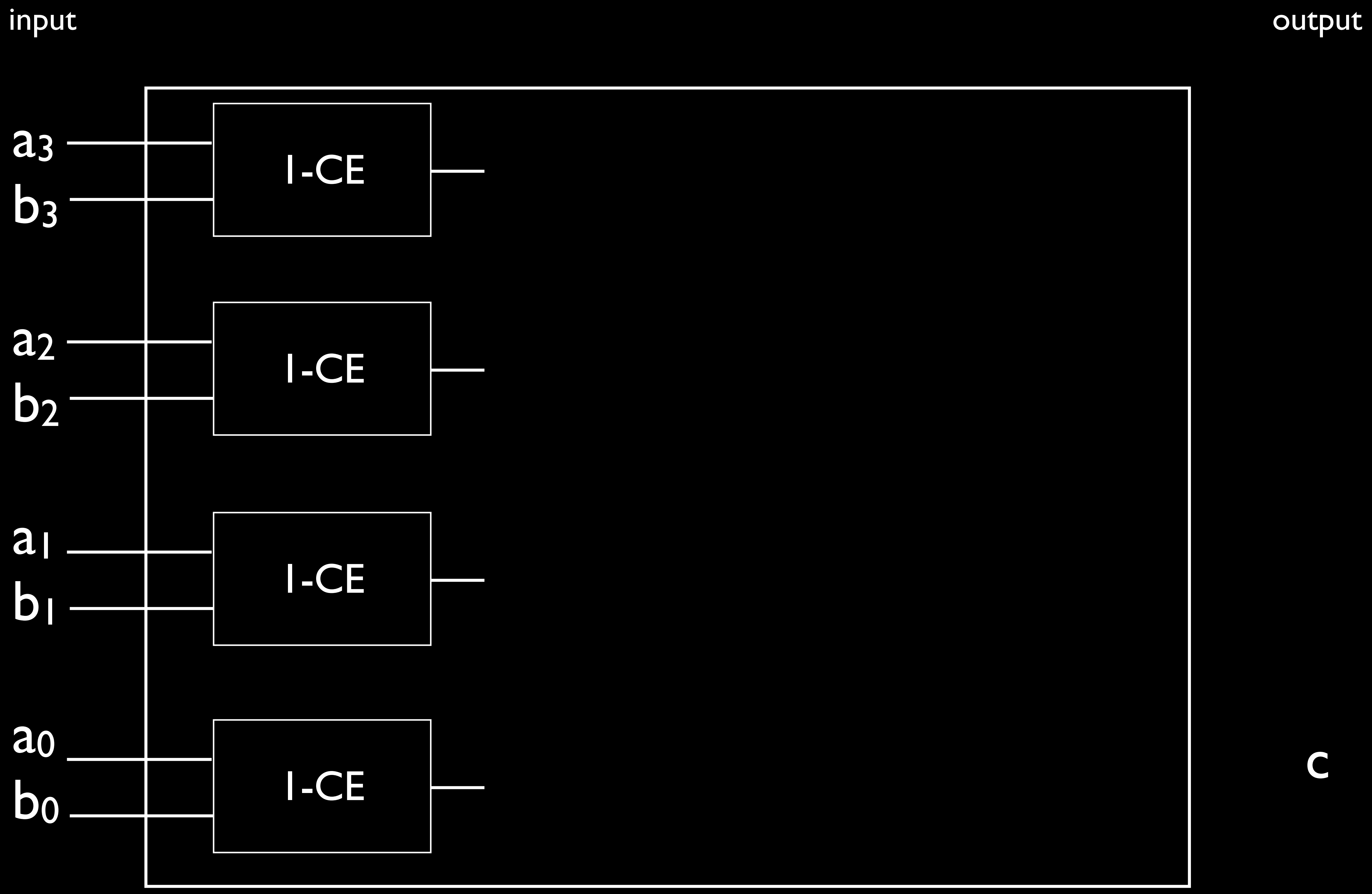
b_1

a_0

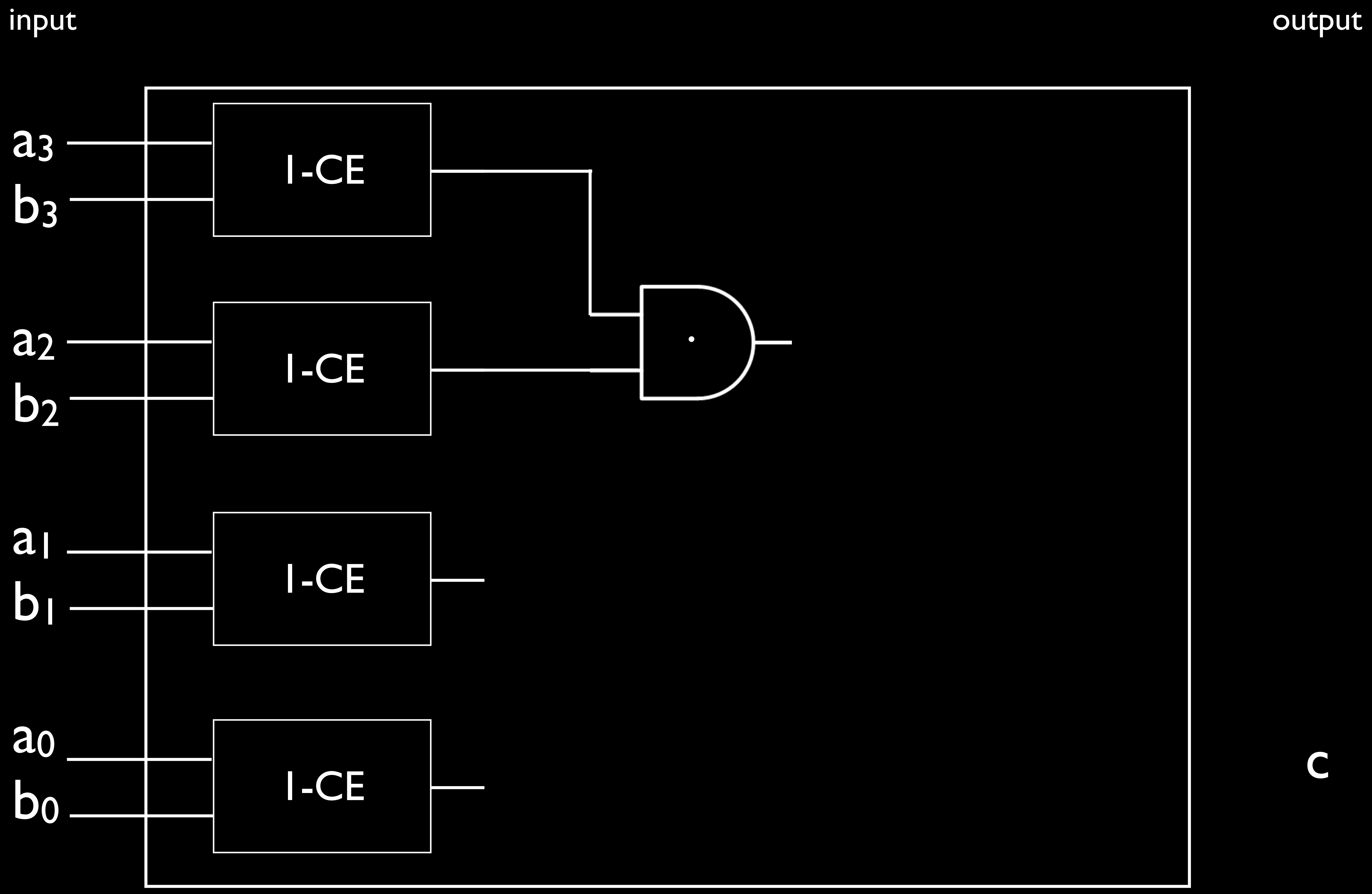
b_0

c

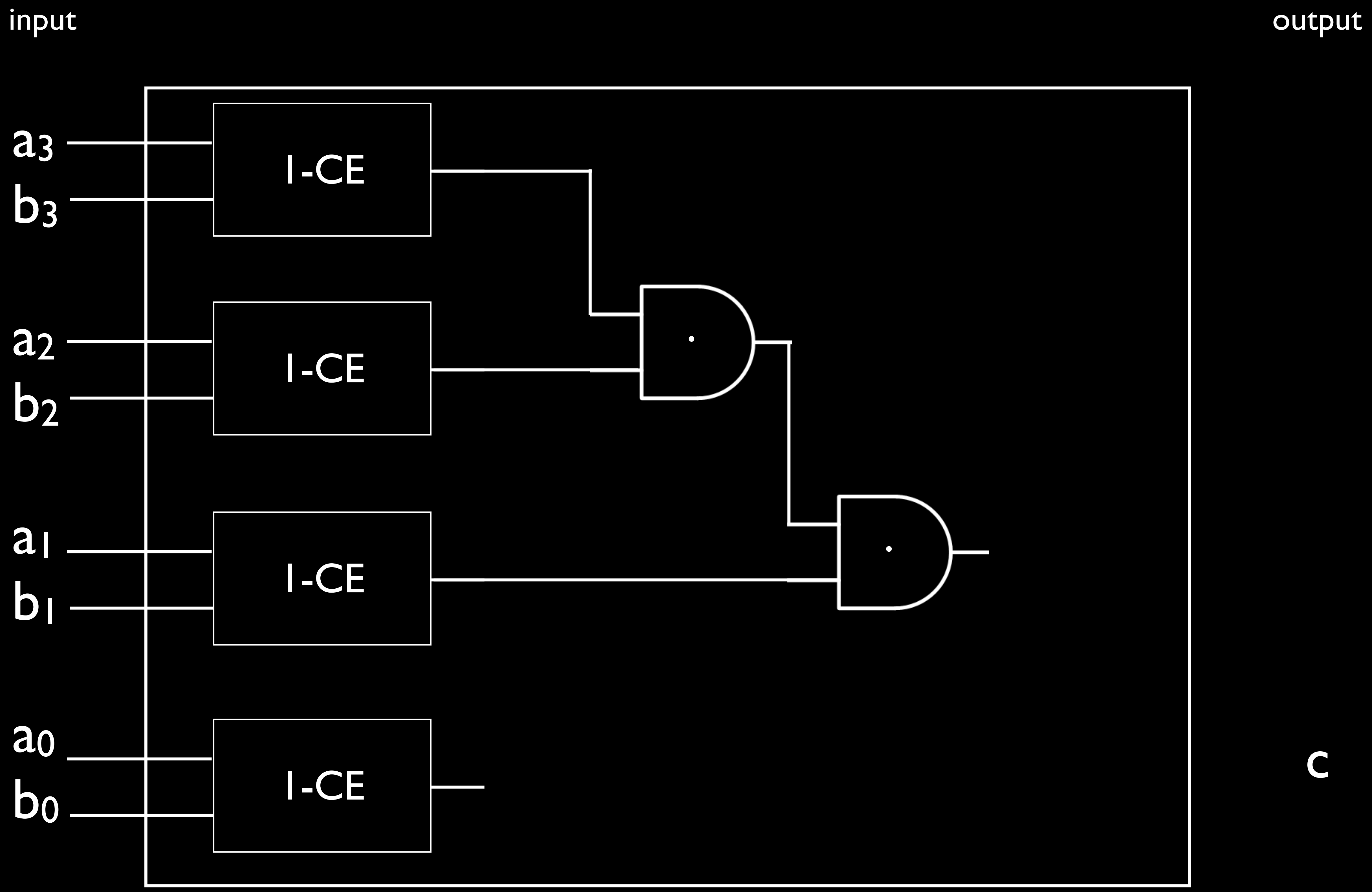
4-Bit Compare for Equality (CE)



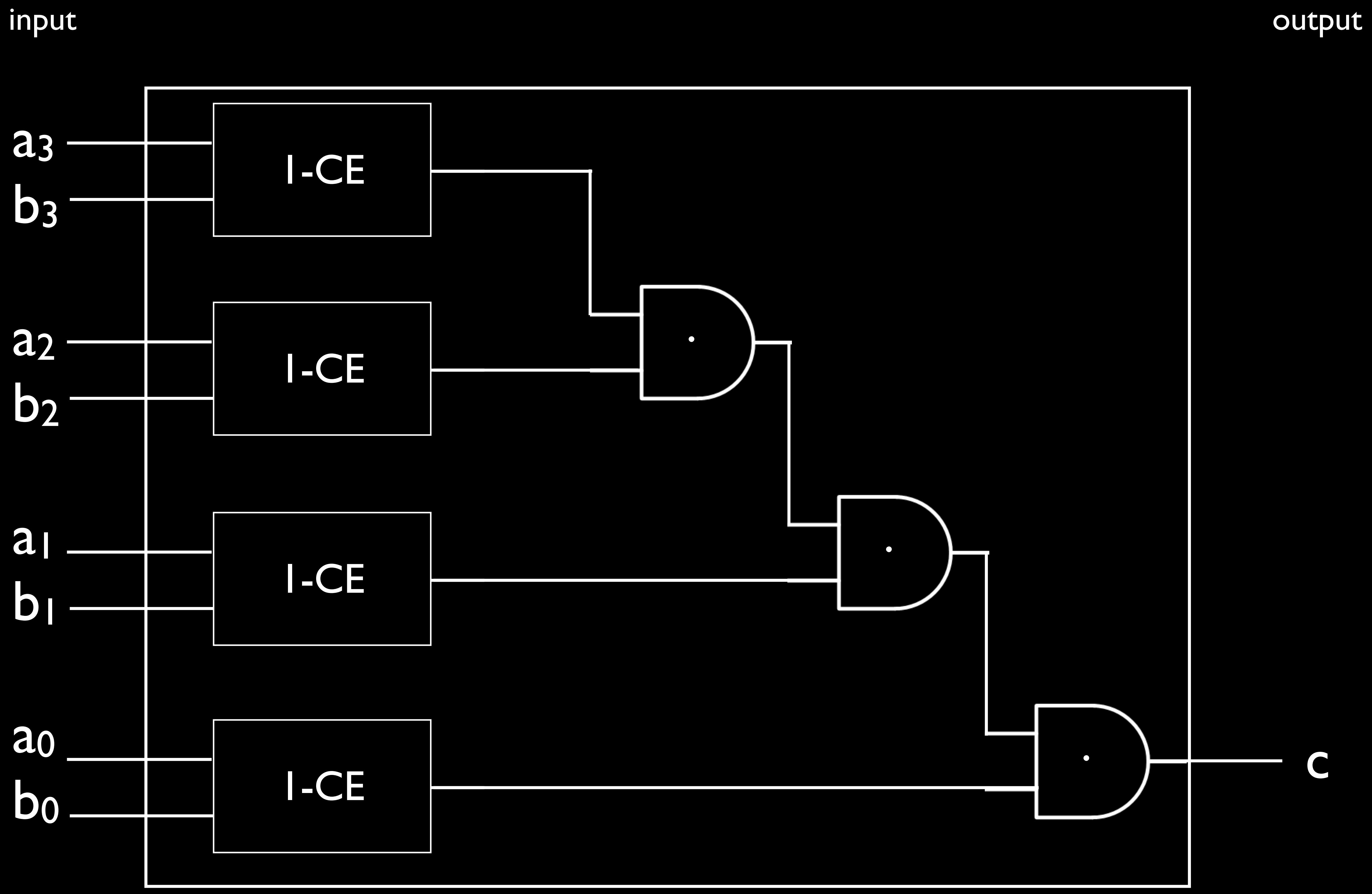
4-Bit Compare for Equality (CE)



4-Bit Compare for Equality (CE)



4-Bit Compare for Equality (CE)



1-Bit Adder

build a circuit that adds two 1-bit numbers

1-Bit Adder

0 + 0 = 0

0 + 1 = 1

1 + 0 = 1

1 + 1 = ?

1-Bit Adder

0 + 0 = 0

0 + 1 = 1

1 + 0 = 1

1 + 1 = 10 *need to carry*

1-Bit Adder

$$0 + 0 = 0$$
$$0 + 1 = 1$$
$$1 + 0 = 1$$
$$1 + 1 = 10$$

input: two digits *a*, *b*

1-Bit Adder

$0 + 0 = 0$
$0 + 1 = 1$
$1 + 0 = 1$
$1 + 1 = 10$

input: two digits a, b and a carry c

1-Bit Adder

0 + 0 = 0

0 + 1 = 1

1 + 0 = 1

1 + 1 = 10

output: sum *d* and carry e

1-Bit Adder

5	101	2^2+2^0
1	001	2^0
---	---	
6		

1-Bit Adder

```
          1
  5      101
  1      001
 ---     ---
  6        0
```

1-Bit Adder

```
             1
  5         101
  1         001
 ---        ---
  6          10
```

1-Bit Adder

```
          1
  5      101
  1      001
 ---     ---
  6      110
```

1-Bit Adder

```
          1
 5       101
 1       001
---      ---
 6       110
```

2^2+2^1

1-Bit Adder

input: digits *a*, *b* and carry *c*

output: sum *d* and carry e

1-Bit Adder

a	b	c	d	e
0	0	0		
0	0	1		
0	1	0		
0	1	1		
1	0	0		
1	0	1		
1	1	0		
1	1	1		

input: digits *a*, *b* and carry *c*
output: sum *d* and carry *e*

