Trees

Announcements

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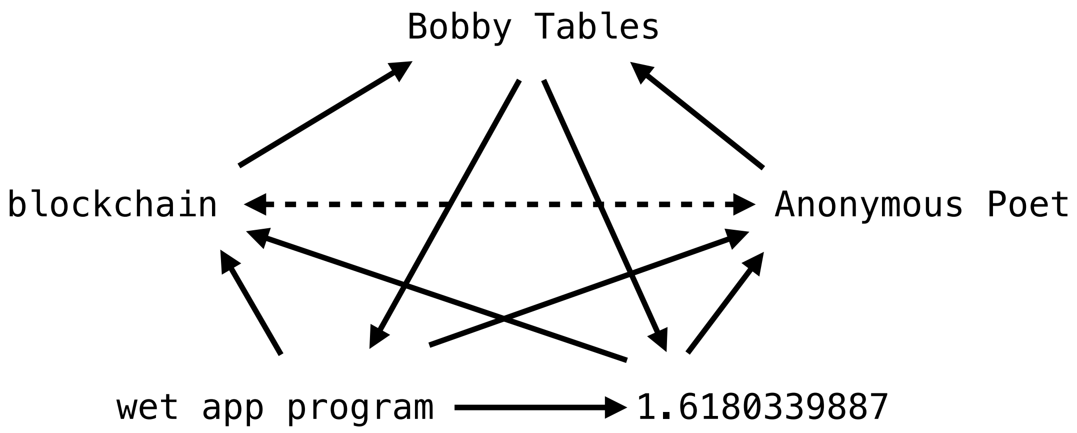

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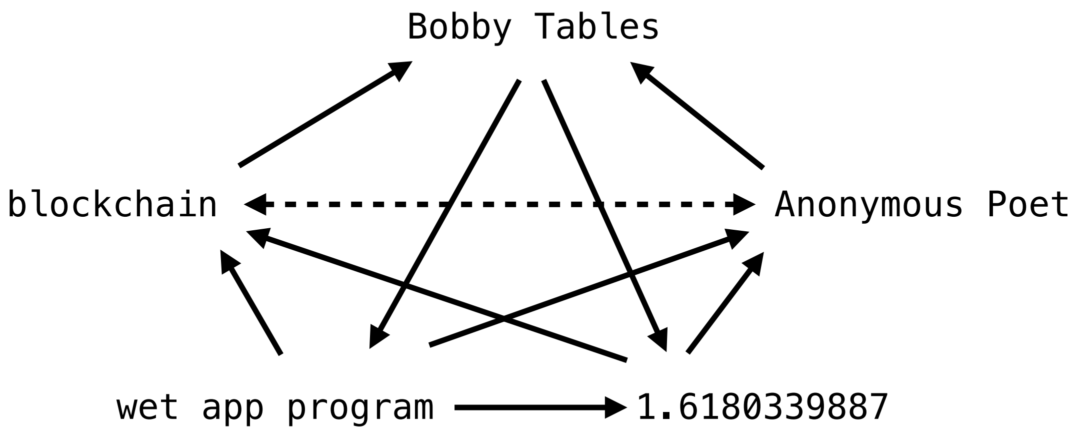

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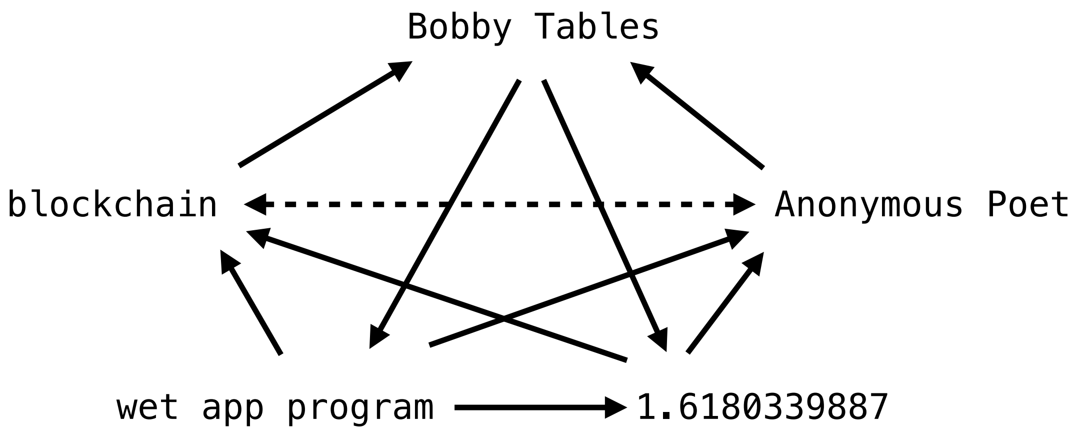

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hog-contest.cs61a.org

Box-and-Pointer Notation

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Lists can contain lists as elements (in addition to anything else)

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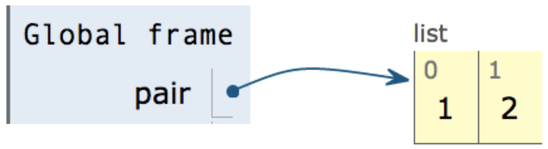

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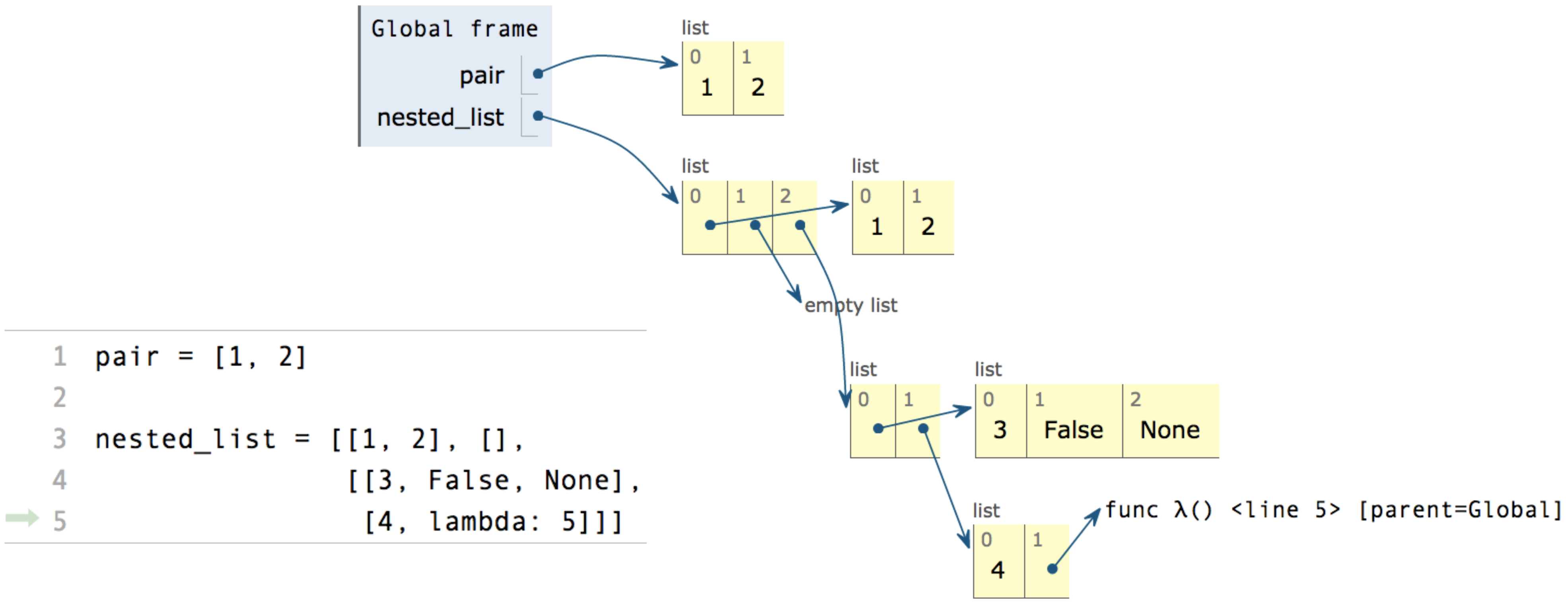

```
1  pair = [1, 2]
2
3  nested_list = [[1, 2], [],
4                 [[3, False, None],
5                  [4, lambda: 5]]]
```

