Tree Recursion

Announcements

Recursive Factorial

factorial (!)

if n == 0
   n! = 1

if n > 0
   n! = n x (n-1) x (n-2) x ... x 1

```python
def factorial(n):
    fact = 1
    i = 1
    while i <= n:
        fact *= i
        i += 1
    return fact
```

```
factorial(5)

1   = 1*1
2   = 2*1!
6   = 3*2!
24  = 4*3!
120 = 5*4!
```

factorial (!)

if n == 0 *base case*
n! = 1

if n > 0 *recursive case*
n! = n x (n-1)!

```python
def factorial(n):
    if n == 0:
        return 1
    else:
        return n * factorial(n-1)
```

```
factorial(3)
```

```
3 * factorial(2)
    2 * factorial(1)
        1 * factorial(0)
```

Order of Recursive Calls

The Cascade Function

(Demo)

http://pythontutor.com/composingprograms.html#code=def%20cascade%28n%29%3A%20%20%20%20%0A%20%20%20%20if%20n%20%3C%2010%3A%20%20%20%20%20%20%20%20%0A%20%20%20%20%20%20%20%20print%28n%29%20%20%20%20%0A%20%20%20%20else%3A%20%20%20%20%20%20%20%20%0A%20%20%20%20%20%20%20%20print%28n%29%20%20%20%20%20%20%20%20%0A%20%20%20%20%20%20%20%20cascade%28n//10%29%20%20%20%20%20%20%20%20%0A%20%20%20%20%20%20%20%20print%28n%29%20%20%20%20%20%20%20%20%0A%20%20%20%20%20%20%20%20%0Acascade%28123%29&cumulative=true&curInstr=0&mode=display&origin=composingprograms.js&py=3&rawInputLstJSON=%5B%5D

The Cascade Function

(Demo)

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def cascade(n):
    if n < 10:
        print(n)
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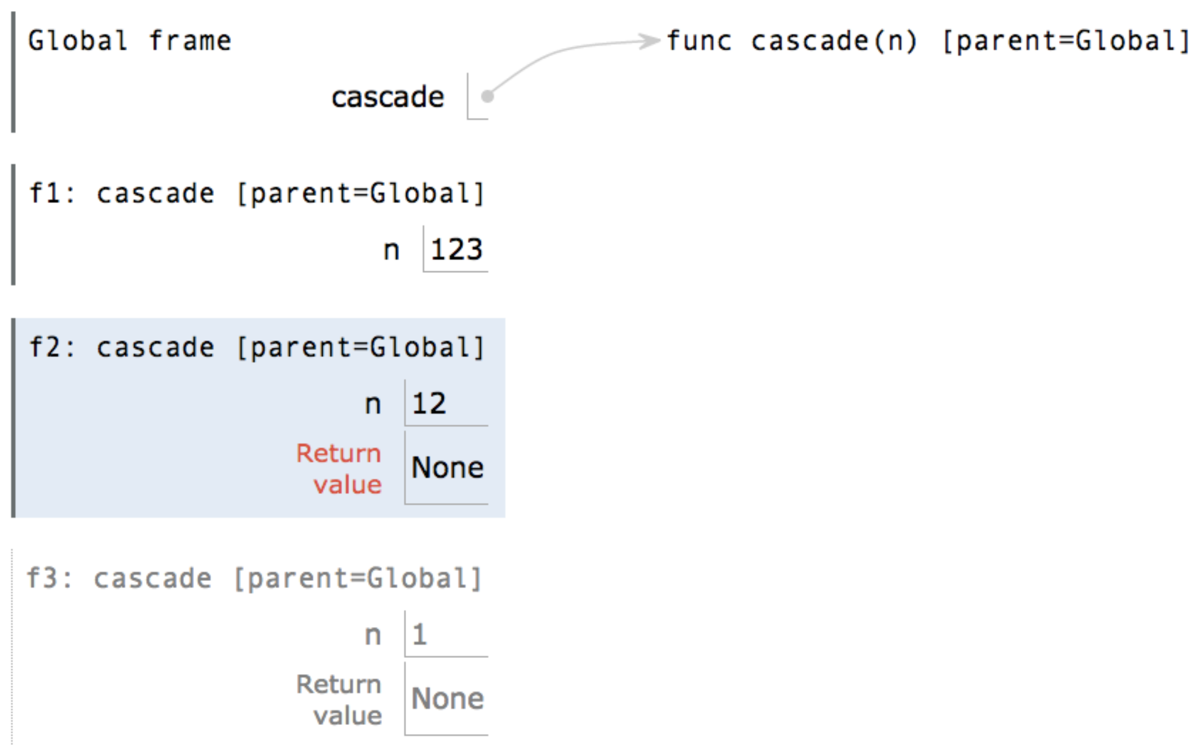
        print(n)
        cascade(n//10)
        print(n)