1-Bit Adder

a	b	c	d	e
0	0	0	0	0
0	0	1		
0	1	0		
0	1	1		
1	0	0		
1	0	1		
1	1	0		
1	1	1		

input: digits *a*, *b* and carry *c*
output: sum *d* and carry *e*

1-Bit Adder

a	b	c	d	e
0	0	0	0	0
0	0	1	1	0
0	1	0		
0	1	1		
1	0	0		
1	0	1		
1	1	0		
1	1	1		

input: digits *a*, *b* and carry *c*

output: sum *d* and carry *e*

1-Bit Adder

a	b	c	d	e
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1		
1	0	0		
1	0	1		
1	1	0		
1	1	1		

input: digits *a*, *b* and carry *c*

output: sum *d* and carry *e*

1-Bit Adder

a	b	c	d	e
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0		
1	0	1		
1	1	0		
1	1	1		

input: digits *a*, *b* and carry *c*
output: sum *d* and carry *e*

1-Bit Adder

a	b	c	d	e
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1		
1	1	0		
1	1	1		

input: digits *a*, *b* and carry *c*
output: sum *d* and carry *e*

1-Bit Adder

a	b	c	d	e
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0		
1	1	1		

input: digits *a*, *b* and carry *c*
output: sum *d* and carry *e*

1-Bit Adder

a	b	c	d	e
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1		

input: digits *a*, *b* and carry *c*
output: sum *d* and carry *e*

1-Bit Adder

a	b	c	d	e
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

input: digits *a*, *b* and carry *c*

output: sum *d* and carry *e*

1-Bit Adder

a	b	c	d	e	sub-expressions (d)	sub-expressions (e)
0	0	0	0	0		
0	0	1	1	0		
0	1	0	1	0		
0	1	1	0	1		
1	0	0	1	0		
1	0	1	0	1		
1	1	0	0	1		
1	1	1	1	1		