pythontutor.com/composingprograms.html#code=pair%20%3D%20[1,%202]%0A%0Anested_list%20%3D%20[[1,%202],%20[],%0A%20%20%20%20%20%20%20%20%20%20%20%20%20%20%20%20[[3,%20False,%20None],
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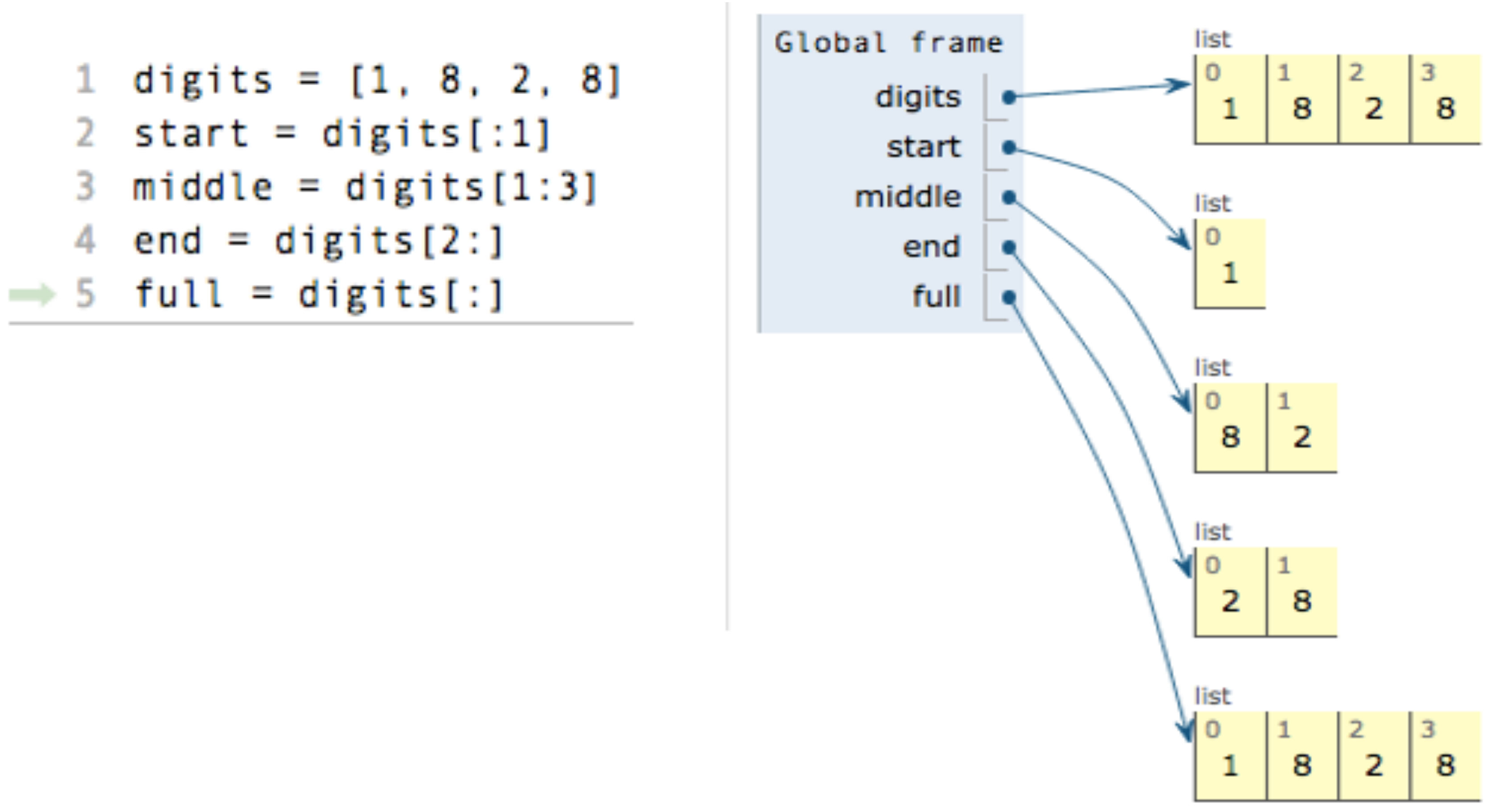
Slicing

`(Demo)`

Slicing Creates New Values



```
digits = [1, 8, 2, 8]
start = digits[:1]
middle = digits[1:3]
end = digits[2:]
full = digits[:]
```

pythontutor.com/composingprograms.html#code=digits%20%3D%20[1,%208,%202,%208]%0Astart%20%3D%20digits[%3A1]%0Amiddle%20%3D%20digits[1%3A3]%0Aend%20%3D%20digits[2%3A]%0Afull%20%3D%20digits[%3A]&cumulative%3Dtrue&curInstr%3D5&mode=display&origin=composingprograms.js&py=3&rawInputLstJSON=[]

Processing Container Values

Sequence Aggregation

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  max(iterable[, key=func]) -> value
  max(a, b, c, ...[, key=func]) -> value

  With a single iterable argument, return its largest item.
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  With a single iterable argument, return its largest item.
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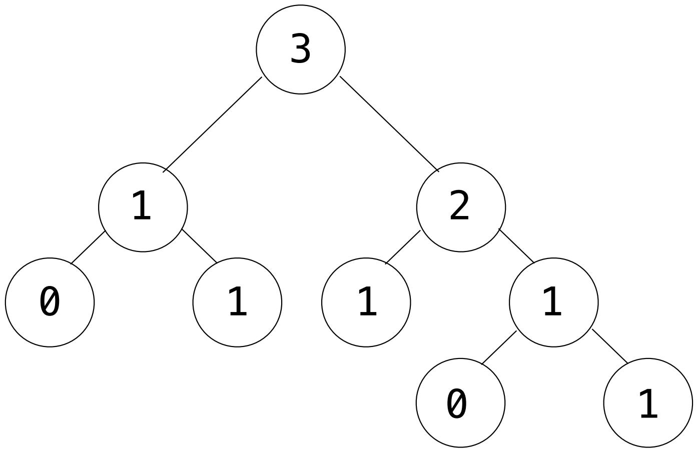
- **all**(iterable) -> bool

  Return True if bool(x) is True for all values x in the iterable.
  If the iterable is empty, return True.

