Binary Numbers

723

Binary Numbers

$$723 = 7 \times 100 + 2 \times 10 + 3 \times 1$$

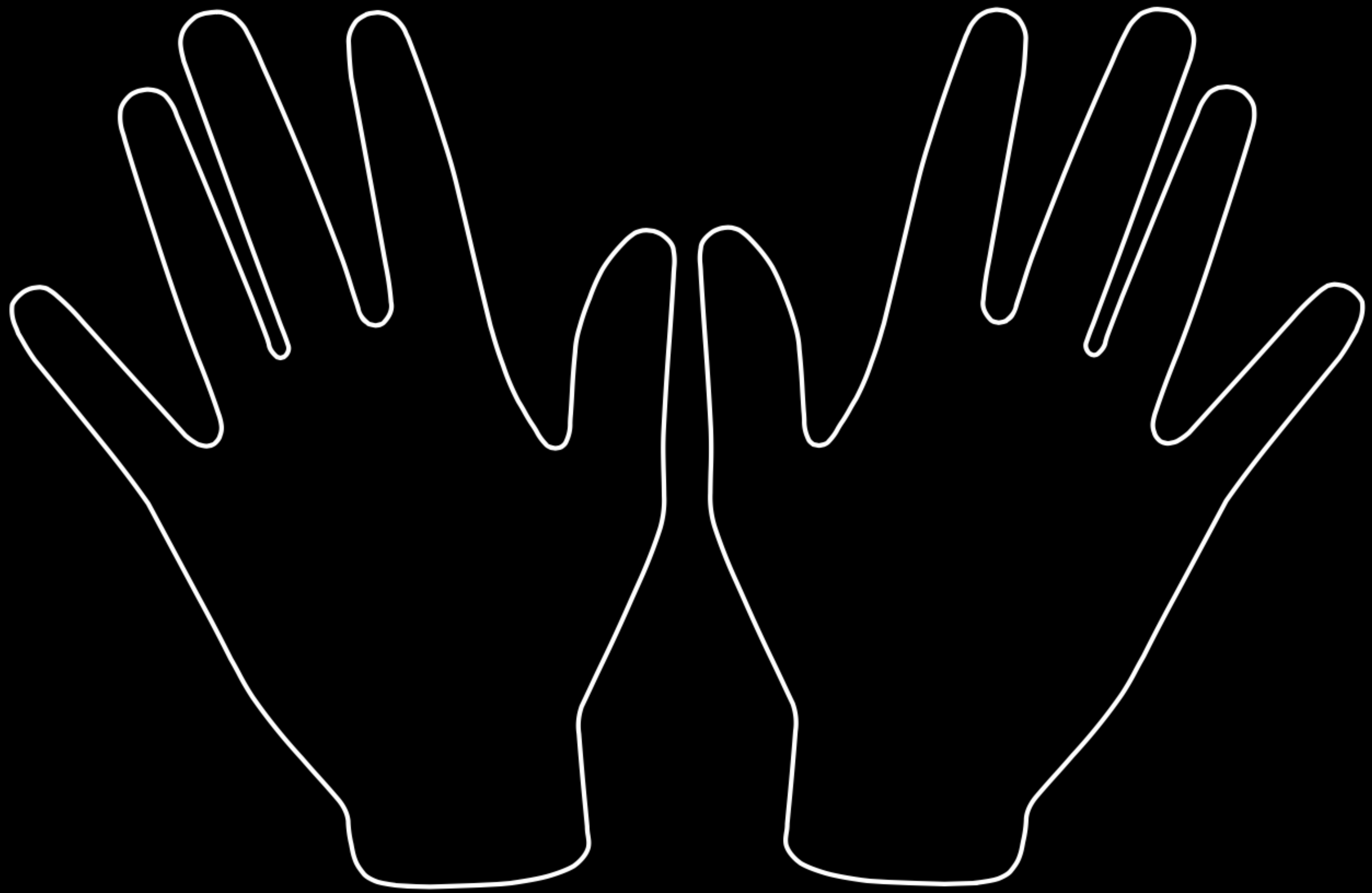
Binary Numbers

$$
\begin{aligned}
723 &= 7 \times 100 + 2 \times 10 + 3 \times 1 \\
&= 7 \times 10^2 + 2 \times 10^1 + 3 \times 10^0
\end{aligned}
$$

Binary Numbers

$$5349 = 5 \times 10^3 + 3 \times 10^2 + 4 \times 10^1 + 9 \times 10^0$$

Binary Numbers



Why base 10?

Binary Numbers

257 (base 8)

$2 \times 8^2 + 5 \times 8^1 + 7 \times 8^0$

$2 \times 64 + 5 \times 8 + 7 \times 1$

175 (base 10)

Binary Numbers

0110 (base 2)

$0 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 0 \times 2^0$

$0 \times 8 + 1 \times 4 + 1 \times 2 + 0 \times 1$

6 (base 10)

Binary Numbers

$2^1 2^0$

00	0
01	1
10	2
11	3

max value = $2^2 - 1$

2-bit binary number

Binary Numbers

000	0
001	1
010	2
011	3
100	4
101	5
110	6
111	7

max value = $2^3 - 1$

3-bit binary number

Binary Numbers

0000	0
0001	1
0010	2
0011	3
0100	4
0101	5
0110	6
0111	7
1000	8
1001	9
1010	10
1011	11
1100	12
1101	13
1110	14
1111	15