1-Bit Adder

a	b	c	d	e	sub-expressions (d)	sub-expressions (e)
0	0	0	0	0		
0	0	1	1	0		
0	1	0	1	0		
0	1	1	0	1		
1	0	0	1	0		
1	0	1	0	1		
1	1	0	0	1		
1	1	1	1	1		

1-Bit Adder

a	b	c	d	e	sub-expressions (d)	sub-expressions (e)
0	0	0	0	0		
0	0	1	1	0	a’ · b’ · c	
0	1	0	1	0		
0	1	1	0	1		
1	0	0	1	0		
1	0	1	0	1		
1	1	0	0	1		
1	1	1	1	1		

1-Bit Adder

a	b	c	d	e	sub-expressions (d)	sub-expressions (e)
0	0	0	0	0		
0	0	1	1	0	$a' \cdot b' \cdot c$	
0	1	0	1	0	$a' \cdot b \cdot c'$	
0	1	1	0	1		
1	0	0	1	0		
1	0	1	0	1		
1	1	0	0	1		
1	1	1	1	1		

1-Bit Adder

a	b	c	d	e	sub-expressions (d)	sub-expressions (e)
0	0	0	0	0		
0	0	1	1	0	$a' \cdot b' \cdot c$	
0	1	0	1	0	$a' \cdot b \cdot c'$	
0	1	1	0	1		
1	0	0	1	0	$a \cdot b' \cdot c'$	
1	0	1	0	1		
1	1	0	0	1		
1	1	1	1	1		

1-Bit Adder

a	b	c	d	e	sub-expressions (d)	sub-expressions (e)
0	0	0	0	0		
0	0	1	1	0	a’·b’·c	
0	1	0	1	0	a’·b·c’	
0	1	1	0	1		
1	0	0	1	0	a·b’·c’	
1	0	1	0	1		
1	1	0	0	1		
1	1	1	1	1	a·b·c	

1-Bit Adder

a	b	c	d	e	sub-expressions (d)	sub-expressions (e)
0	0	0	0	0		
0	0	1	1	0	a’·b’·c	
0	1	0	1	0	a’·b·c’	
0	1	1	0	1		a’·b·c
1	0	0	1	0	a·b’·c’	
1	0	1	0	1		a·b’·c
1	1	0	0	1		a·b·c’
1	1	1	1	1	a·b·c	a·b·c

1-Bit Adder

a	b	c	d	e	sub-expressions (d)	sub-expressions (e)
0	0	0	0	0		
0	0	1	1	0	$a' \cdot b' \cdot c$	
0	1	0	1	0	$a' \cdot b \cdot c'$	
0	1	1	0	1		$a' \cdot b \cdot c$
1	0	0	1	0	$a \cdot b' \cdot c'$	
1	0	1	0	1		$a \cdot b' \cdot c$
1	1	0	0	1		$a \cdot b \cdot c'$
1	1	1	1	1	$a \cdot b \cdot c$	$a \cdot b \cdot c$

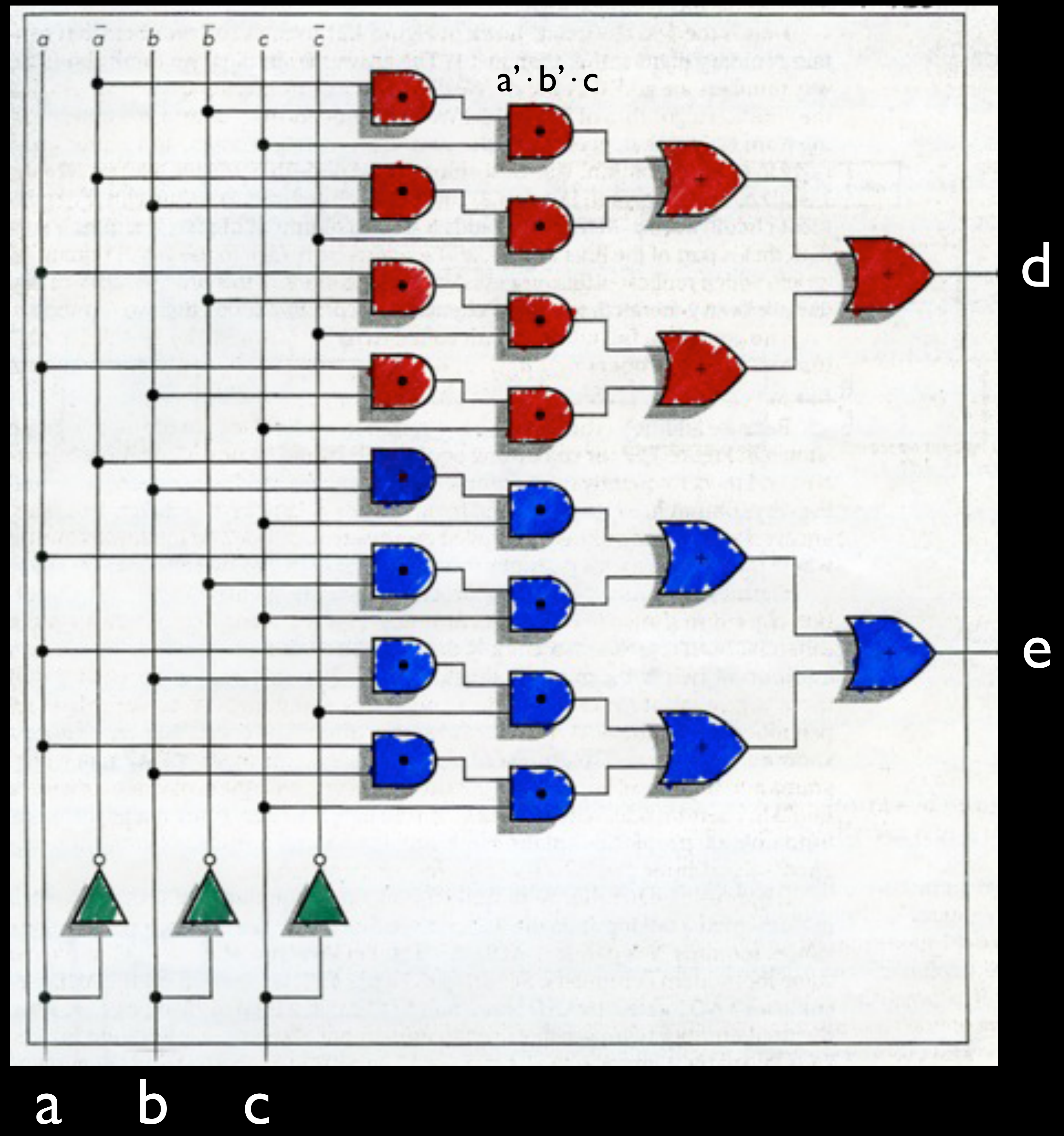
$$d = (a' \cdot b' \cdot c) + (a' \cdot b \cdot c') + (a \cdot b' \cdot c') + (a \cdot b \cdot c)$$

1-Bit Adder

a	b	c	d	e	sub-expressions (d)	sub-expressions (e)
0	0	0	0	0		
0	0	1	1	0	$a' \cdot b' \cdot c$	
0	1	0	1	0	$a' \cdot b \cdot c'$	
0	1	1	0	1		$a' \cdot b \cdot c$
1	0	0	1	0	$a \cdot b' \cdot c'$	
1	0	1	0	1		$a \cdot b' \cdot c$
1	1	0	0	1		$a \cdot b \cdot c'$
1	1	1	1	1	$a \cdot b \cdot c$	$a \cdot b \cdot c$

$$e = (a' \cdot b \cdot c) + (a \cdot b' \cdot c) + (a \cdot b \cdot c') + (a \cdot b \cdot c)$$