cascade(123)
```



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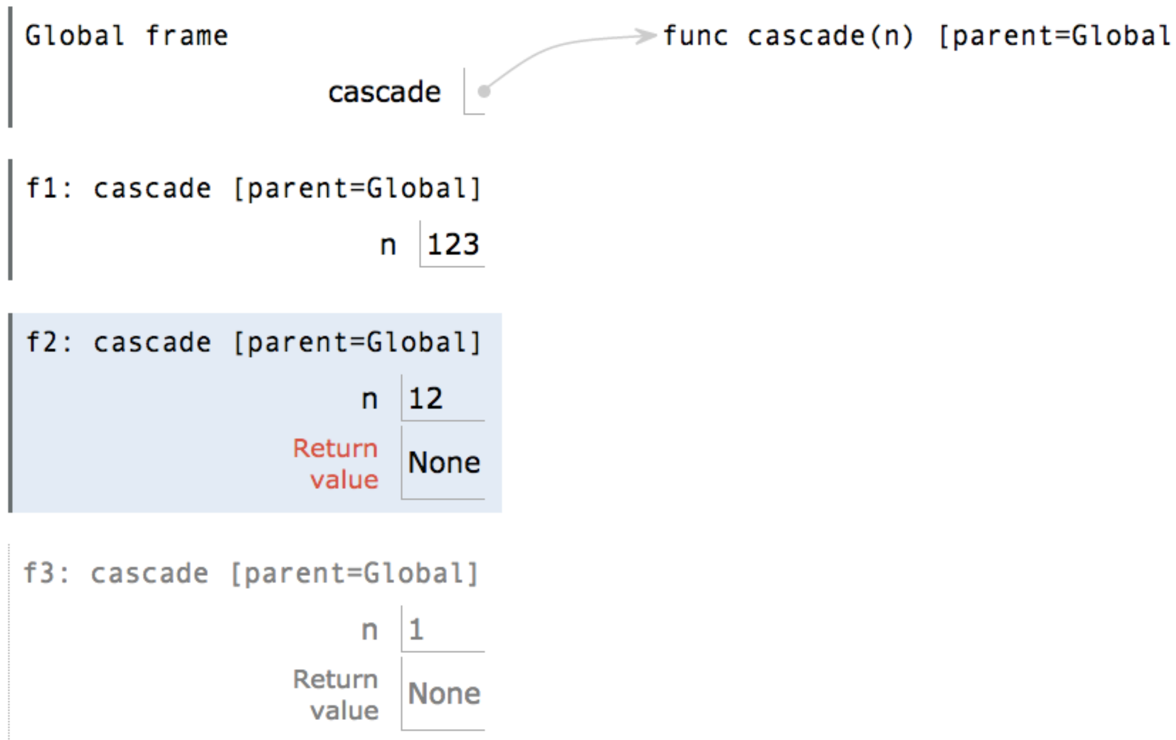

Program output:

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123