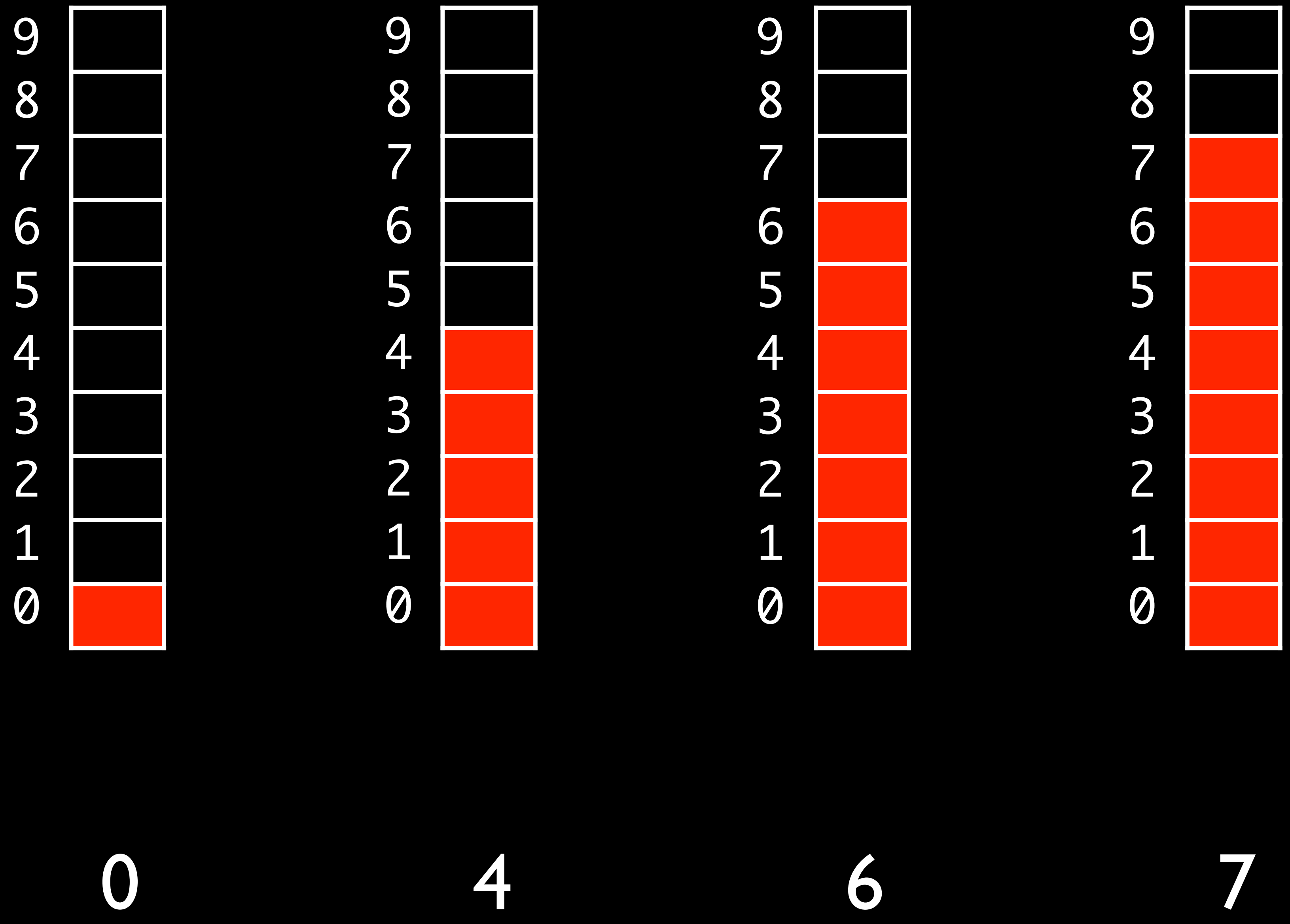
max value = $2^4 - 1$

4-bit binary number

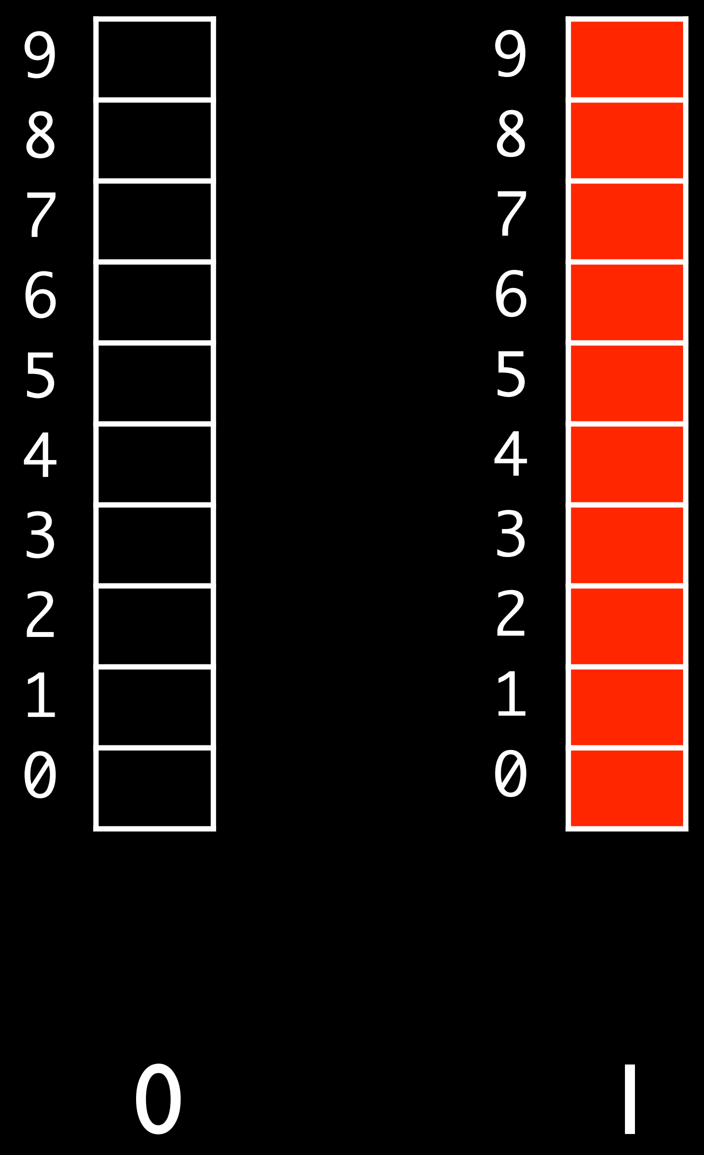
Binary Numbers (why?)

reliability!

Binary Numbers (why?)



Binary Numbers (why?)



Binary Numbers

How do we encode negative numbers?

use left-most bit to represent sign

0 = “+”

1 = “-”

Binary Numbers

sign $2^1 2^0$

000	0
001	1
010	2
011	3
100	
101	
110	
111	

3-bit *signed* binary number

Binary Numbers

sign	2^1	2^0	
0	0	0	0
0	0	1	1
0	1	0	2
0	1	1	3
1	0	0	-0 ???
1	0	1	-1
1	1	0	-2
1	1	1	-3

3-bit *signed* binary number

Binary Numbers (two's complement)

1. start with an unsigned 4-bit binary number where left-most bit is 0
   - `0110 = 6`

Binary Numbers (two’s complement)

1. start with an unsigned 4-bit binary number where left-most bit is 0
   - `0110 = 6`
2. complement your binary number (flip bits)
   - `1001`

Binary Numbers (two's complement)

1. start with an unsigned 4-bit binary number where left-most bit is 0
    - `0110 = 6`
2. complement your binary number (flip bits)
    - `1001`
3. add one to your binary number
    - `1010 = -6`

Binary Numbers (two's complement)

positive		complement	+1	negative
0	000			0
1	001			-1
2	010			-2
3	011			-3

3-bit *signed* binary number

Binary Numbers (two's complement)

positive		complement	+1	negative
0	000			0
1	001	→ 110	→ 111	-1
2	010			-2
3	011			-3

3-bit *signed* binary number

Binary Numbers (two's complement)

positive		complement	+1	negative
0	000			0
1	001	→ 110	→ 111	-1
2	010	→ 101	→ 110	-2
3	011			-3

3-bit *signed* binary number

Binary Numbers (two's complement)

positive		complement	+1	negative
0	000			0
1	001	→ 110	→ 111	-1
2	010	→ 101	→ 110	-2
3	011	→ 100	→ 101	-3

3-bit *signed* binary number

Binary Numbers (two's complement)

positive		complement	+1	negative
0	000	→ 111	→	0
1	001	→ 110	→ 111	-1
2	010	→ 101	→ 110	-2
3	011	→ 100	→ 101	-3

3-bit *signed* binary number

Binary Numbers (two's complement)

positive		complement	+1	negative	
0	000 →	111 →	000	0	!!!
1	001 →	110 →	111	-1	
2	010 →	101 →	110	-2	
3	011 →	100 →	101	-3	

3-bit *signed* binary number

Binary Numbers (two's complement)

positive		complement	+1	negative
0	000	111	000	0
1	001	110	111	-1
2	010	101	110	-2
3	011	100	101	-3

we lost a number?

Binary Numbers (two's complement)

positive			complement	+1	negative
0	000	→	111	→ 000	0
1	001	→	110	→ 111	-1
2	010	→	101	→ 110	-2
3	011	→	100	→ 101	-3
				100	

we lost a number?

Binary Numbers (two's complement)

complement -1

$\longleftarrow \quad \longleftarrow$ `100`

Binary Numbers (two's complement)

complement		-1	
←	011	←	100

010	-2
011	-1
100	
101	+1
110	+2

Binary Numbers (two's complement)

$$100 \xleftarrow{\text{complement}} 011 \xleftarrow{-1} 100$$

Binary Numbers (two's complement)

complement -1

4 100 ⟵ 011 ⟵ 100

Binary Numbers (two's complement)

positive		complement	+1	negative
0	000	→ 111	→ 000	0
1	001	→ 110	→ 111	-1
2	010	→ 101	→ 110	-2
3	011	→ 100	→ 101	-3
			100	-4

n-bit *unsigned* binary numbers: $0 \ldots 2^n - 1$

Binary Numbers (two's complement)

positive		complement	+1	negative
0	000 →	111 →	000	0
1	001 →	110 →	111	-1
2	010 →	101 →	110	-2
3	011 →	100 →	101	-3
			100	-4

n-bit *signed* binary numbers: $-2^{n-1} \ldots 2^{n-1}-1$

Binary Numbers (two's complement)

```
  0010      2
  0010      2
+ ----    + -
  0100      4
```

summing unsigned binary numbers is easy

Binary Numbers (two's complement)

```
  0010      2
  1010     -2
+ ----    + -
  1100      0      ?
```

summing signed binary numbers

Binary Numbers (two's complement)

```
  0011      3
  1011     -3
+ ----    + -
  1110      0      ?
```

summing signed binary numbers

Binary Numbers (two's complement)

```
0010 -> 1101 -> 1110

       0010      2
                -2
     + ----   + -
                 0
```

summing signed (2's complement) binary numbers

Binary Numbers (two's complement)

```
0010 -> 1101 -> 1110
```

```
  0010       2
  1110      -2
+ ----     + -
  0000       0
```

summing signed (2's complement) binary numbers

Binary Numbers (two's complement)

```
0011 -> 1100 -> 1101

               0011        3
                          -3
             + ----     + -
                           0
```

summing signed (2's complement) binary numbers

Binary Numbers (two's complement)

```
0011 -> 1100 -> 1101
```

```
  0011      3
  1101     -3
+ ----    + -
  0000      0
```

summing signed (2's complement) binary numbers

Binary Numbers (decoding two's complement)