Trees

Tree Abstraction



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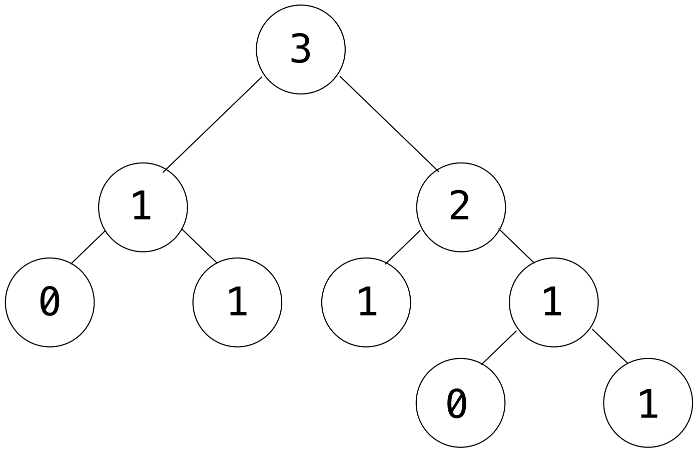

Recursive description **(wooden trees):**

Relative description **(family trees):**

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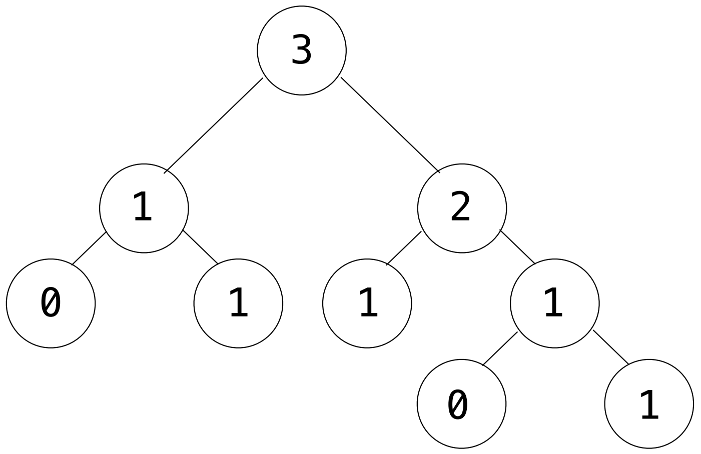

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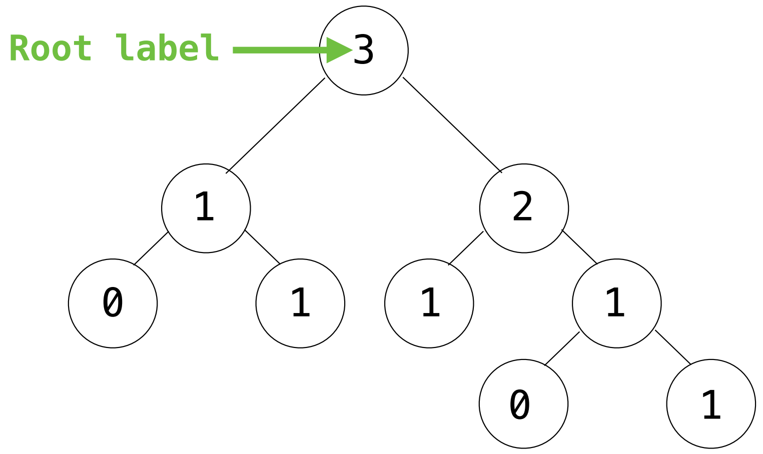

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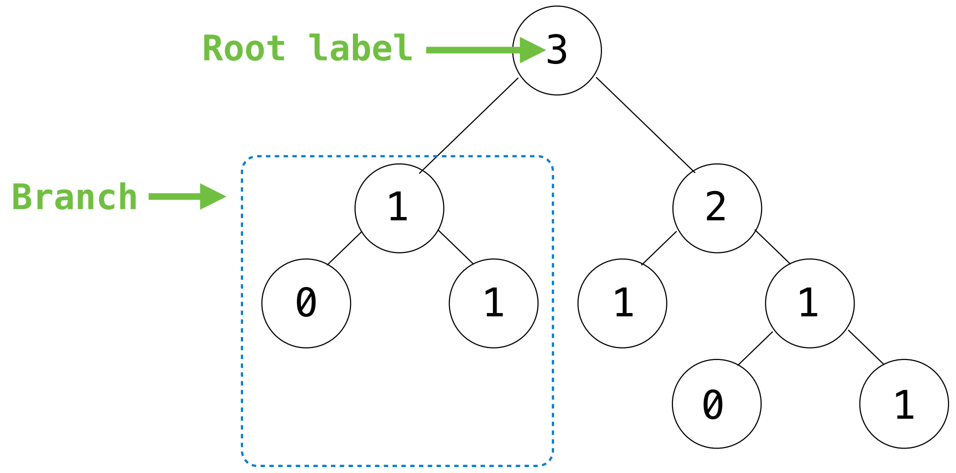

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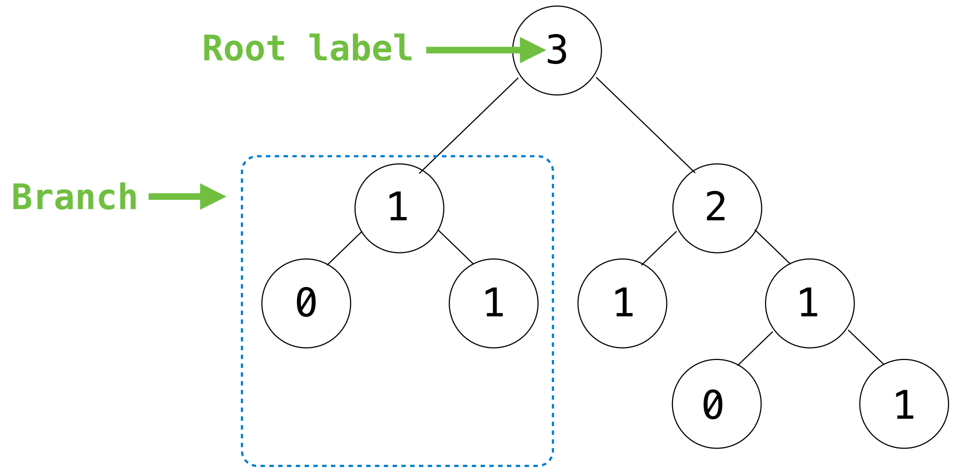