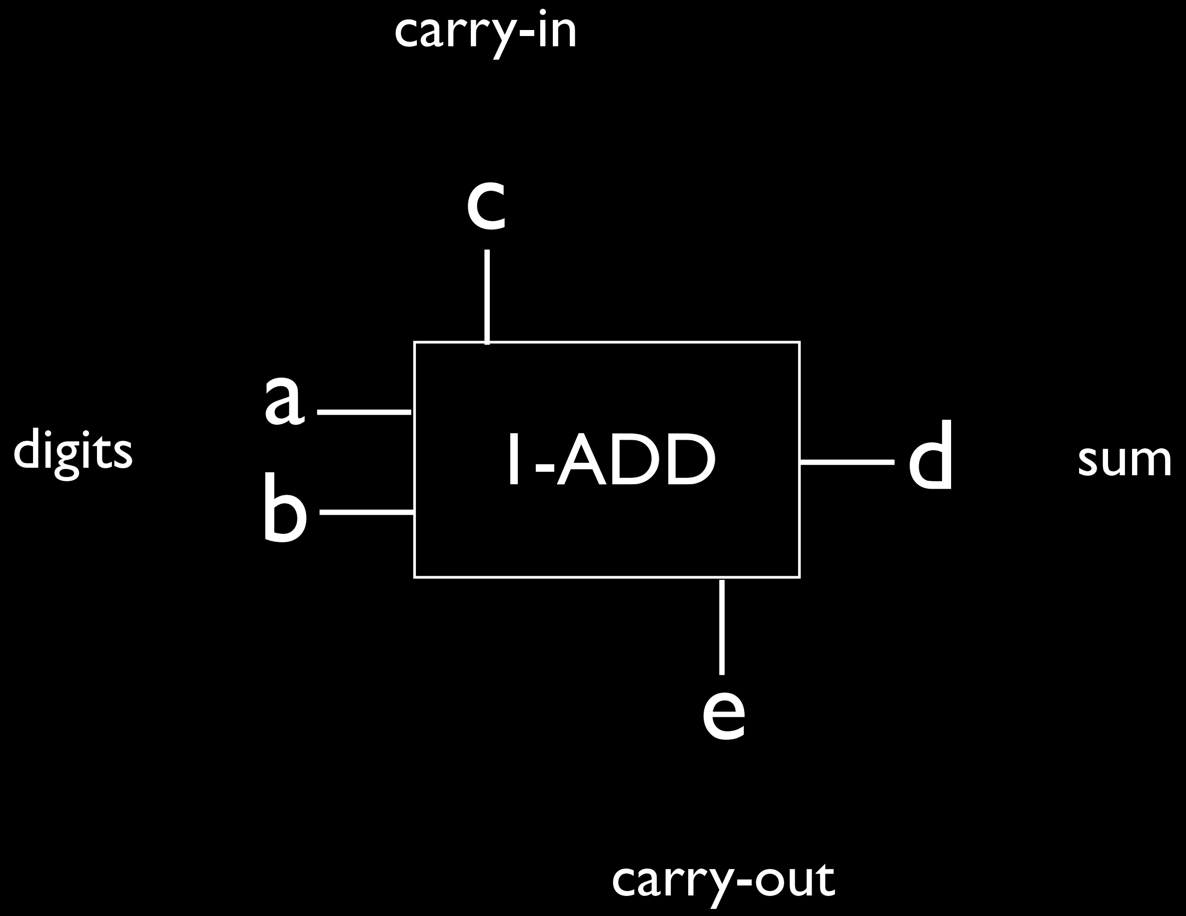
1-Bit Adder



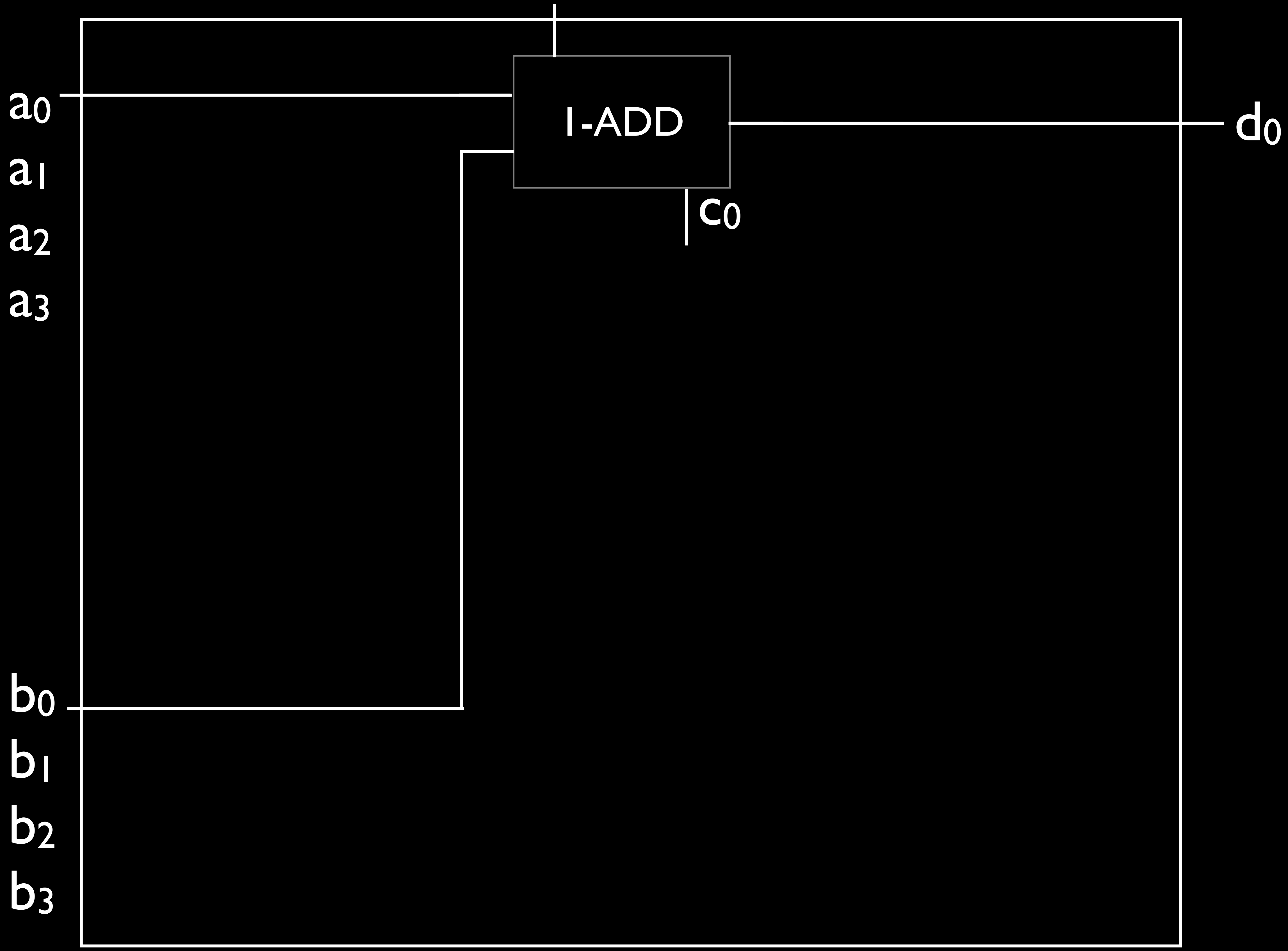
4-Bit Adder

build a circuit that adds two 4-bit numbers

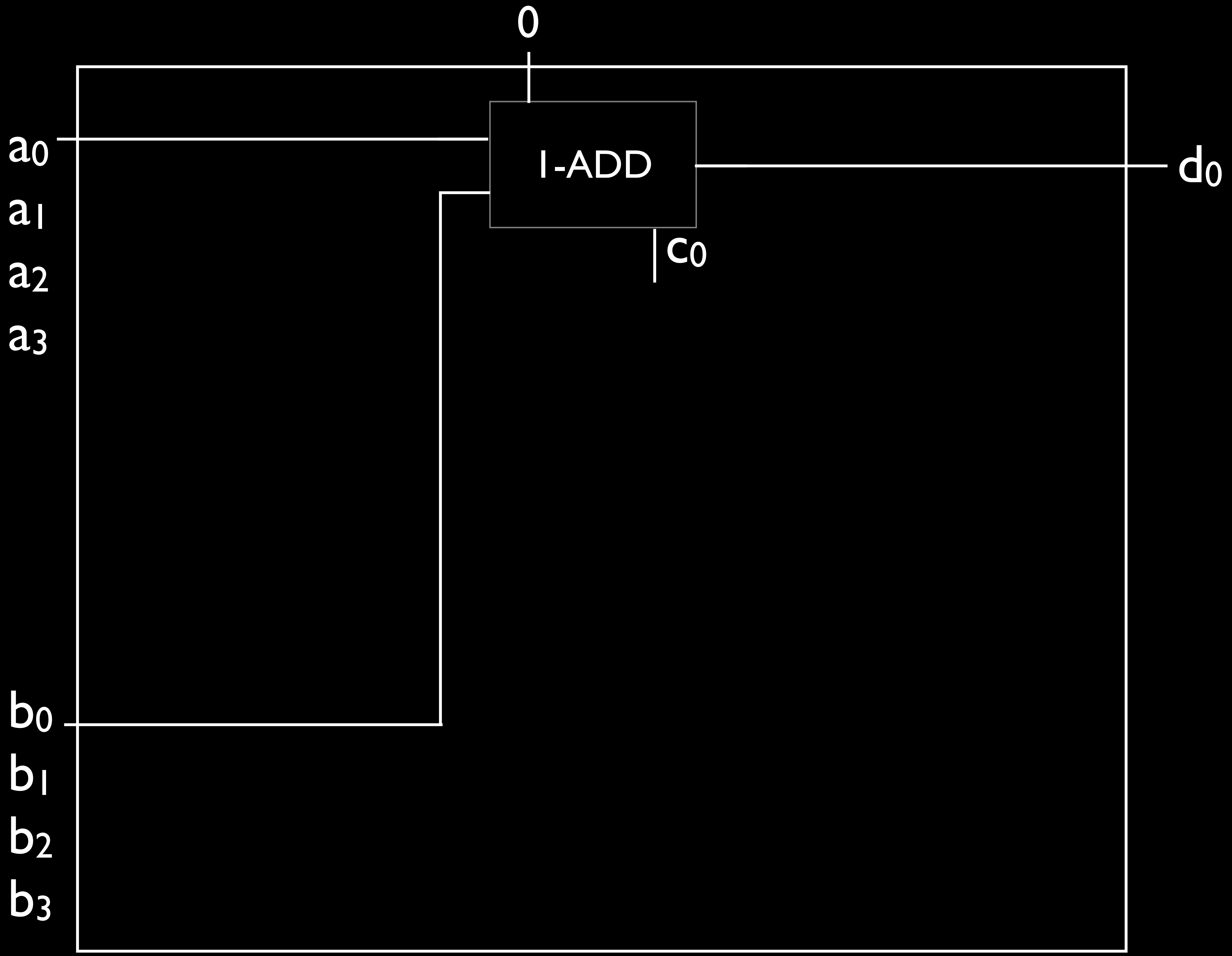
4-Bit Adder



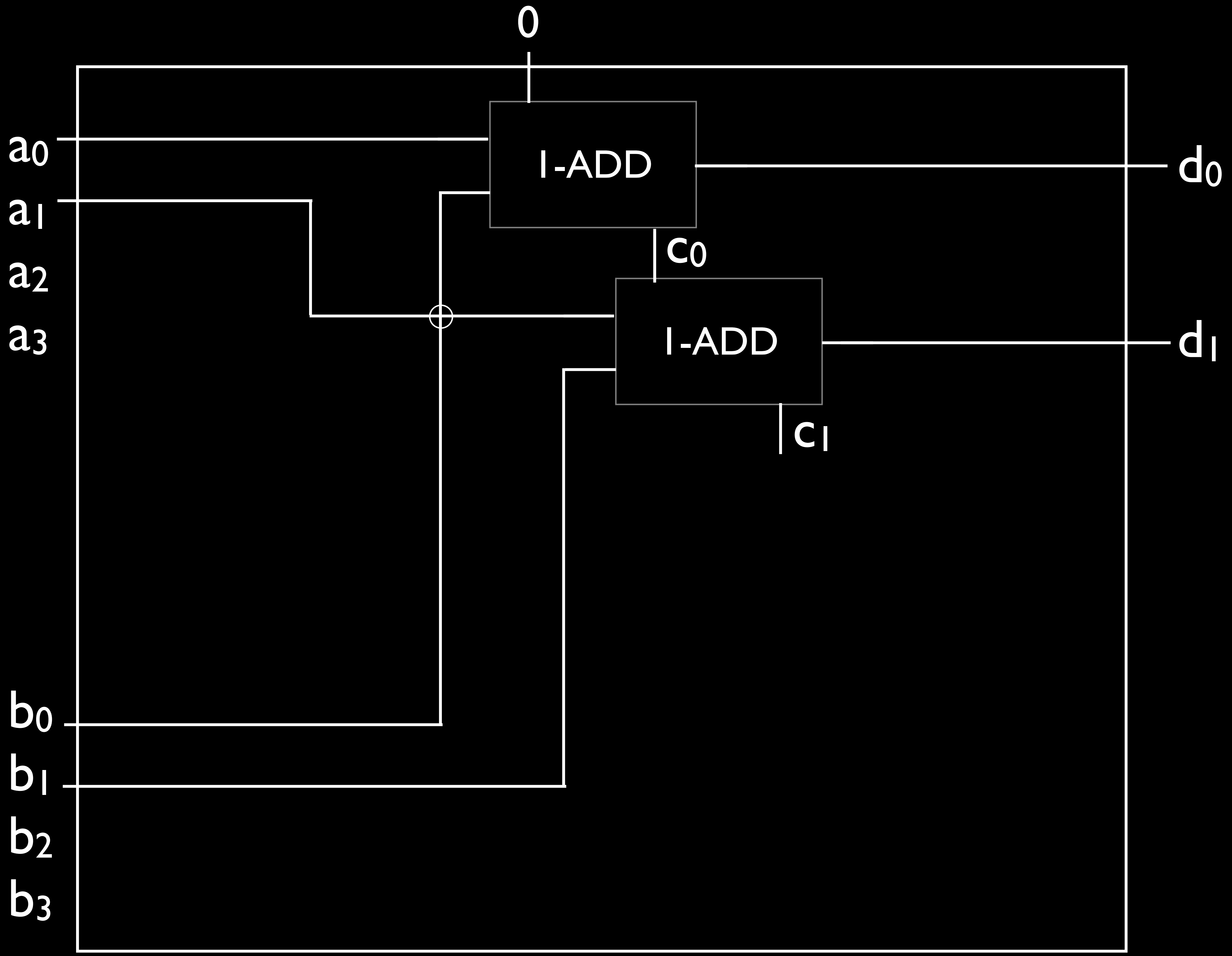
4-Bit Adder