`0111 = ?`

4-bit signed (two's complement) binary number

Binary Numbers (decoding two's complement)

`0111 = 7`

4-bit signed (two's complement) binary number

Binary Numbers (decoding two's complement)

`1011 = ?`

4-bit signed (two's complement) binary number

Binary Numbers (decoding two's complement)

subtract 1

`1011    1010`

4-bit signed (two's complement) binary number

Binary Numbers (decoding two's complement)

	subtract 1	complement
1011	1010	0101

4-bit signed (two's complement) binary number

Binary Numbers (decoding two's complement)

	subtract 1	complement	
1011	1010	0101	5

4-bit signed (two's complement) binary number

Binary Numbers (decoding two's complement)

`1011 = -5`

4-bit signed (two's complement) binary number

How do we encode fractional numbers?

Binary Numbers

$$\pm \text{ mantissa} \times \text{base}^{\pm \text{ exponent}}$$

Boolean Logic (variables)

1 = True

0 = False

Boolean Logic (truth tables)

a	**b**	**a** *and* **b**
1	1	1
1	0	0
0	1	0
0	0	0

a *and* **b**

Boolean Logic (truth tables)

a	**b**	**a** *or* **b**
1	1	1
1	0	1
0	1	1
0	0	0

a *or* **b**

Boolean Logic (truth tables)

a	*not* **a**
1	0
0	1

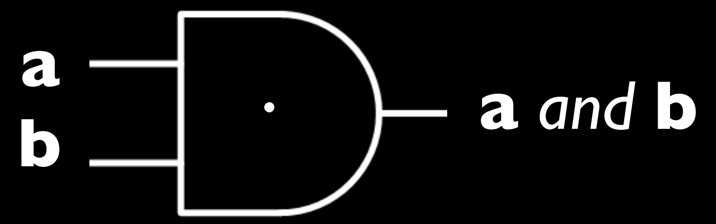
not **a**

Boolean Logic (truth tables)

input (boolean variable)	*output* (boolean variable)
a, **b**	**a** *and* **b**
	a *or* **b**
	not **a**

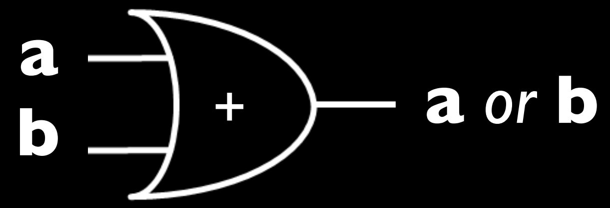
Gates

a	b	a *and* b
1	1	1
1	0	0
0	1	0
0	0	0



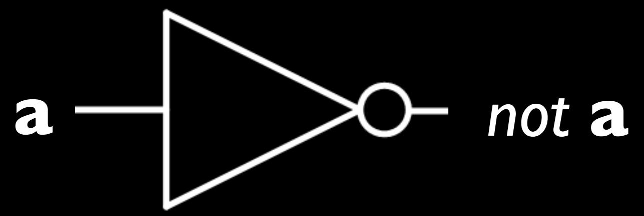
Gates

a	**b**	**a** *or* **b**
1	1	1
1	0	1
0	1	1
0	0	0

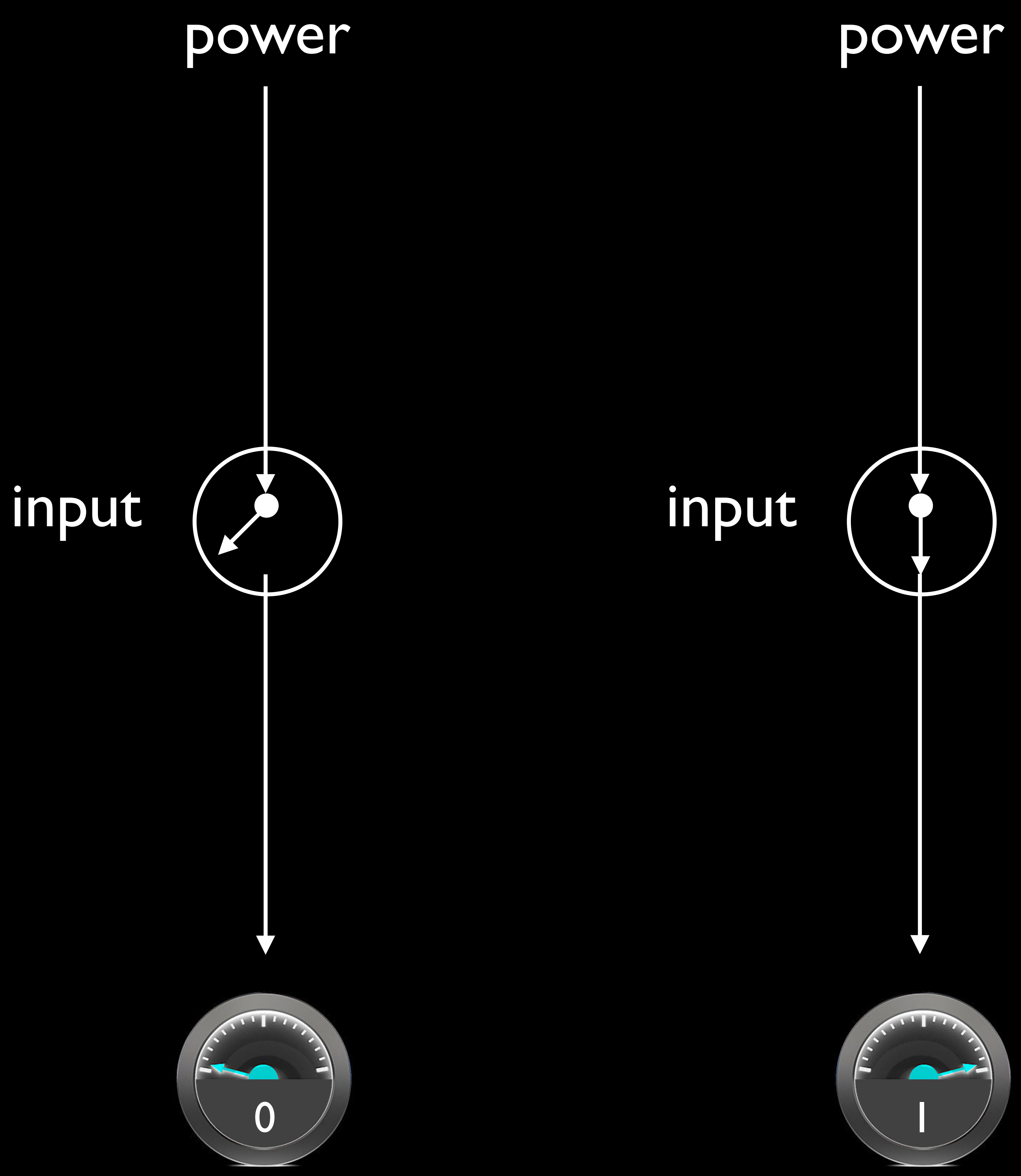


Gates

a	*not* **a**
1	0
0	1



Building Gates (transistors)