12
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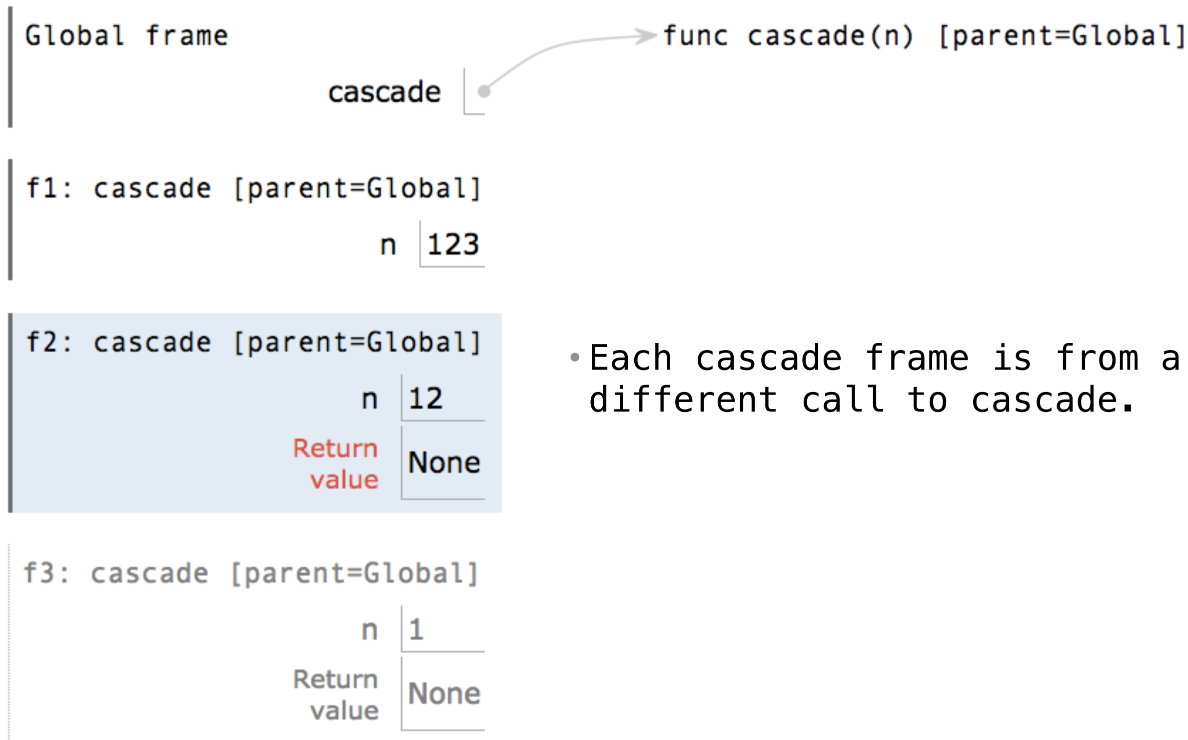

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