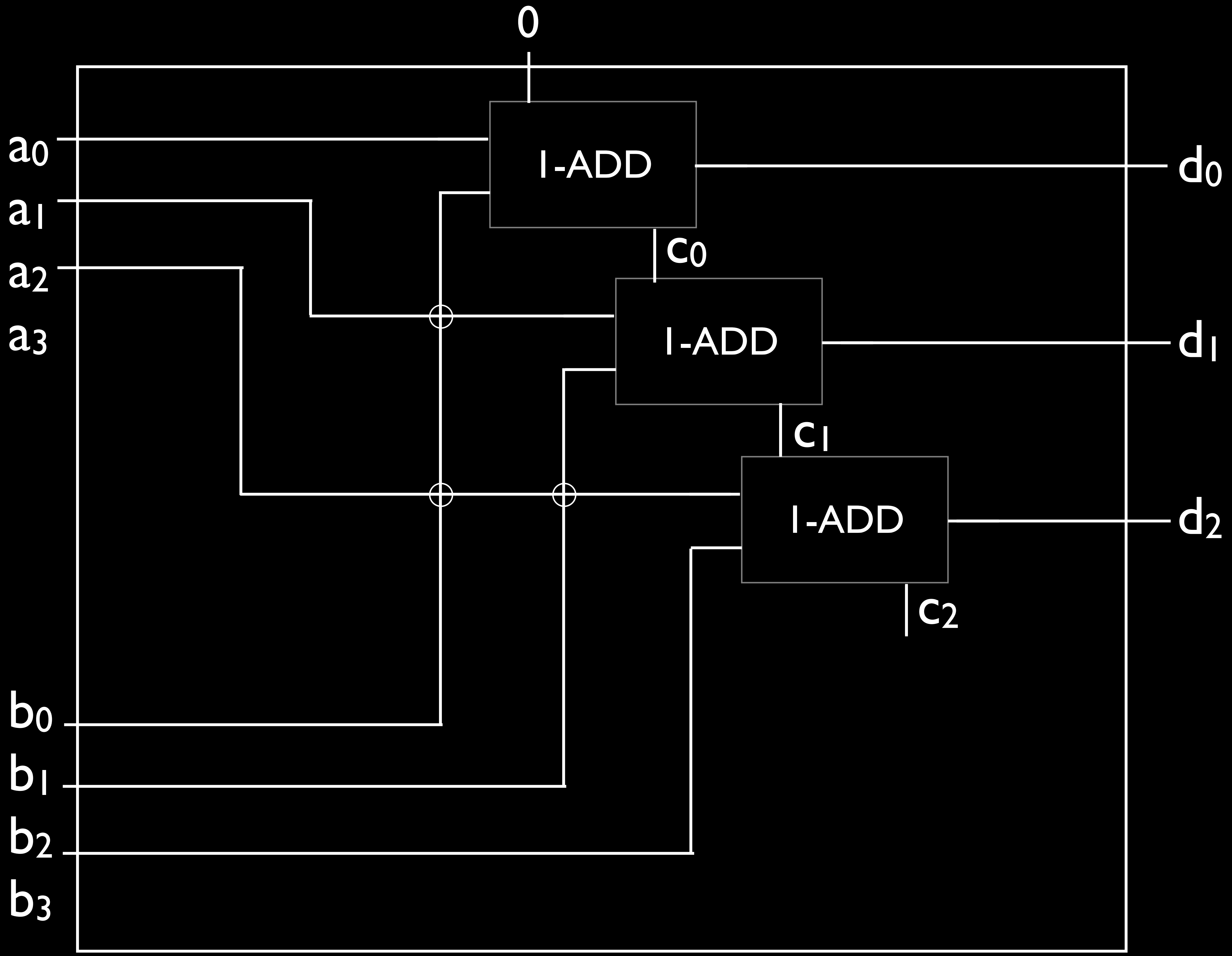
4-Bit Adder



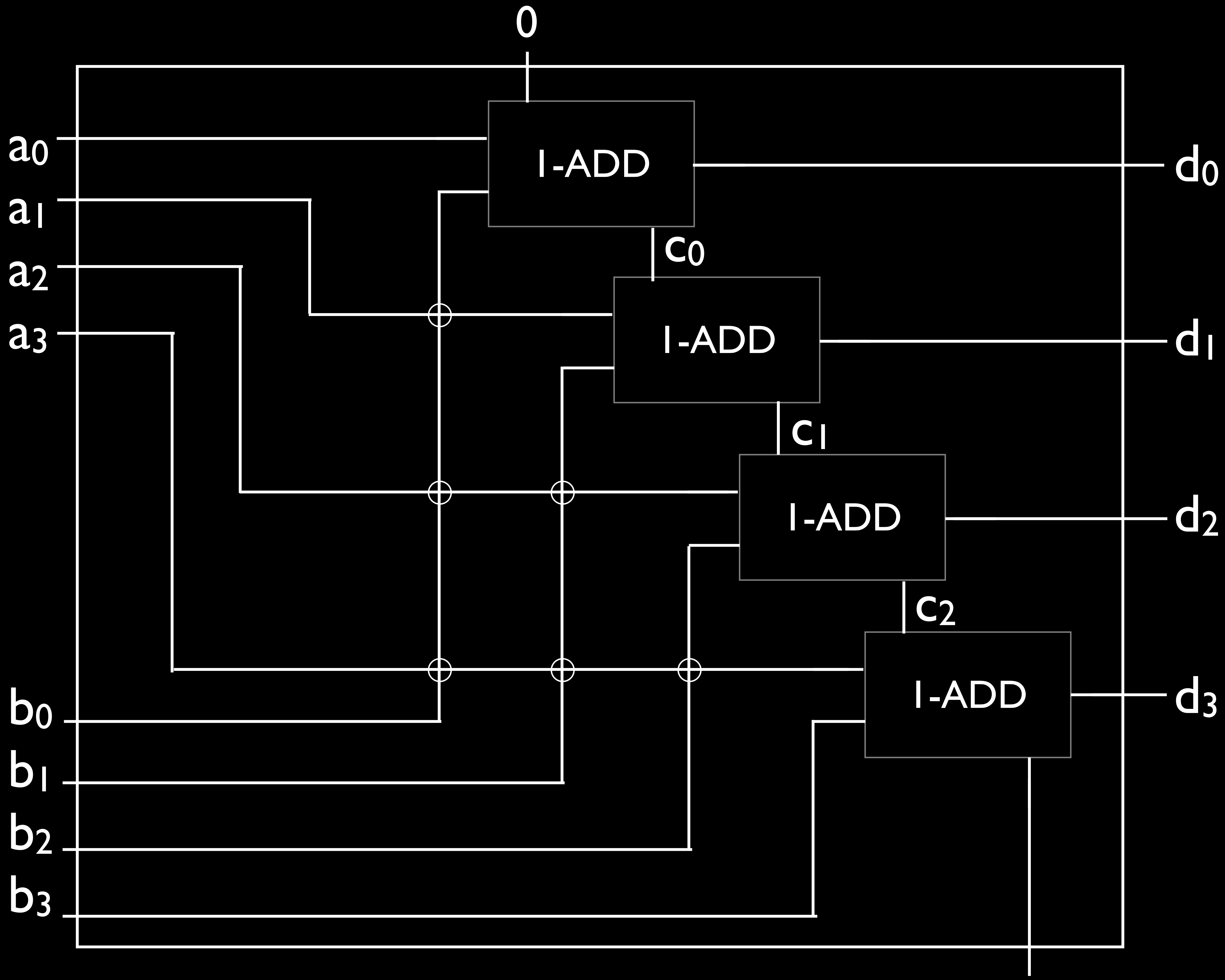
4-Bit Adder



4-Bit Adder



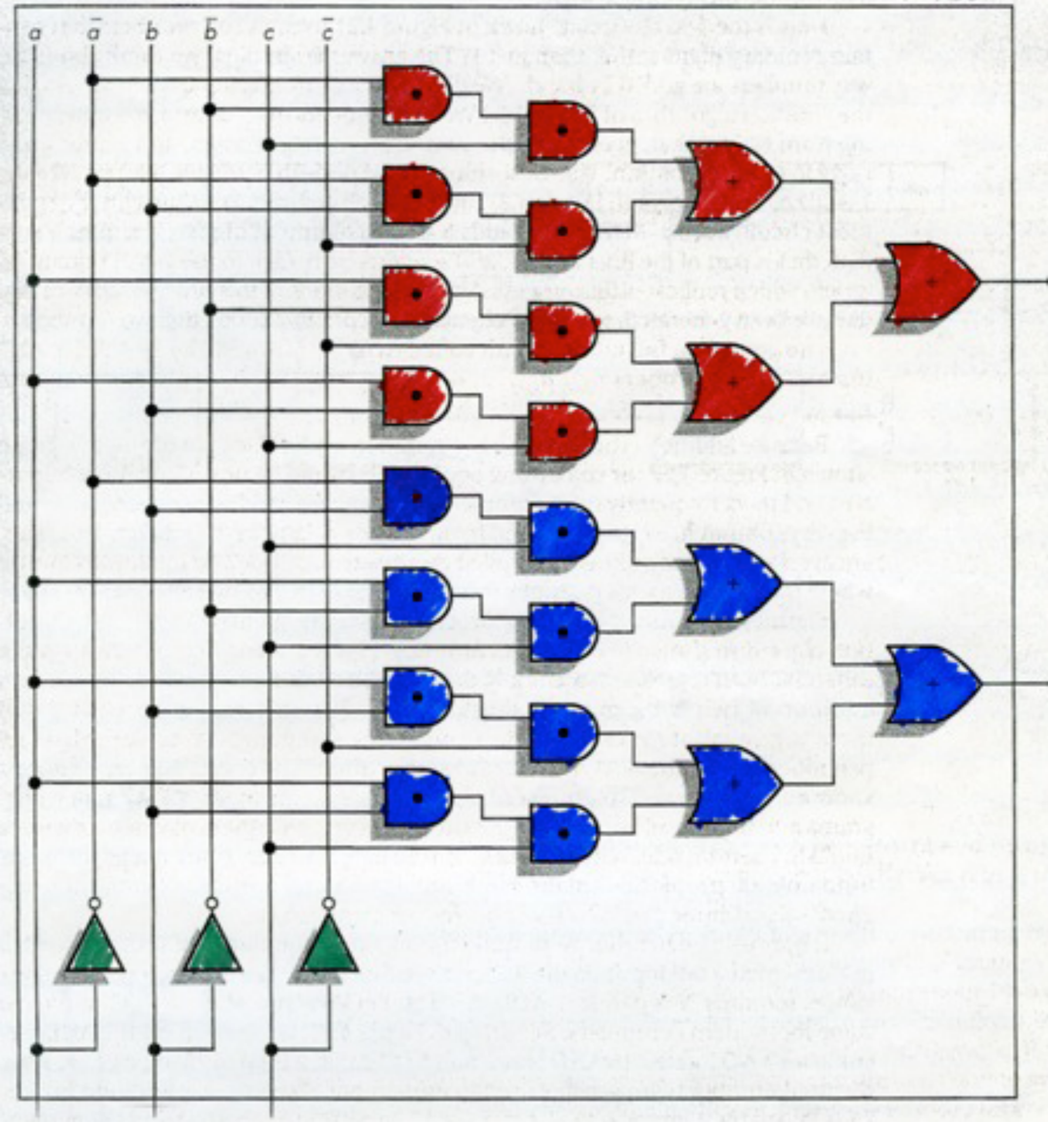
4-Bit Adder



32-bit Adder

96 = 32x3 *not* gates
512 = 32x16 *and* gates
192 = 32x6 *or* gates

800 gates

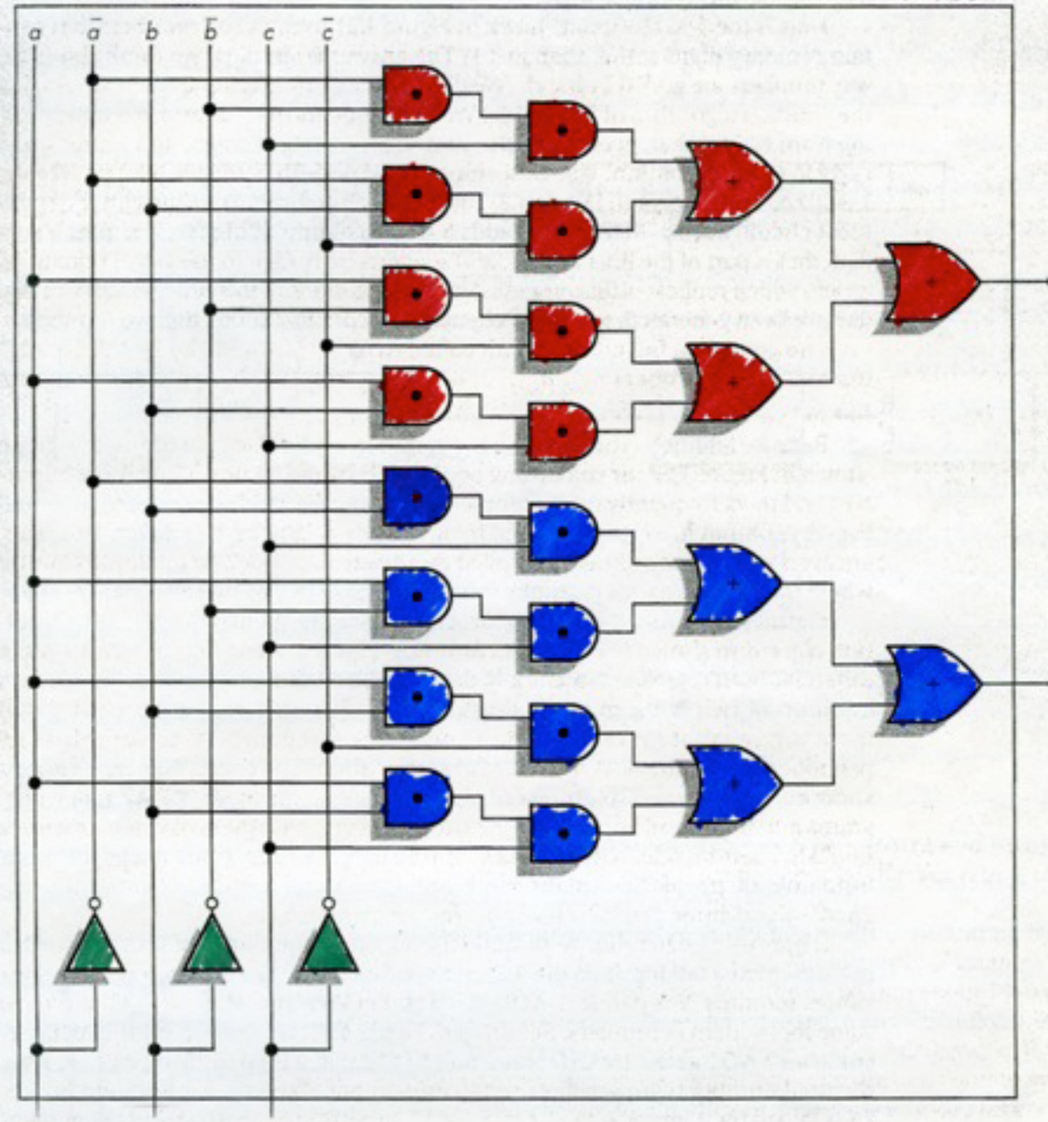