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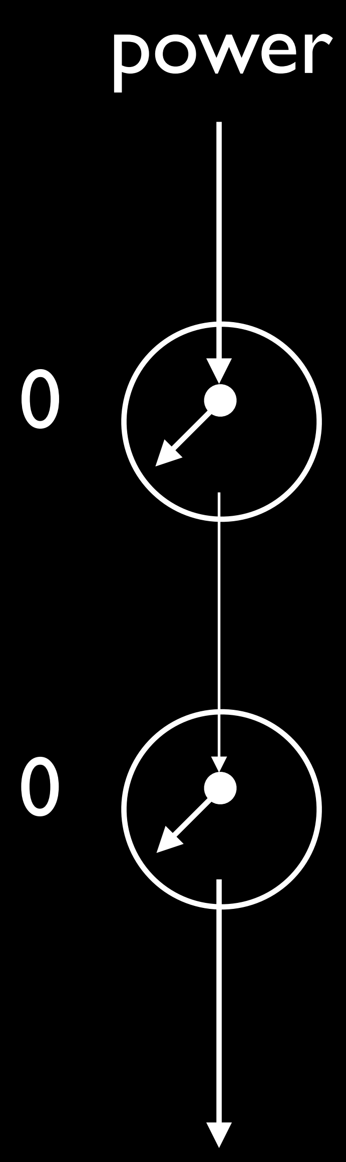

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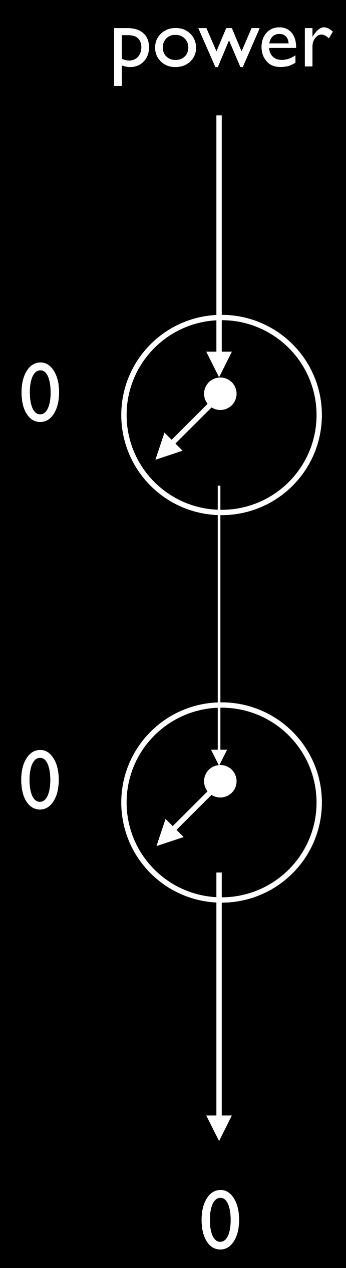

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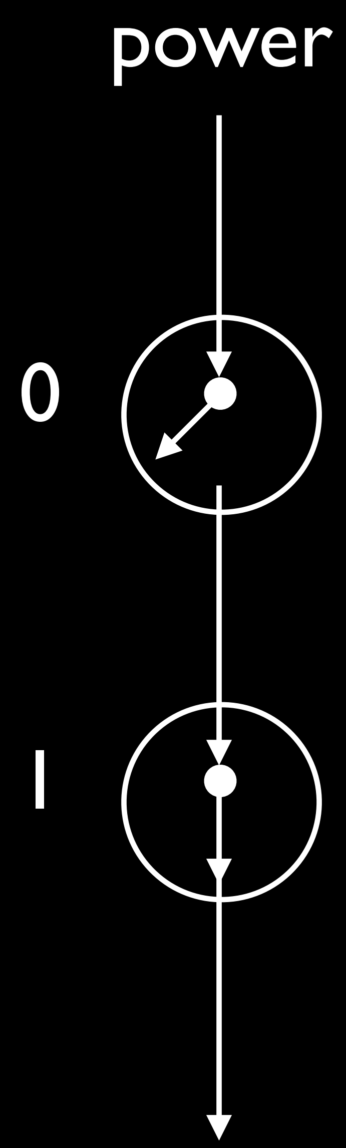
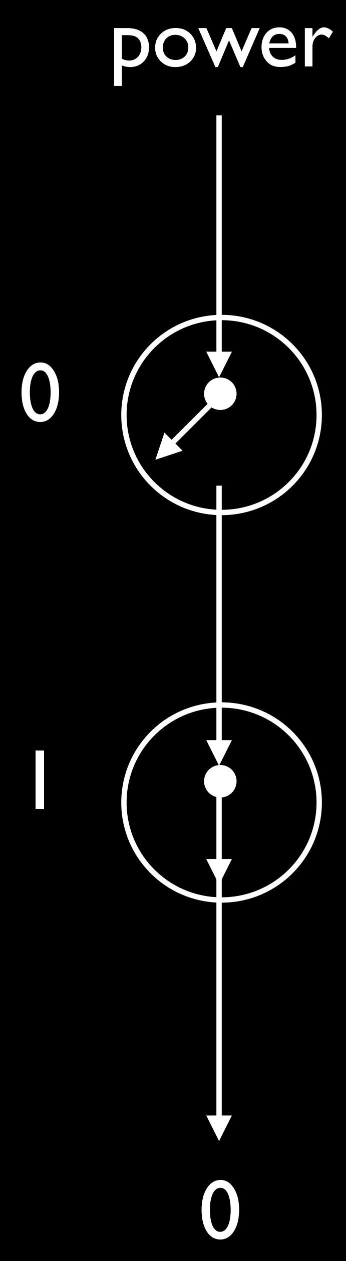


Building Gates (transistors)



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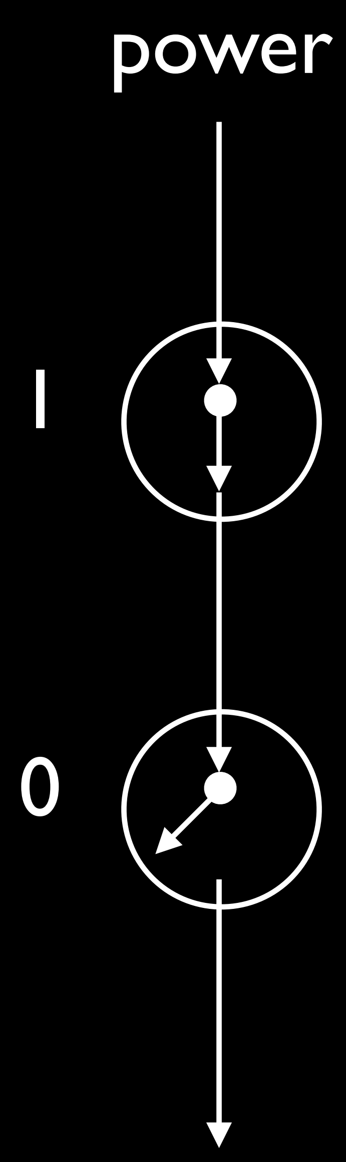
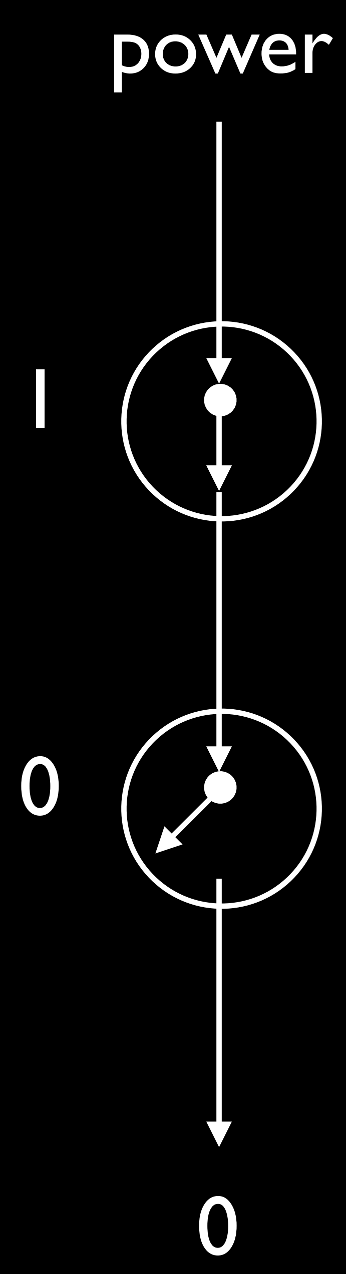
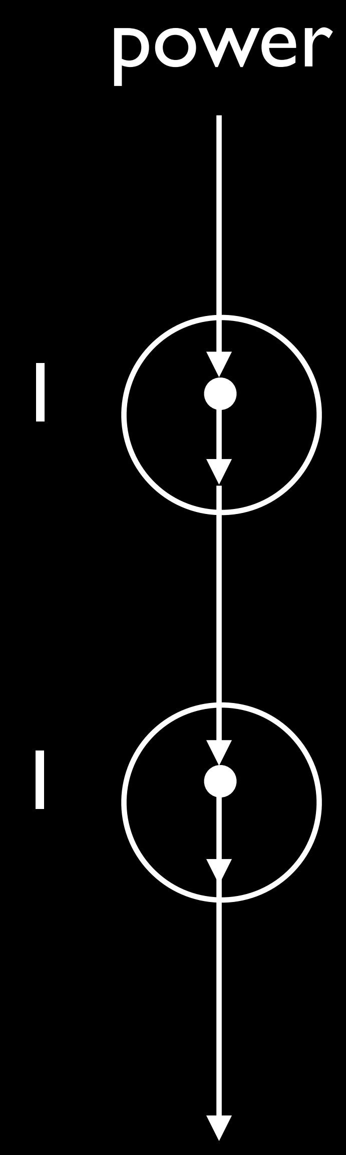