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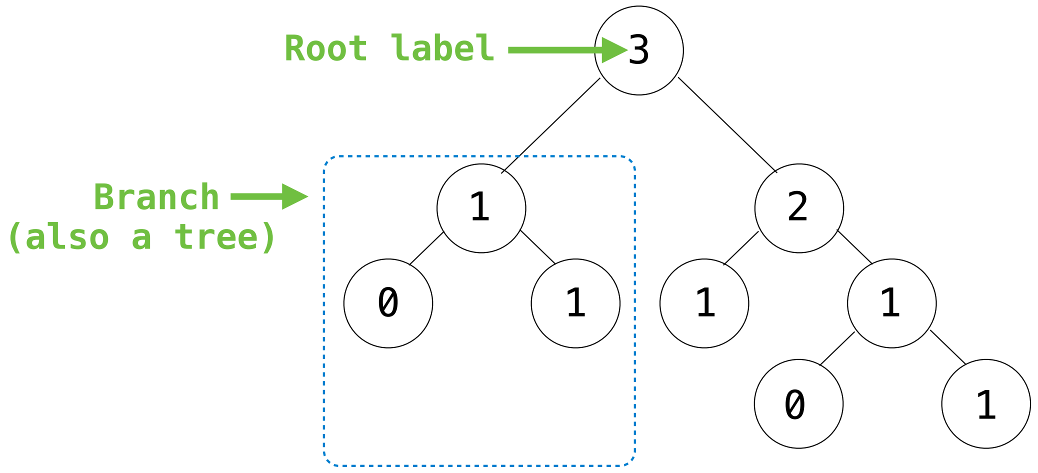

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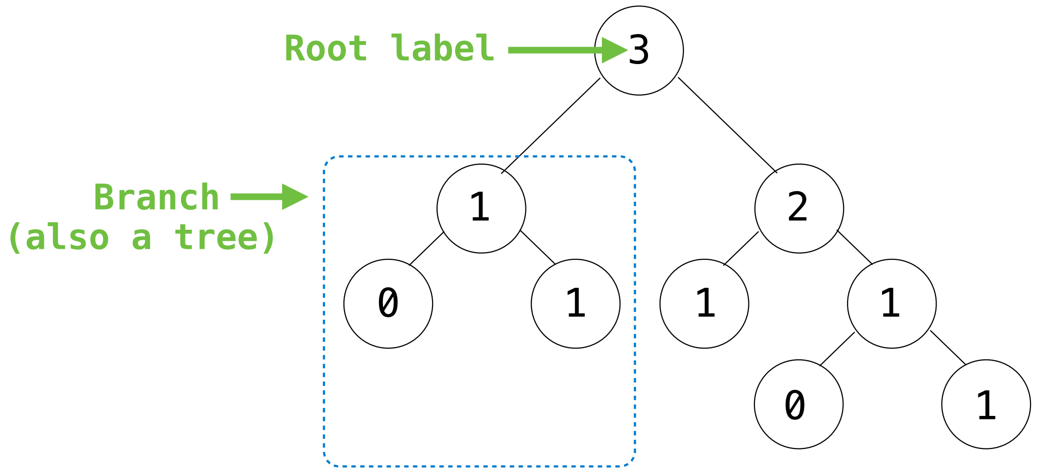

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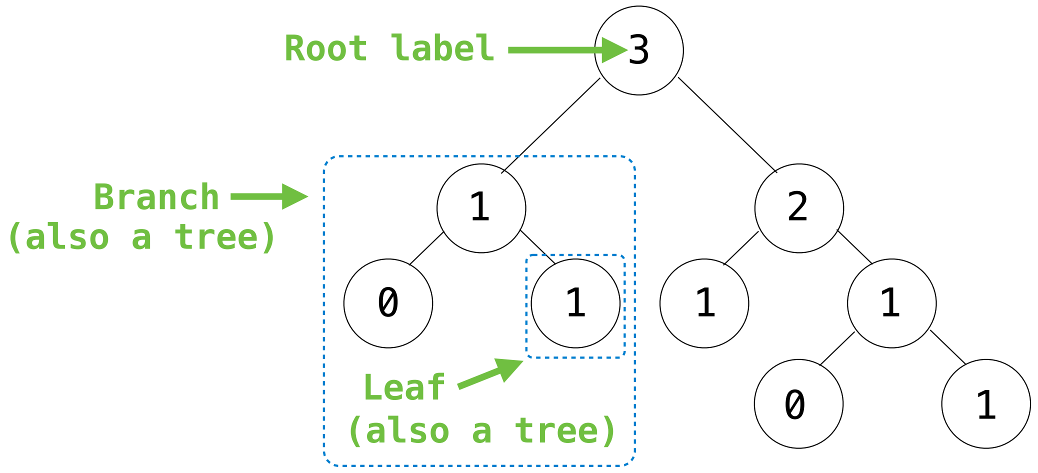

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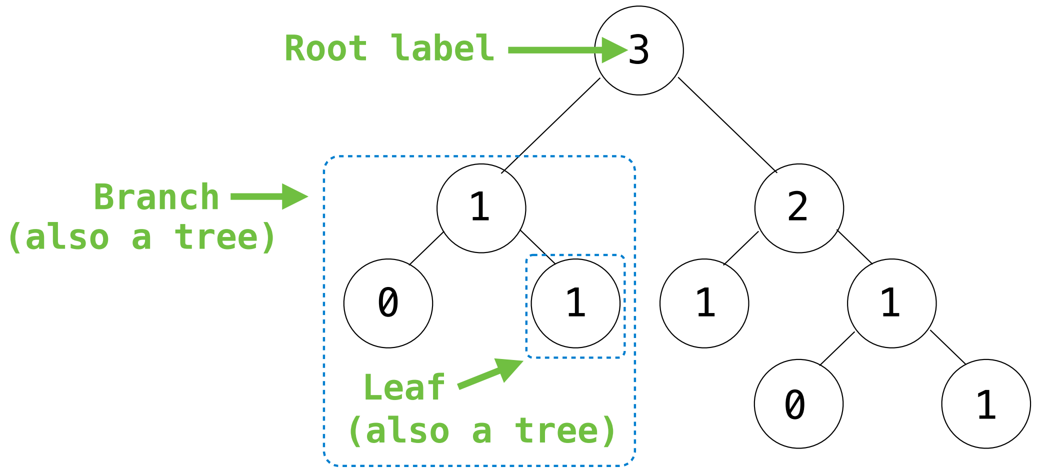

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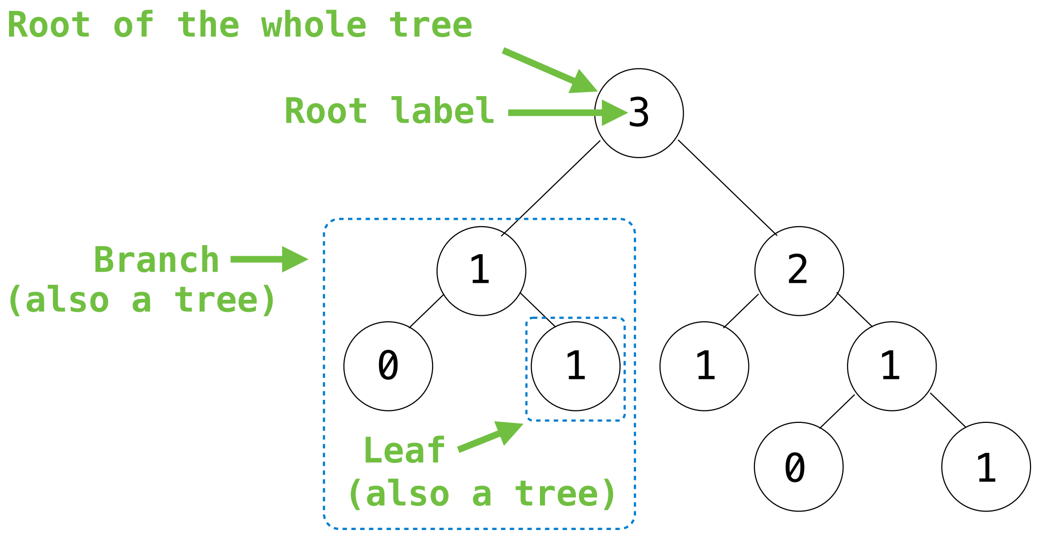