32-bit Adder

96 = 32x3 *not* gates
512 = 32x16 *and* gates
192 = 32x6 *or* gates

800 gates

1,504 = 96+1024+384 transistors



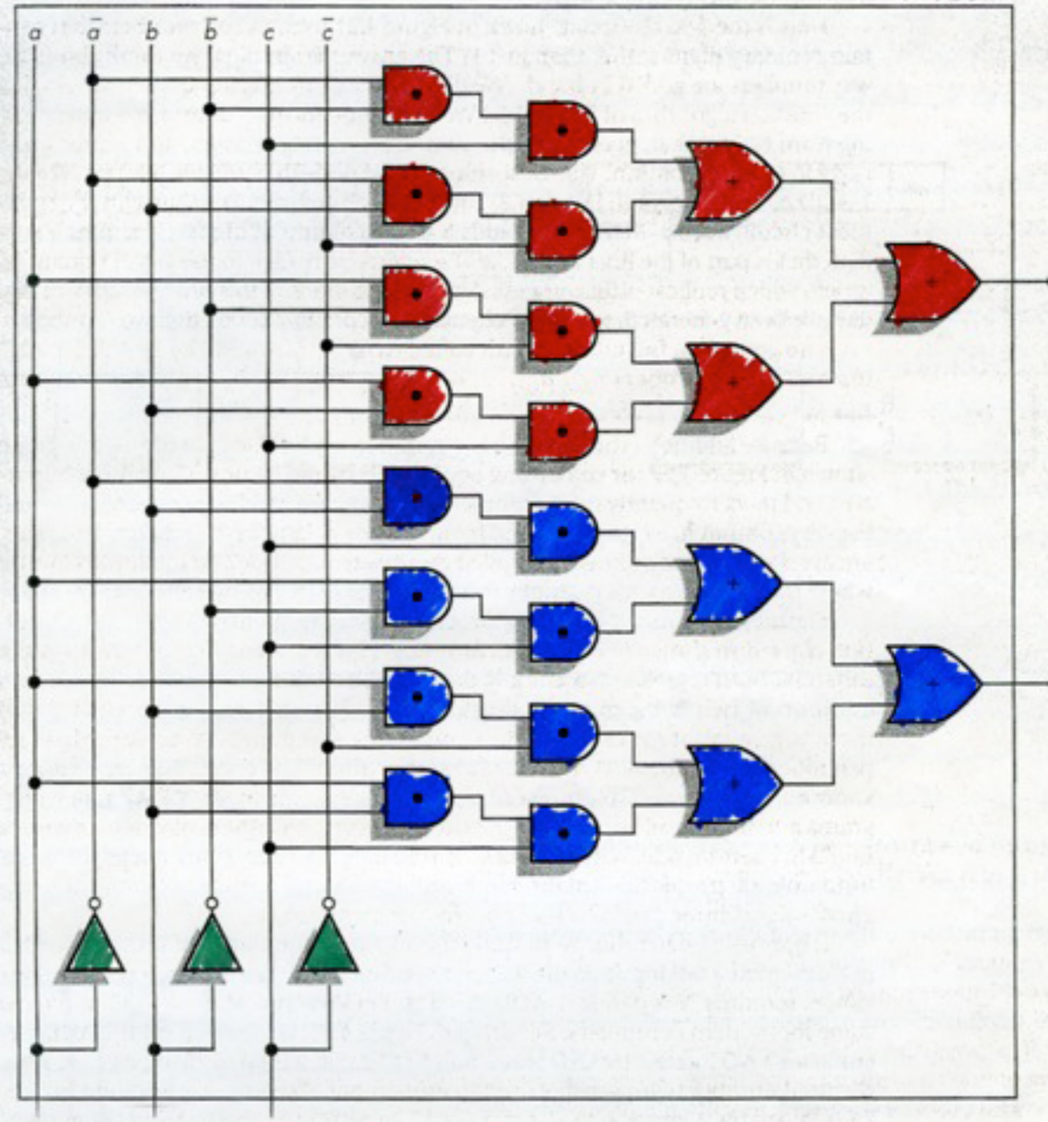
32-bit Adder

96 = 32x3 *not* gates
512 = 32x16 *and* gates
192 = 32x6 *or* gates

800 gates

1,504 = 96+1024+384 transistors

1945: refrigerator-sized computer
2015: .-sized computer



Equals 0

build a circuit that determines if an 8-bit number is 0

Equals 0

$$a_7\ a_6\ a_5\ a_4\ a_3\ a_2\ a_1\ a_0 = 00000000$$

Equals 0

output

a	b	c	d	e	f	g	h	i

⋮ ⋮

How many rows?