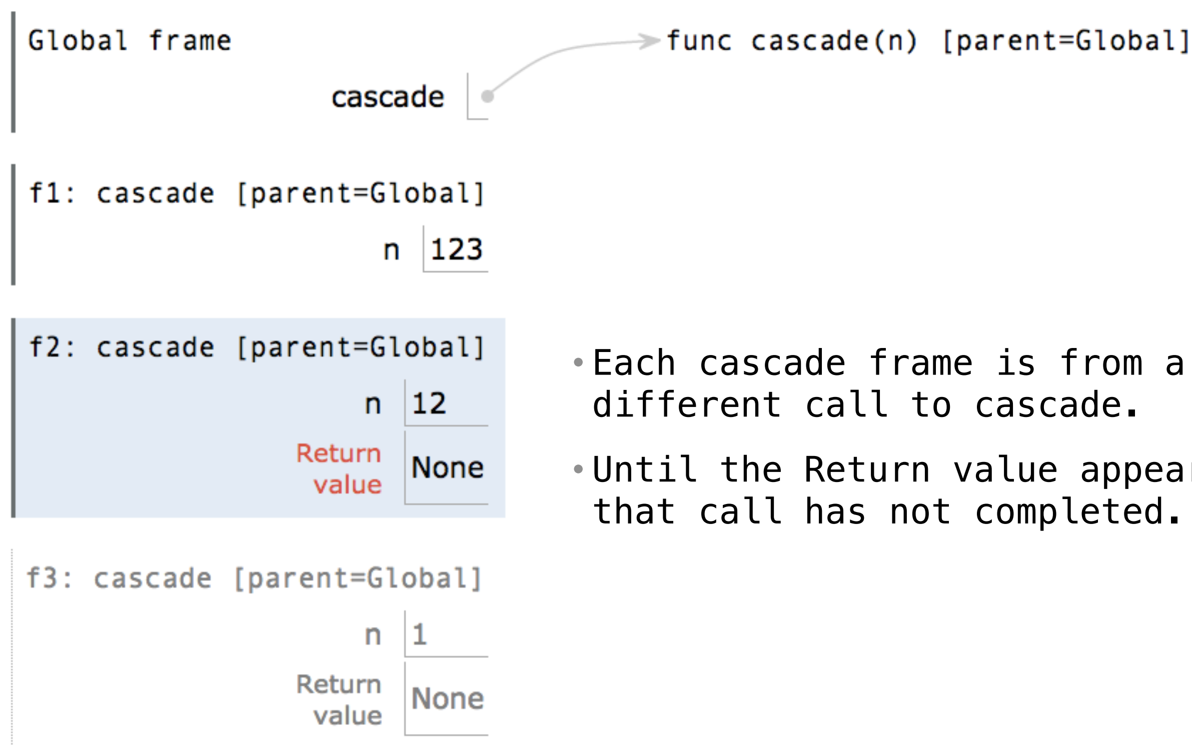

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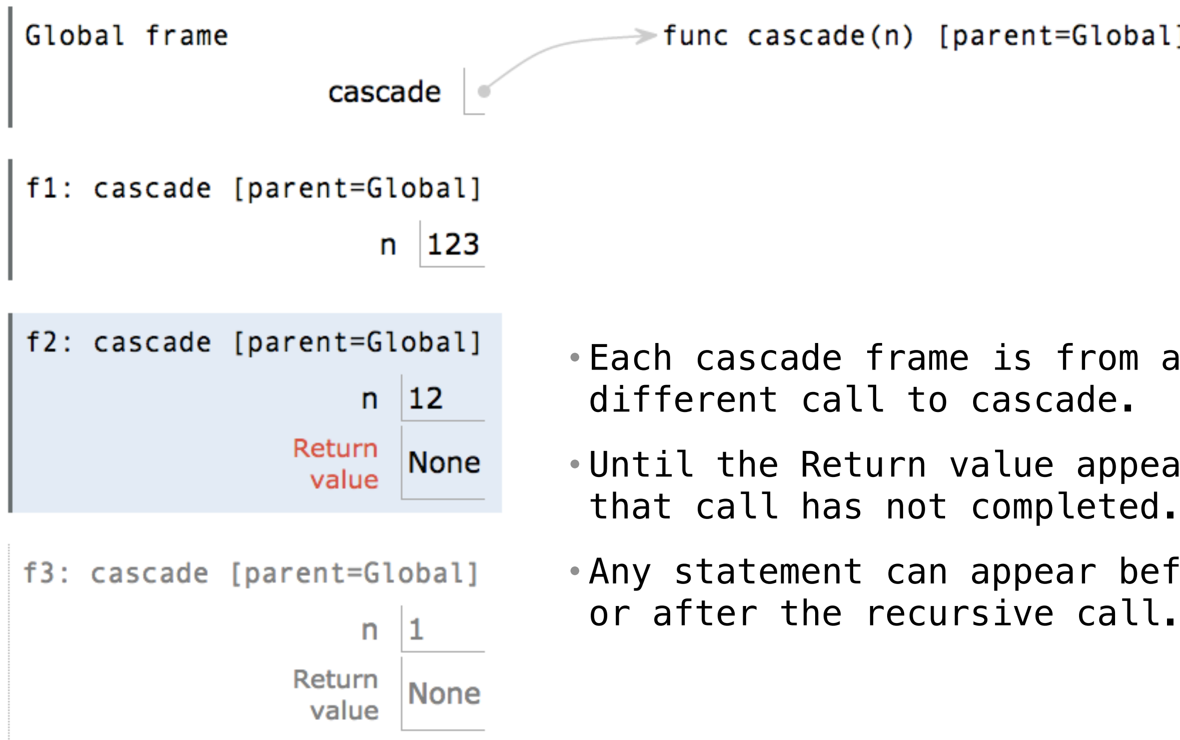