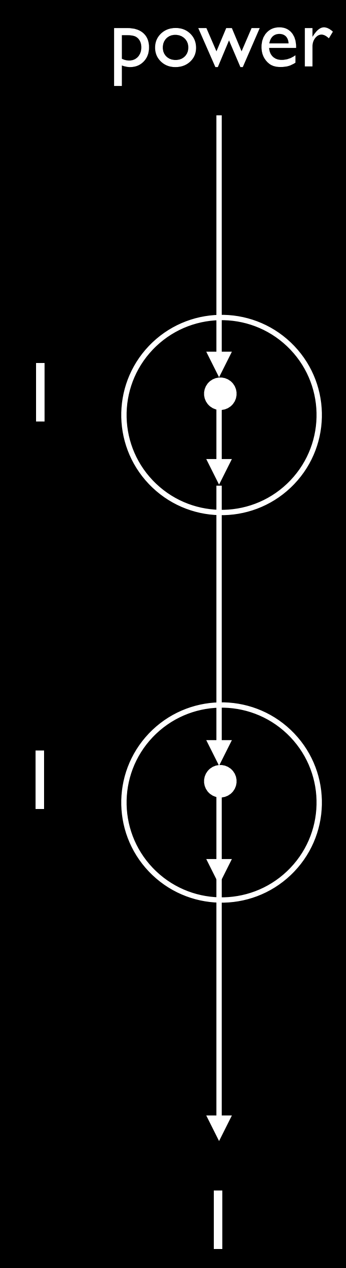
Building Gates (transistors)



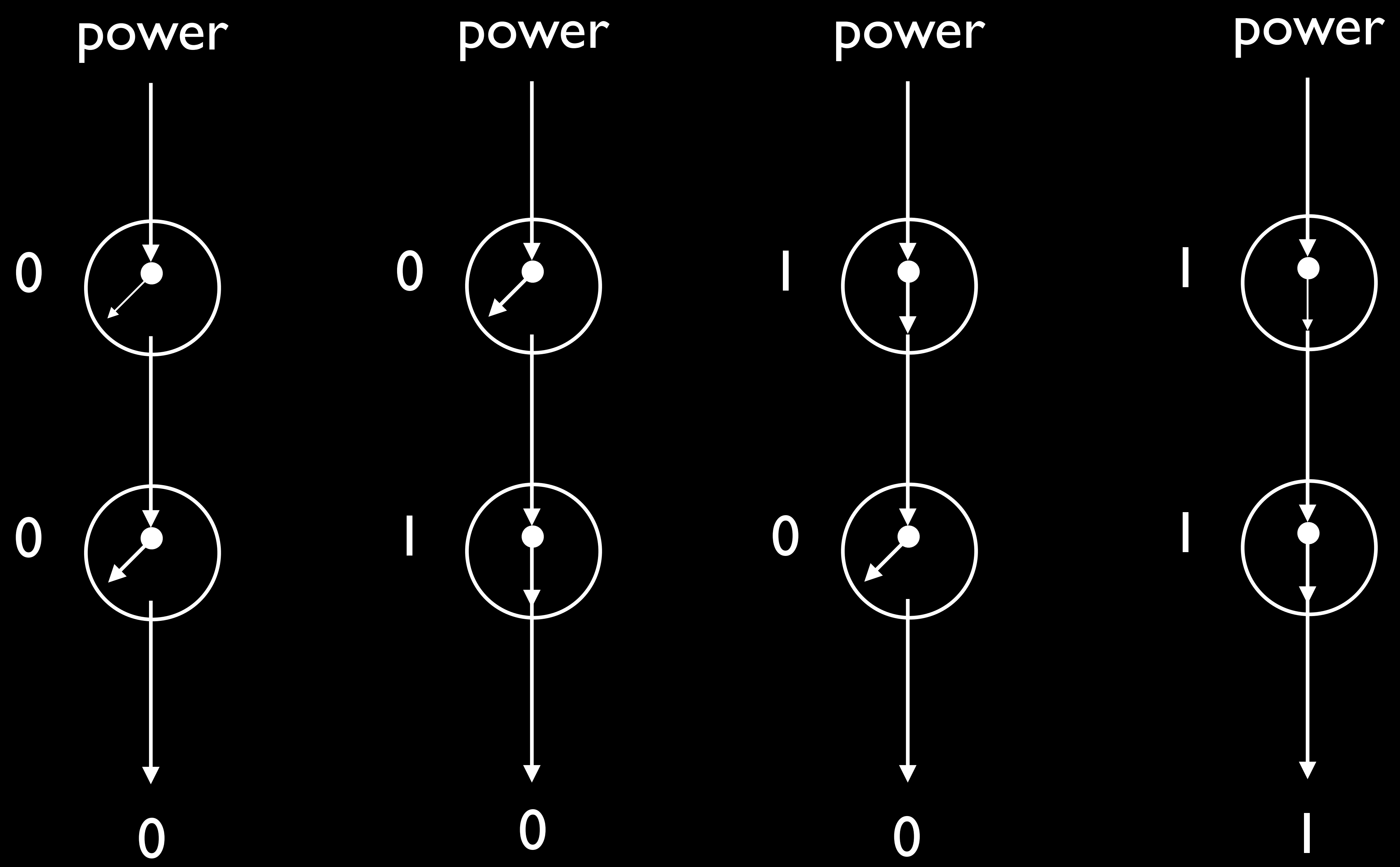
Building Gates (transistors)