Equals 0

a	b	c	d	e	f	g	h	i
0	0	0	0	0	0	0	0	1

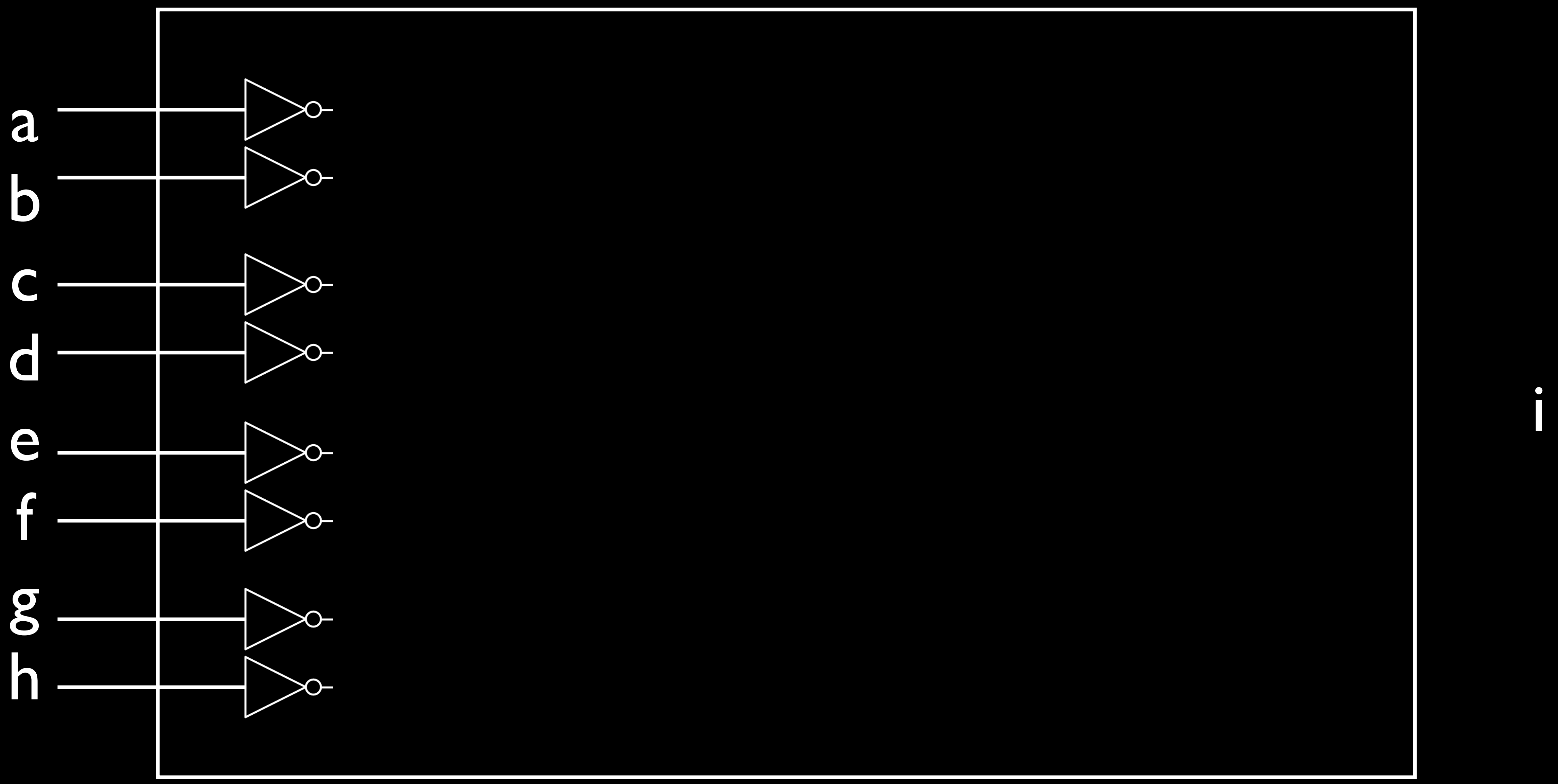
Only one row has an output of 1

Equals 0

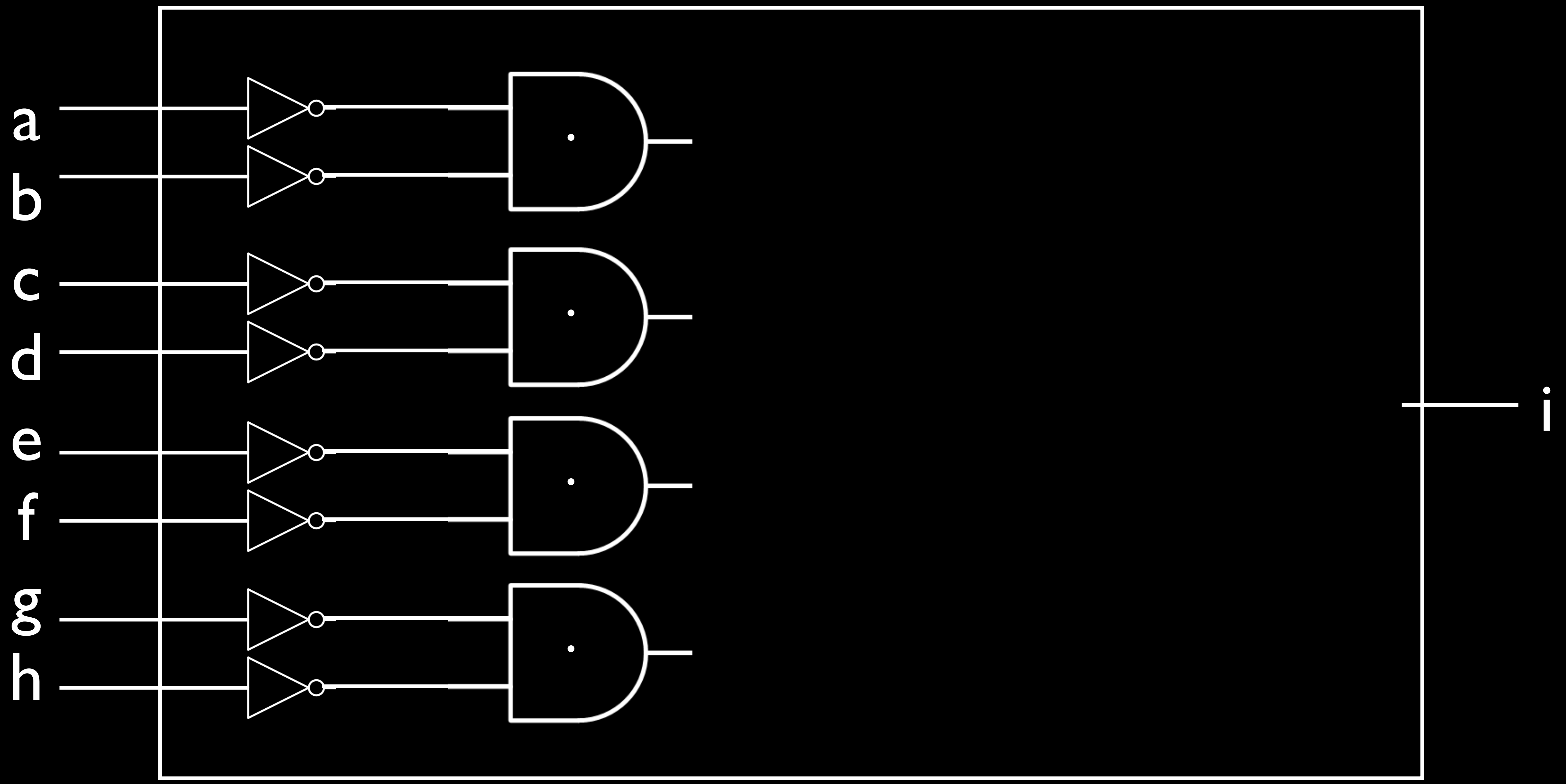
a	b	c	d	e	f	g	h	i
0	0	0	0	0	0	0	0	1

$$i = a' \cdot b' \cdot c' \cdot d' \cdot e' \cdot f' \cdot g' \cdot h'$$

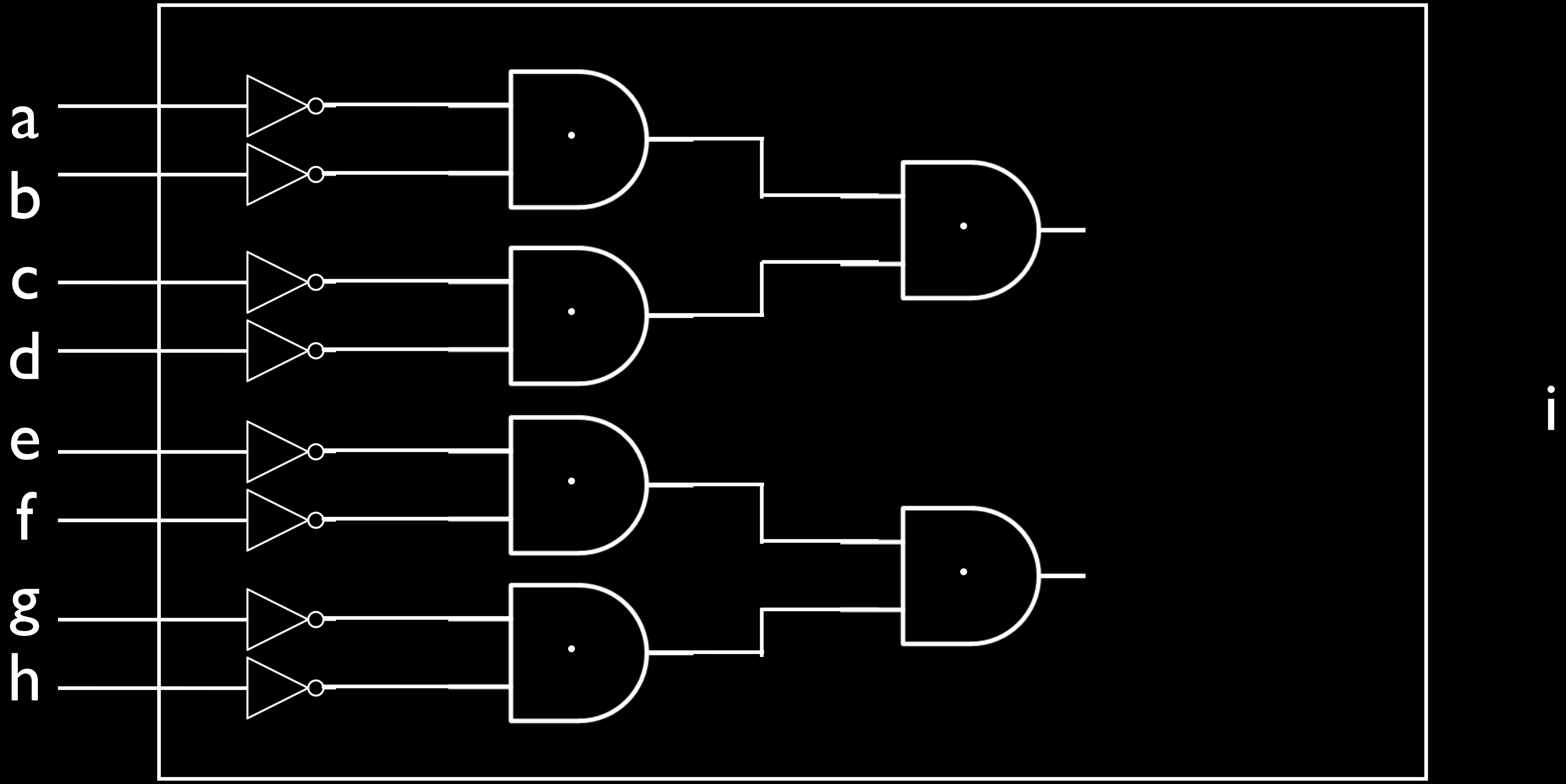
Equals 0



Equals 0



Equals 0



Equals 0