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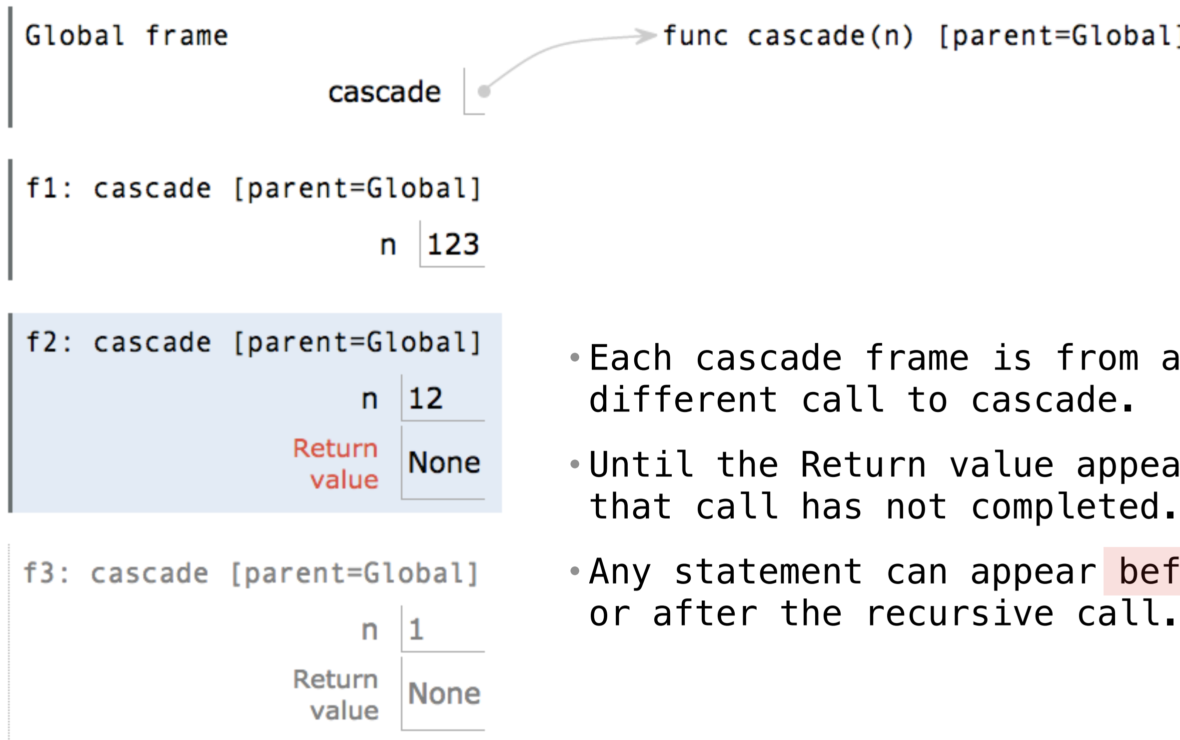

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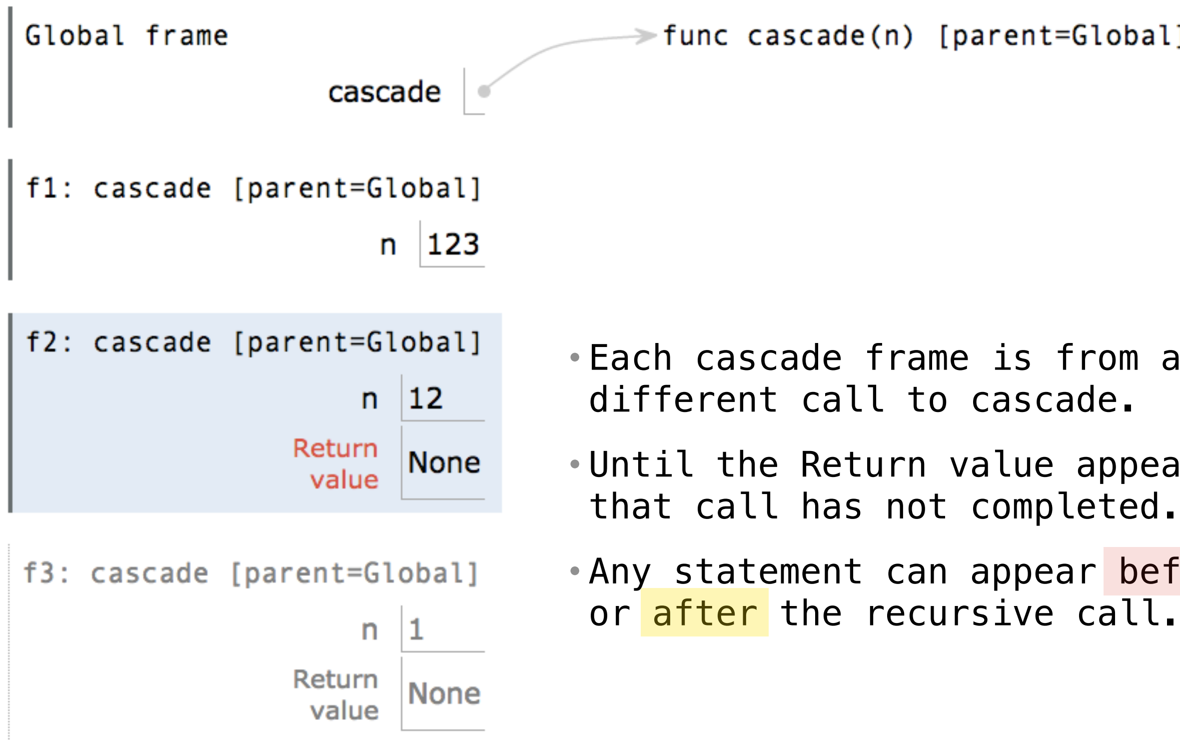