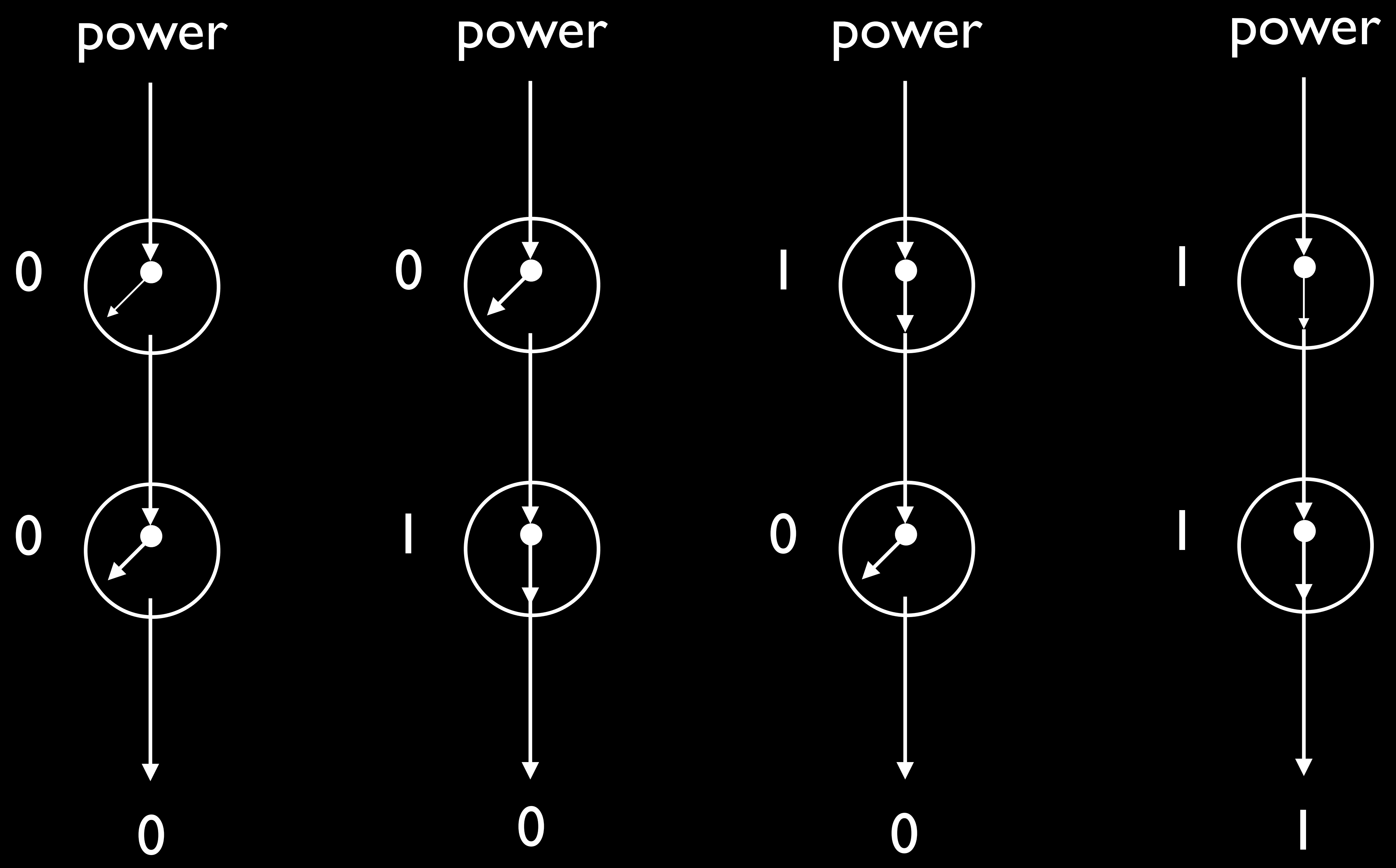
Building Gates (transistors)



Building Gates (transistors)

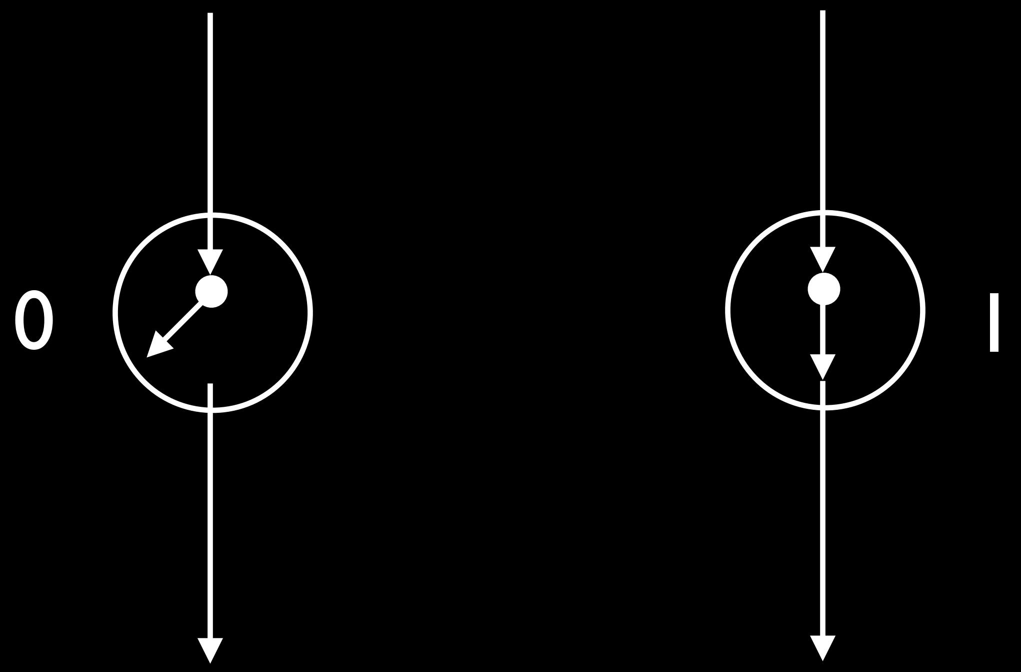


Building Gates (transistors)



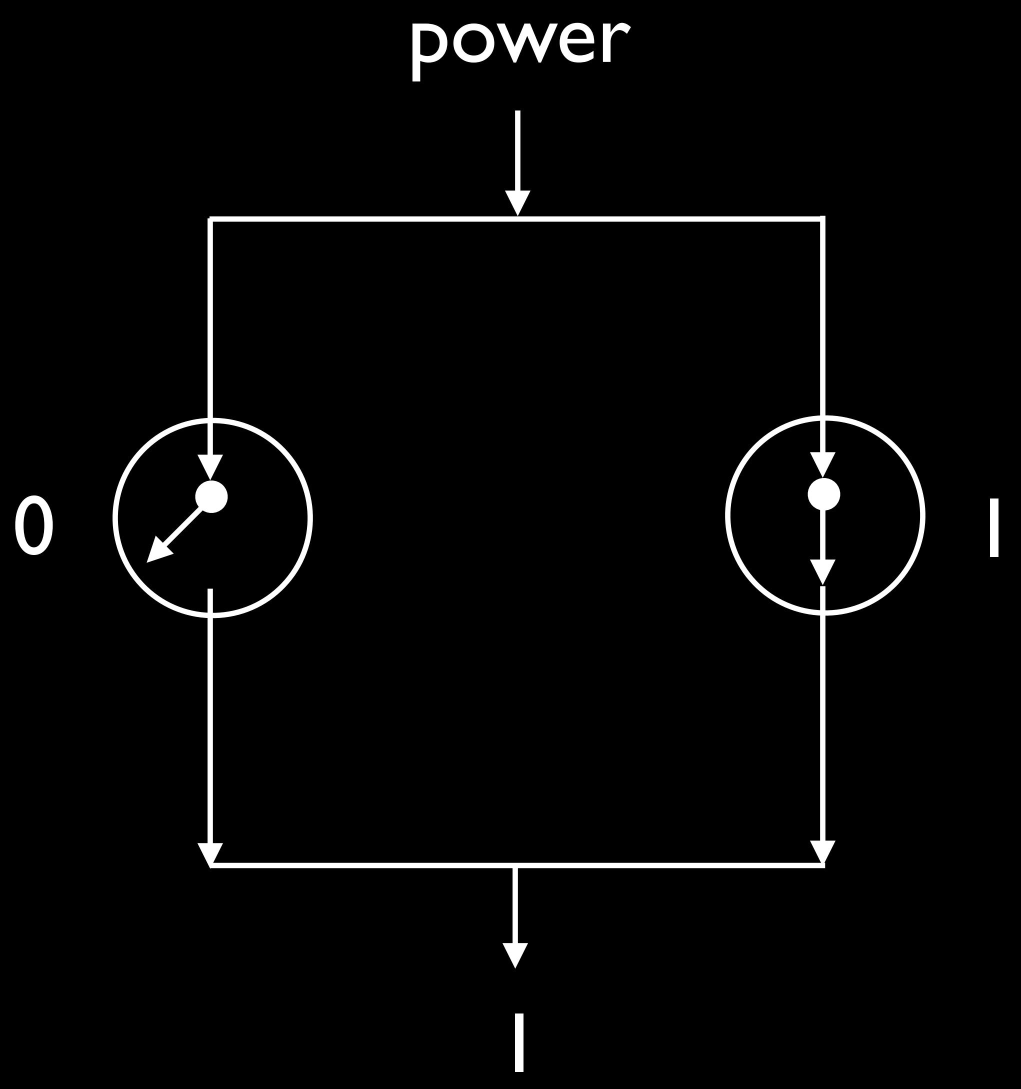
AND gate

Building Gates (transistors)