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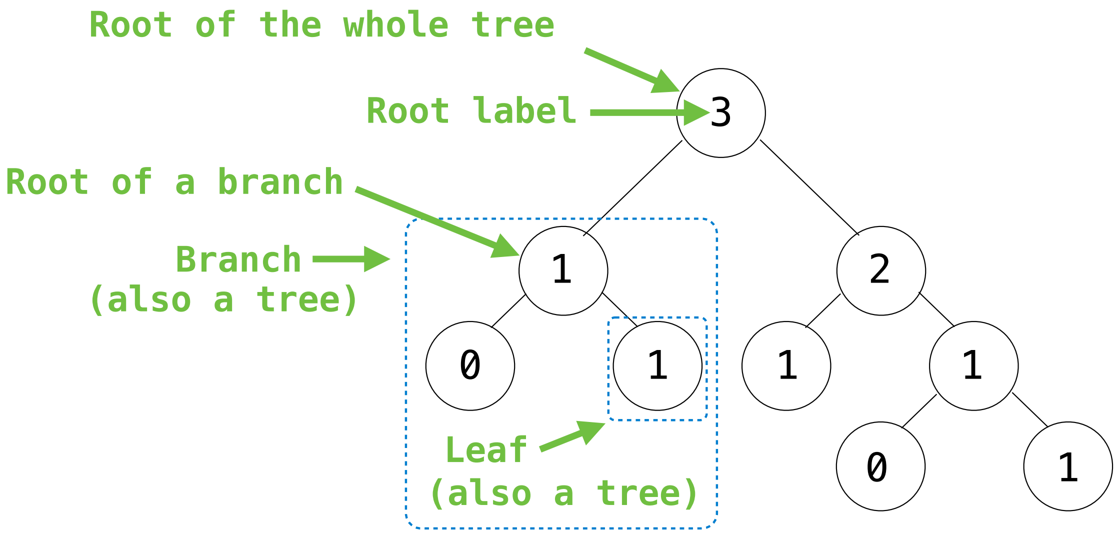

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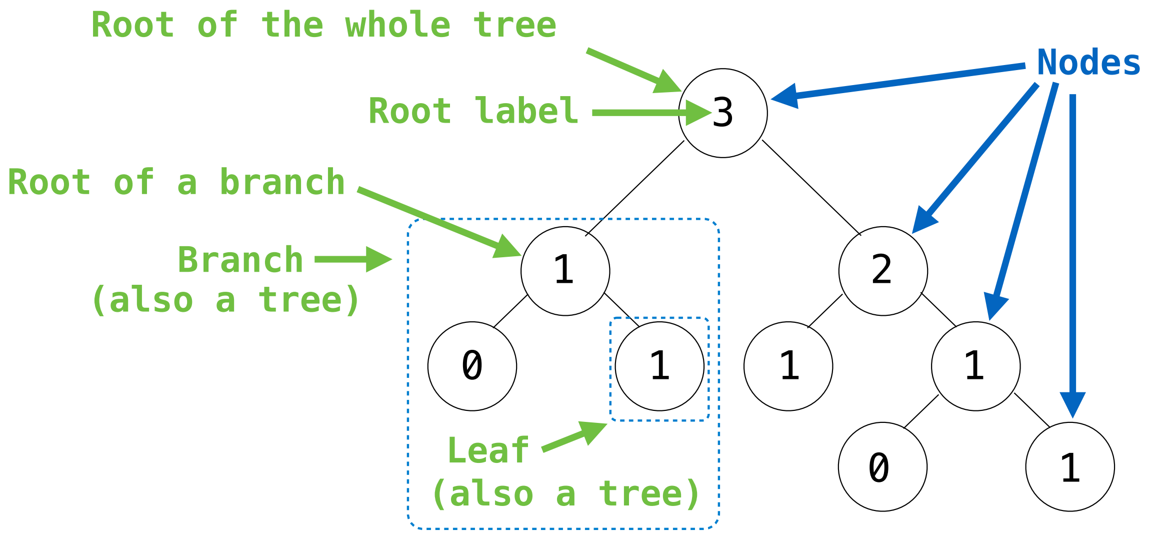

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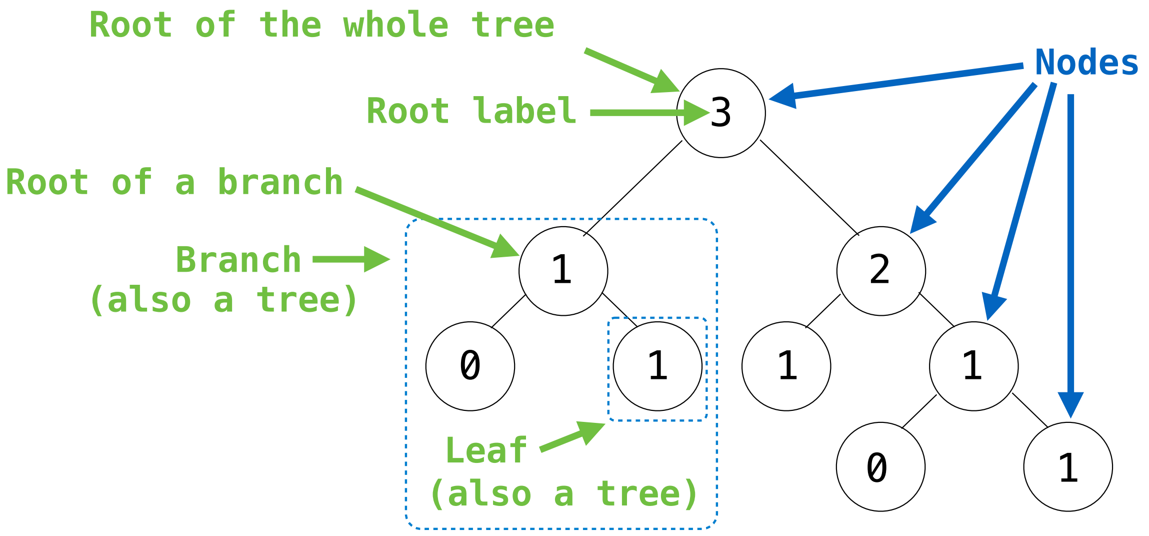