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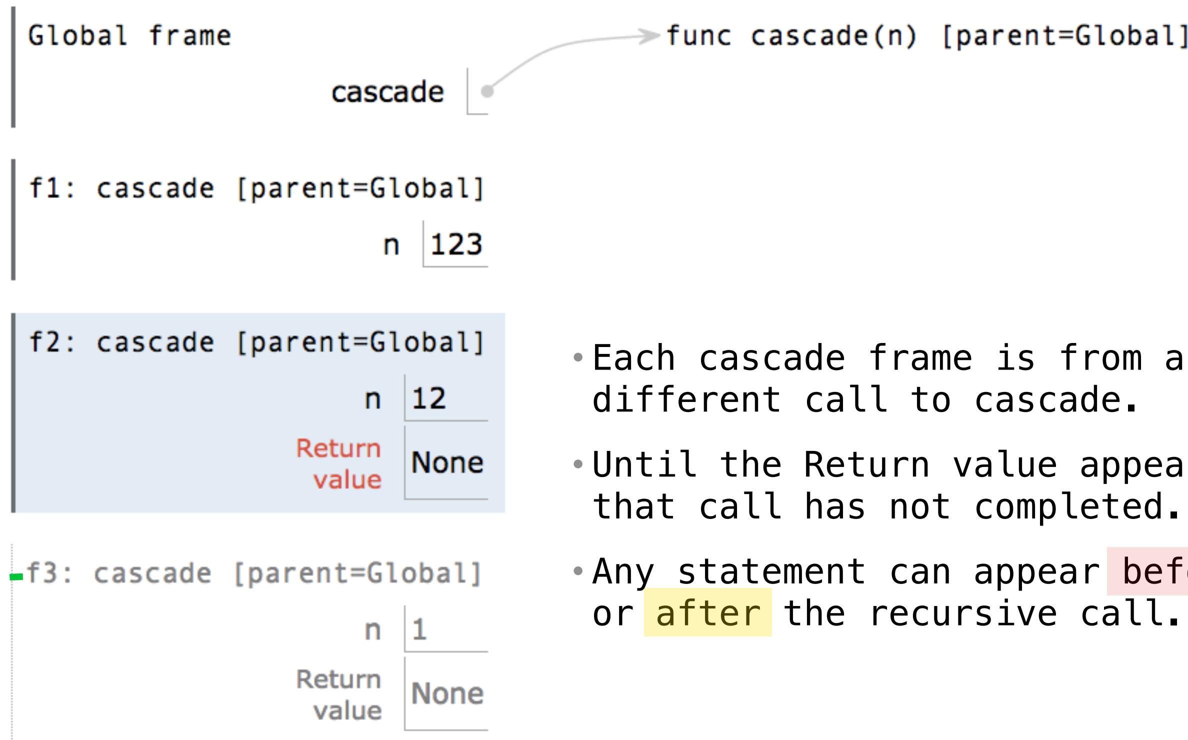

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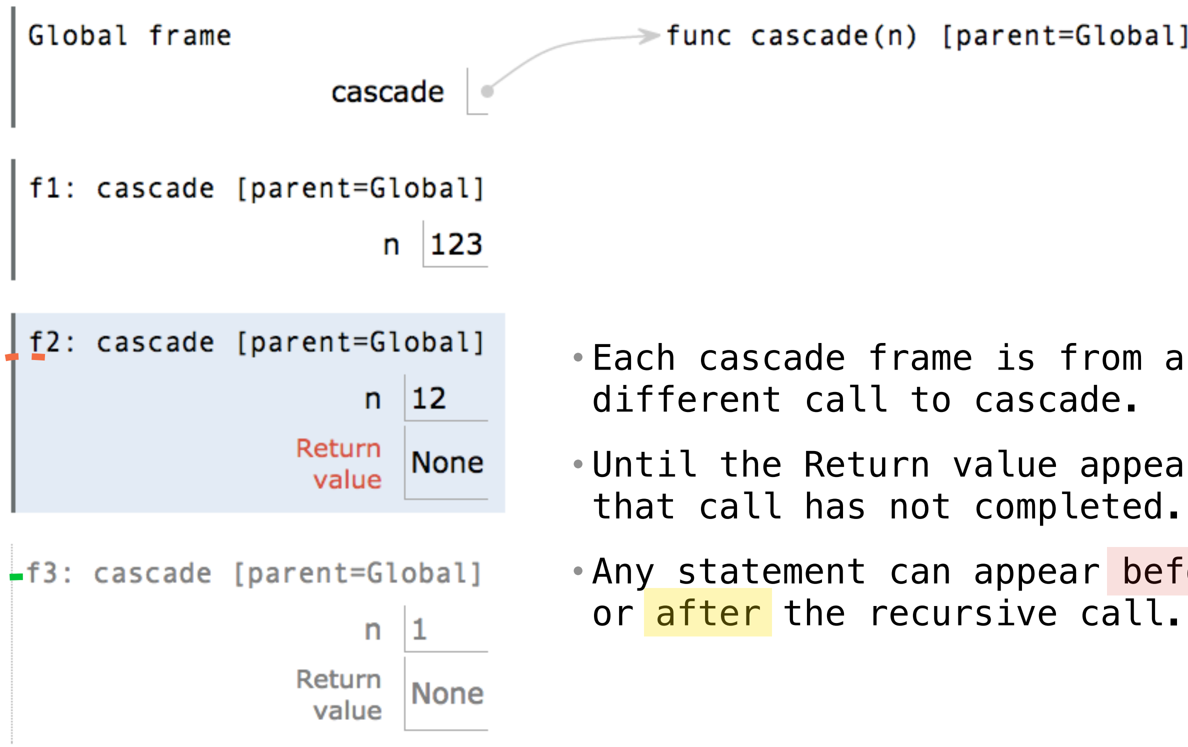

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Two Definitions of Cascade

(Demo)

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```