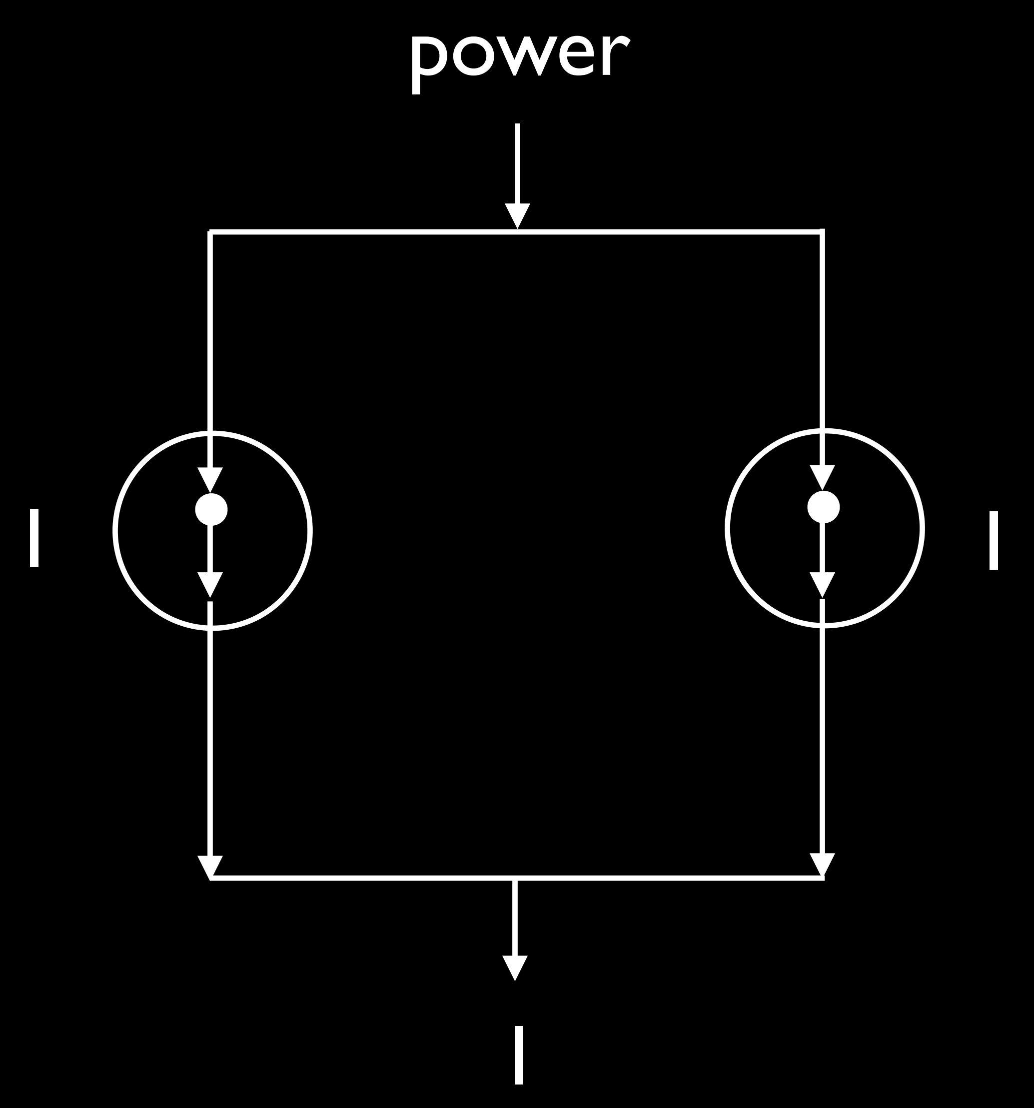
OR gate

Building Gates (transistors)



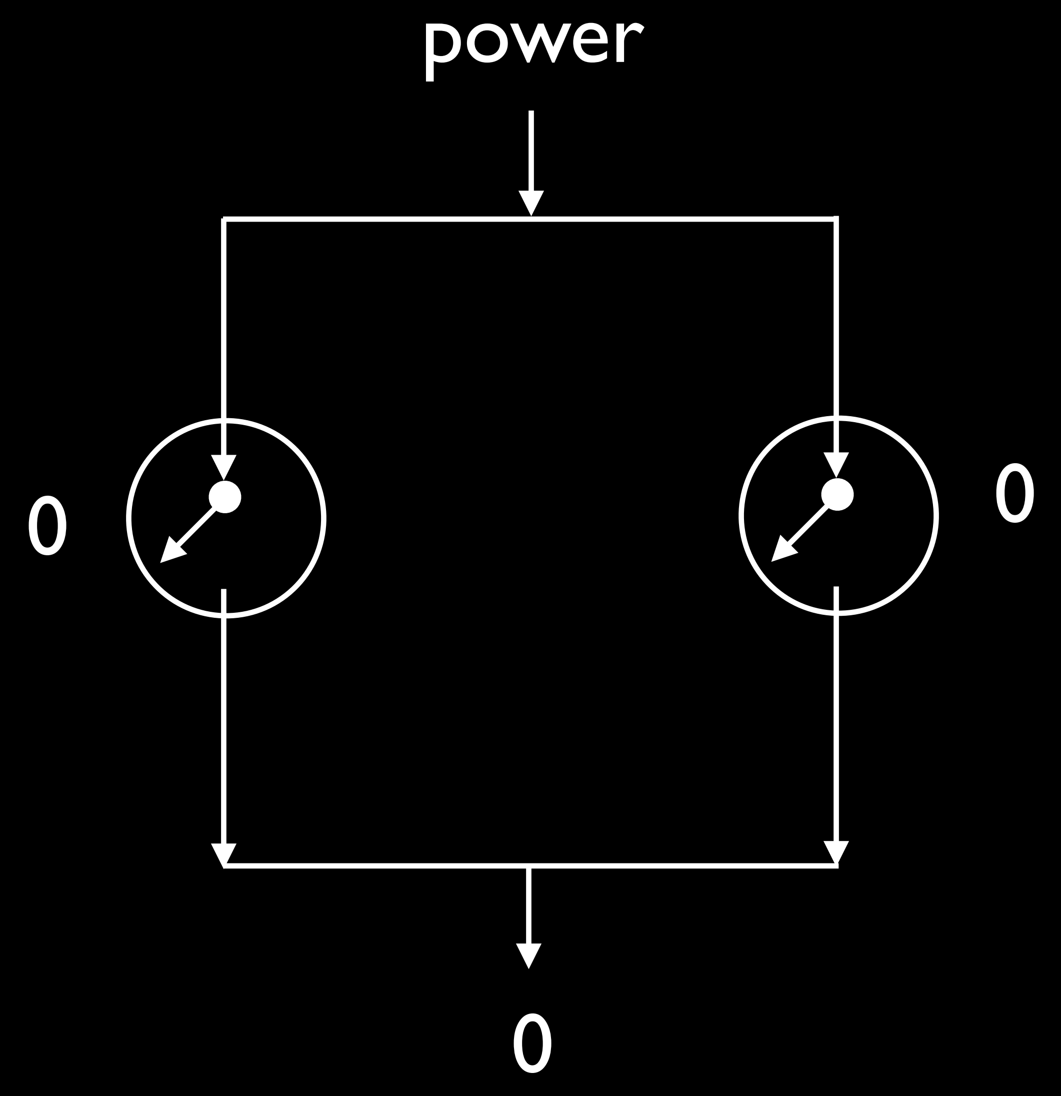
OR gate

Building Gates (transistors)