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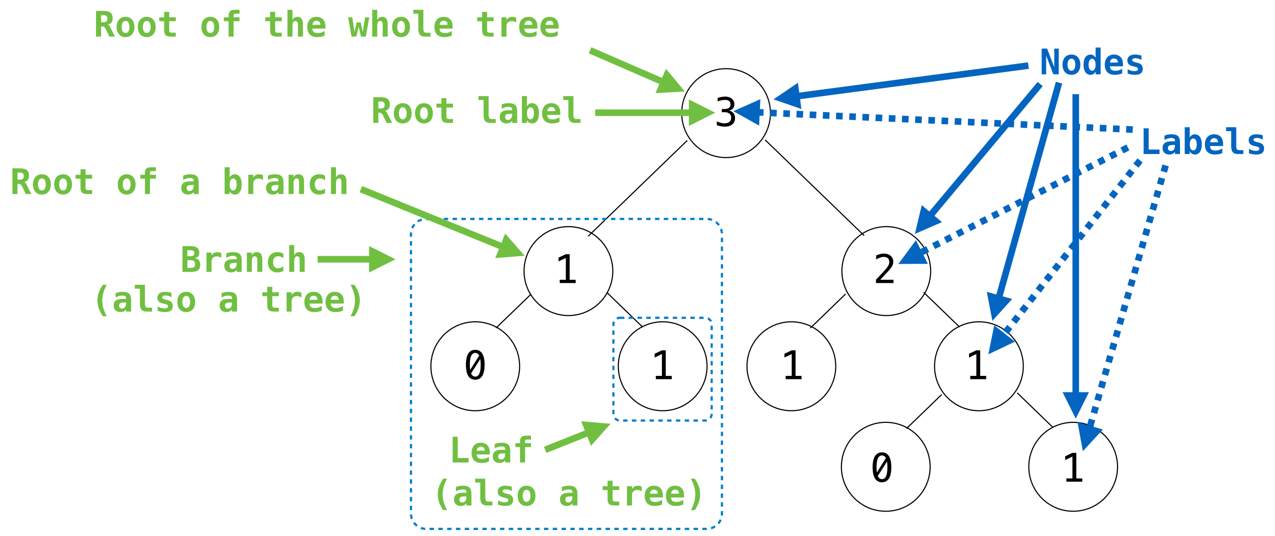

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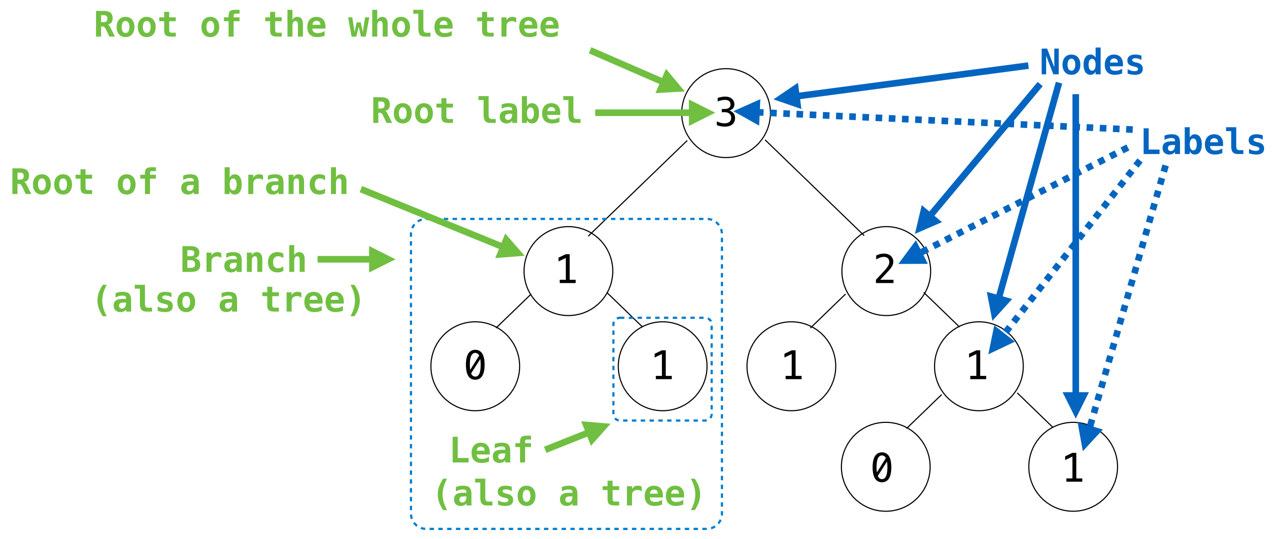

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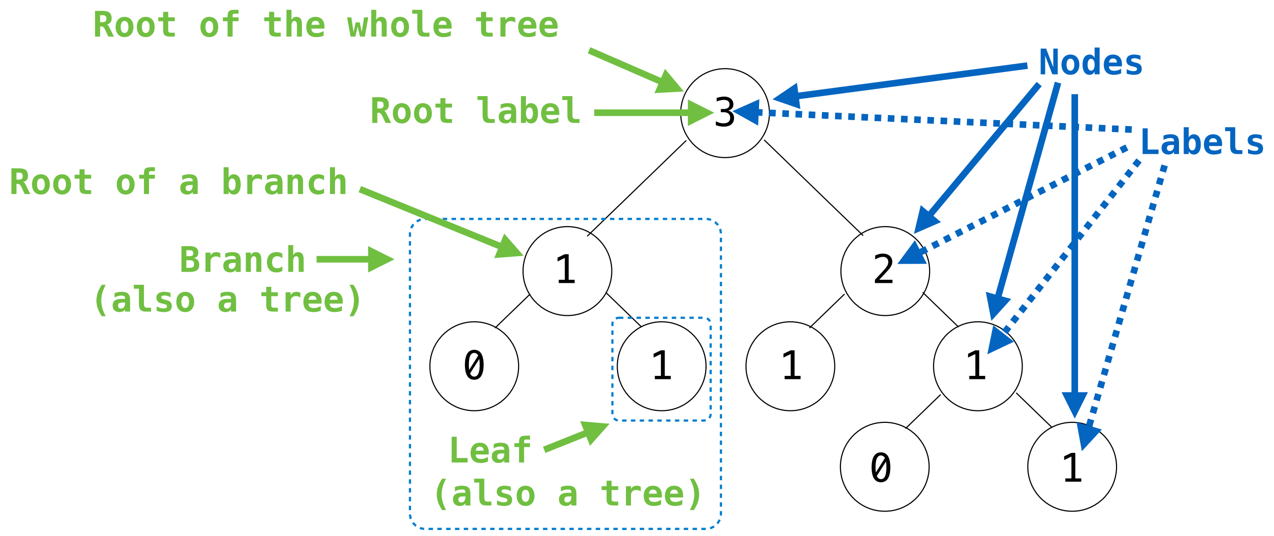

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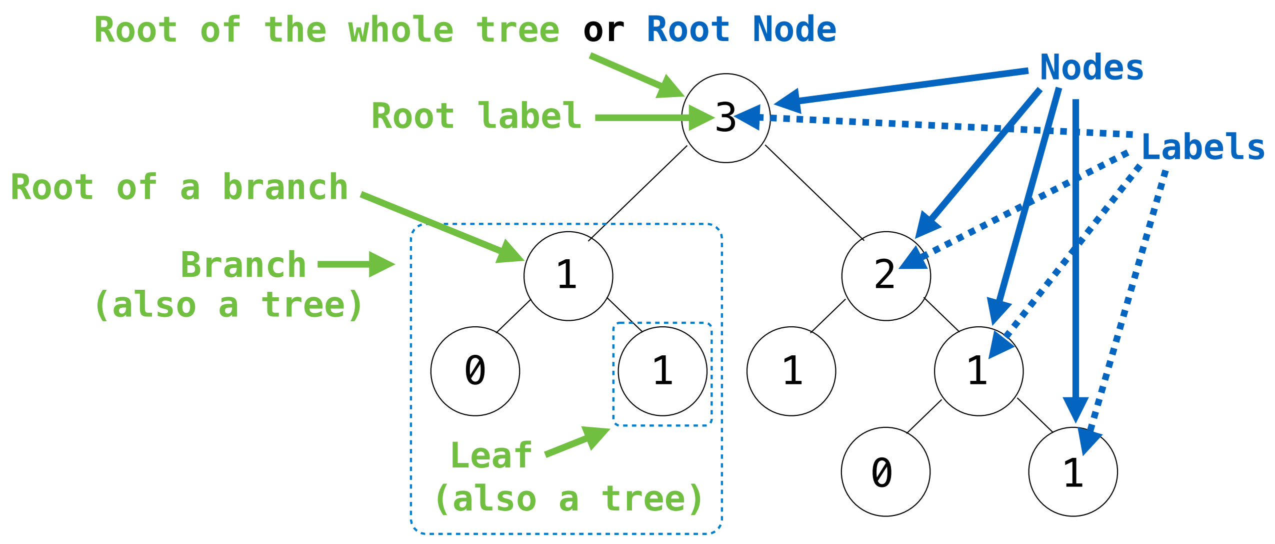

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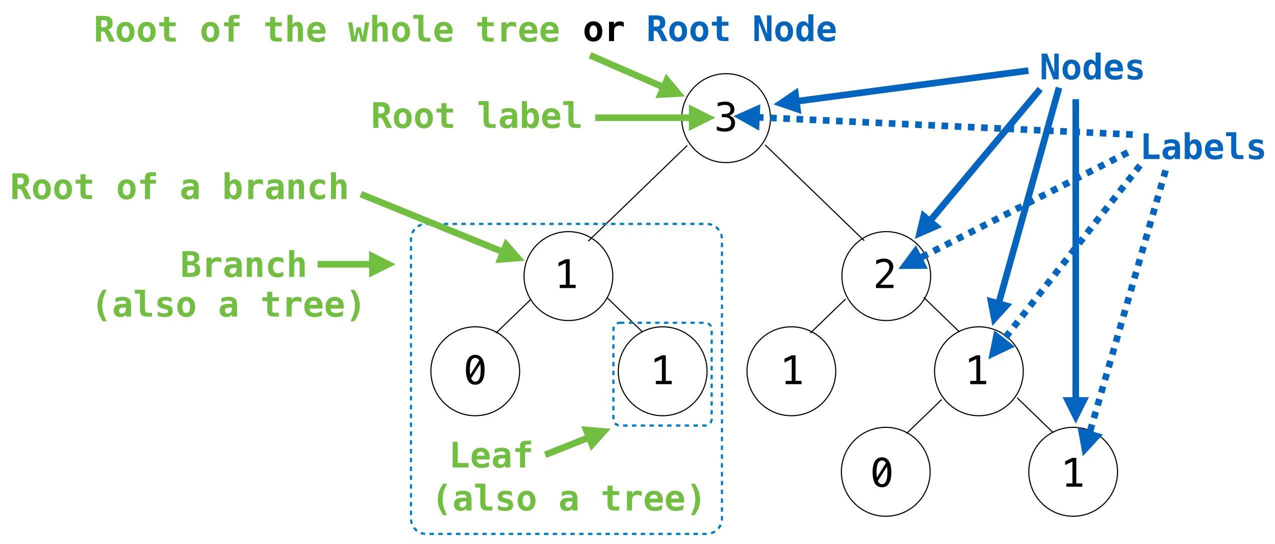

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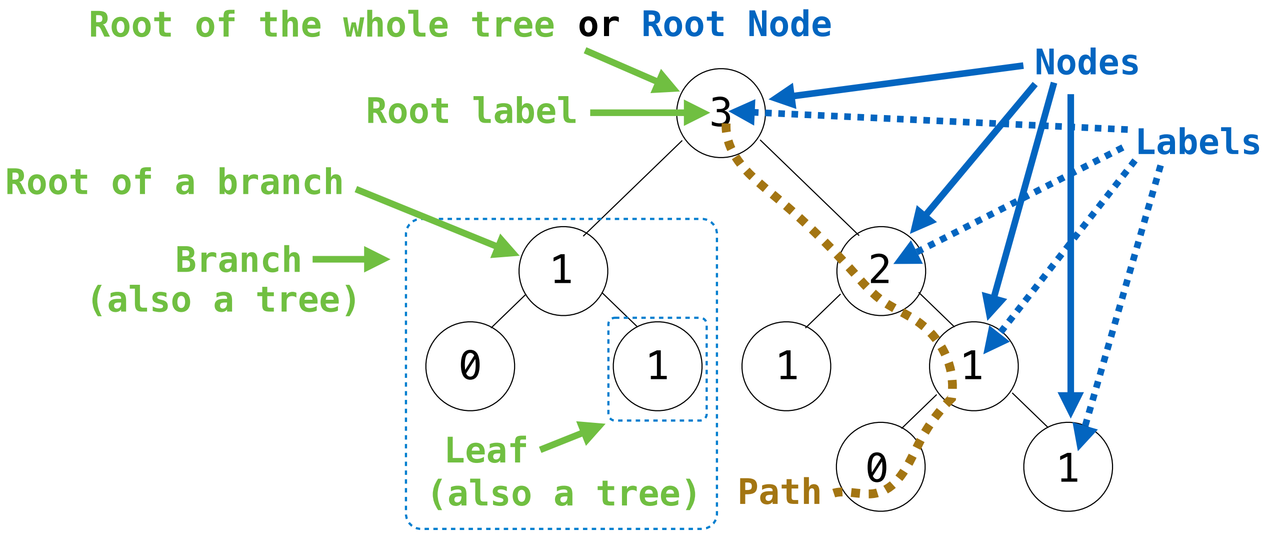

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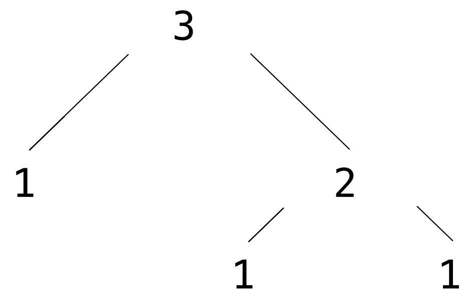

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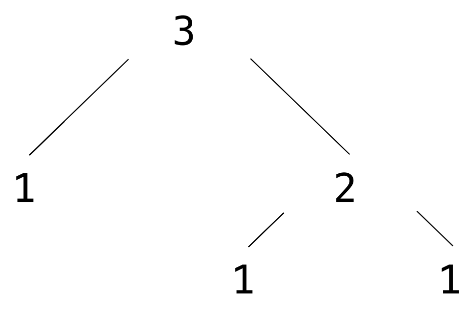