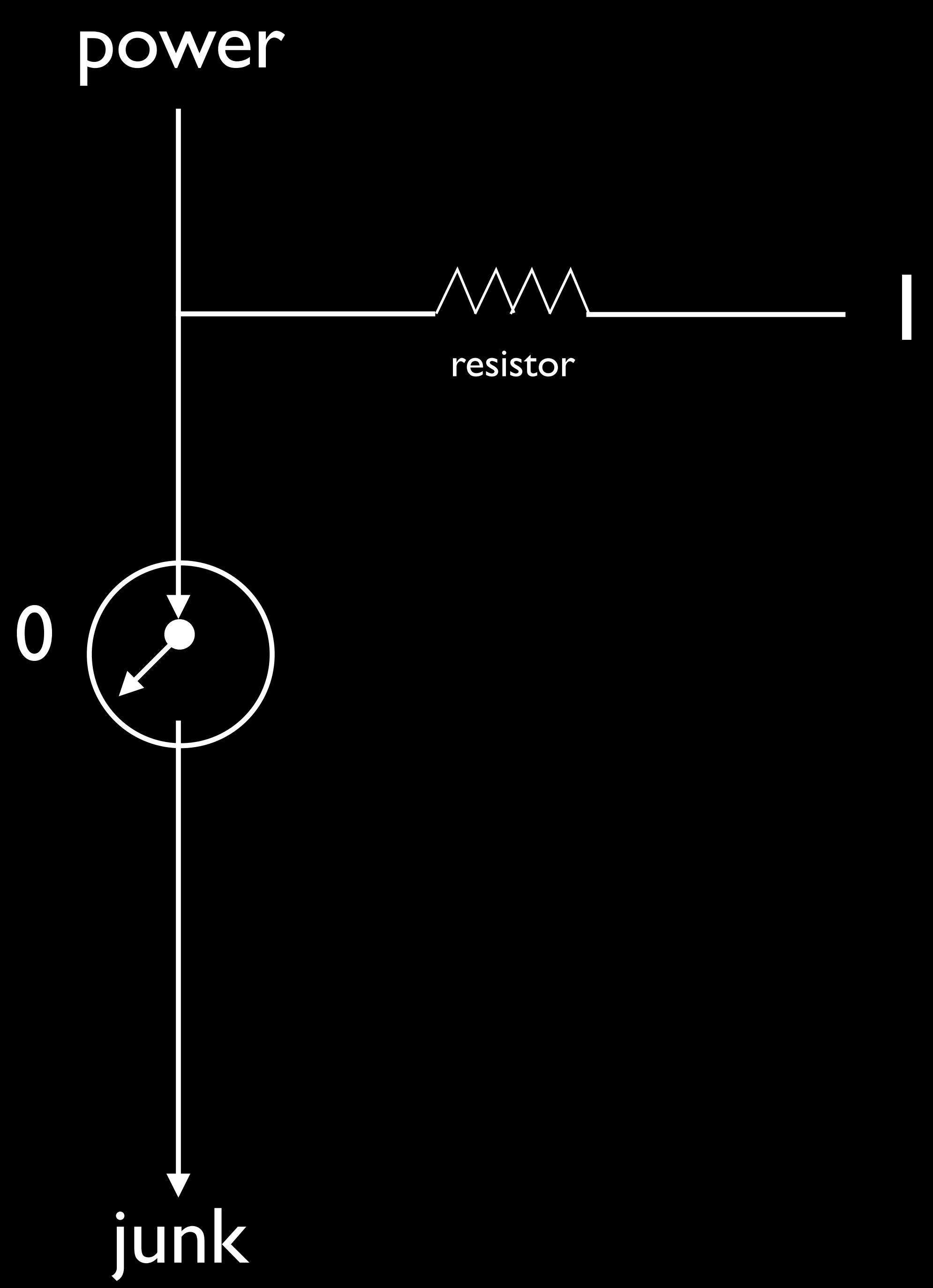
OR gate

Building Gates (transistors)



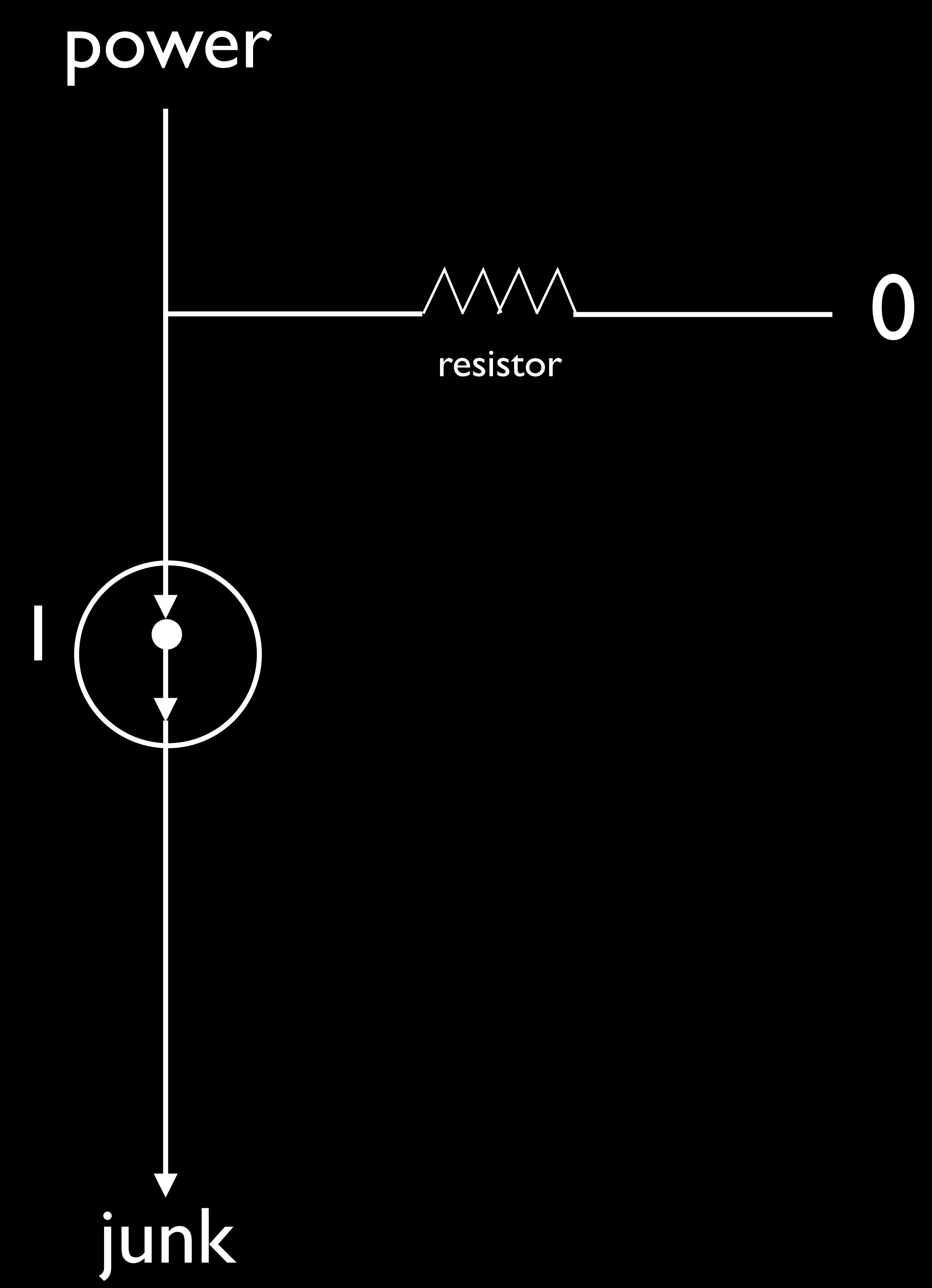
OR gate

Building Gates (transistors)



NOT gate

Building Gates (transistors)



NOT gate