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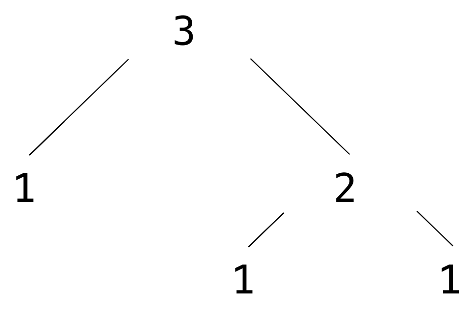

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def tree(label, branches=[]):
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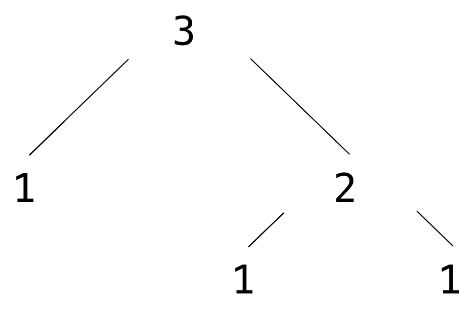

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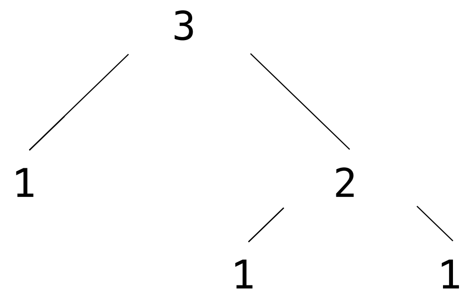

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def label(tree):
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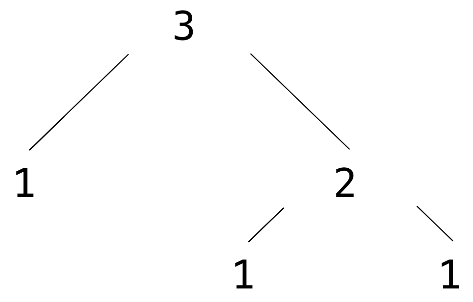

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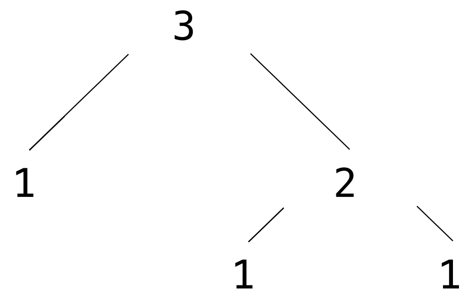

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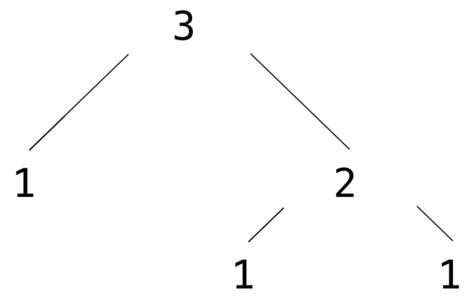

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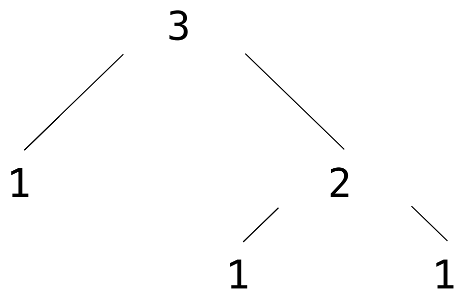

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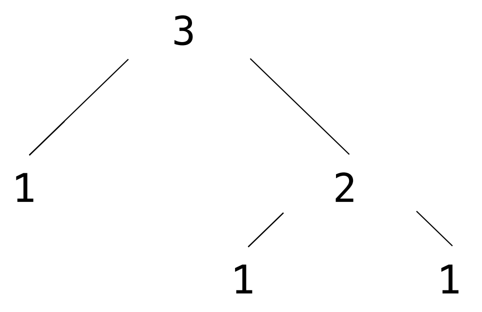

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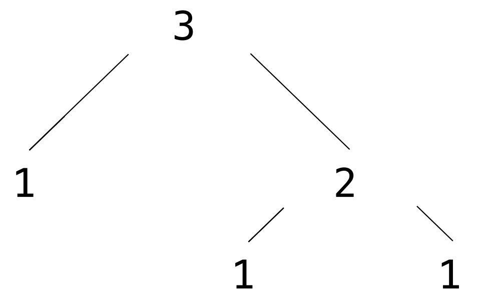

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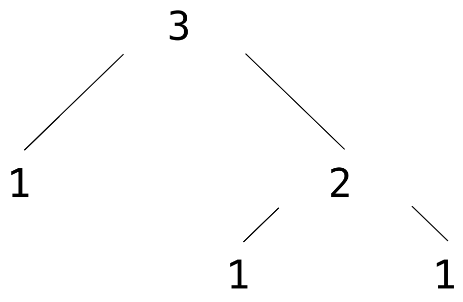

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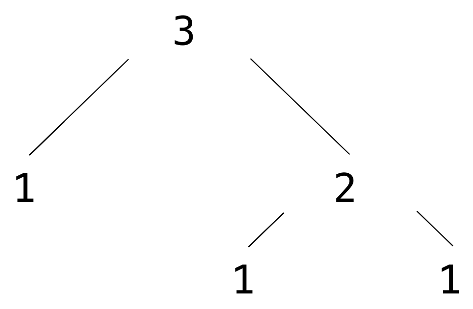

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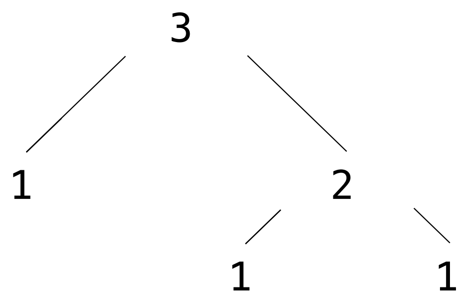

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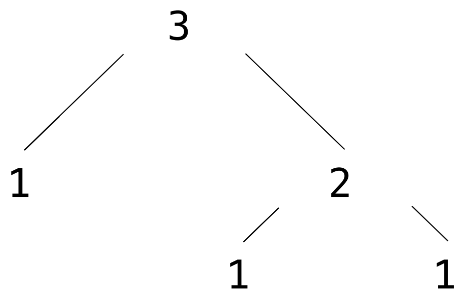

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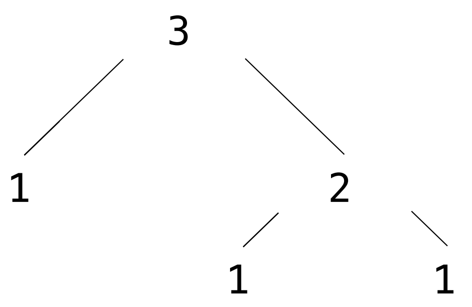

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(Demo)

Tree Processing

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(Demo)

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Example: Printing Trees

`(Demo)`

Example: Summing Paths

(Demo)