```
def cascade(n):
    print(n)
    if n >= 10:
        cascade(n//10)
        print(n)
```

Two Definitions of Cascade

(Demo)

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        cascade(n//10)
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```

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def cascade(n):
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    if n >= 10:
        cascade(n//10)
        print(n)
```

- If two implementations are equally clear, then shorter is usually better

Two Definitions of Cascade

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```

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- In this case, the longer implementation is more clear (at least to me)

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- If two implementations are equally clear, then shorter is usually better
- In this case, the longer implementation is more clear (at least to me)
- When learning to write recursive functions, put the base cases first
- Both are recursive functions, even though only the first has typical structure

Example: Inverse Cascade

Inverse Cascade

```
Write a function that prints an inverse cascade:
```

Inverse Cascade

Write a function that prints an inverse cascade:

```
1
12
123
1234
123
12
1
```

Inverse Cascade

Write a function that prints an inverse cascade:

```
1
12
123
1234
123
12
1
```

```
def inverse_cascade(n):
    grow(n)
    print(n)
    shrink(n)
```

Inverse Cascade

Write a function that prints an inverse cascade:

```
1
12
123
1234
123
12
1
```

```
def inverse_cascade(n):
    grow(n)
    print(n)
    shrink(n)

def f_then_g(f, g, n):
    if n:
        f(n)
        g(n)
```

Inverse Cascade

Write a function that prints an inverse cascade:

```
1
12
123
1234
123
12
1
```

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def inverse_cascade(n):
    grow(n)
    print(n)
    shrink(n)

def f_then_g(f, g, n):
    if n:
        f(n)
        g(n)

grow =   lambda n: f_then_g(                    )
shrink = lambda n: f_then_g(                    )
```

Inverse Cascade

```
Write a function that prints an inverse cascade:
```

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1
12
123
1234
123
12
1
```

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def inverse_cascade(n):
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    print(n)
    shrink(n)

def f_then_g(f, g, n):
    if n:
        f(n)
        g(n)

grow =   lambda n: f_then_g(grow,  print,  n//10)
shrink = lambda n: f_then_g(print, shrink, n//10)
```

Tree Recursion

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Tree-shaped processes arise whenever executing the body of a recursive function makes more than one recursive call

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http://en.wikipedia.org/wiki/File:Fibonacci.jpg

Tree Recursion

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n: 0, 1, 2, 3, 4, 5, 6, 7, 8,

http://en.wikipedia.org/wiki/File:Fibonacci.jpg

Tree Recursion

Tree-shaped processes arise whenever executing the body of a recursive function makes more than one recursive call

n:	0, 1, 2, 3, 4, 5, 6, 7, 8,
fib(n):	0, 1, 1, 2, 3, 5, 8, 13, 21,

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Tree Recursion

Tree-shaped processes arise whenever executing the body of a recursive function makes more than one recursive call

n:	0, 1, 2, 3, 4, 5, 6, 7, 8, ... , 35
fib(n):	0, 1, 1, 2, 3, 5, 8, 13, 21,

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```
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fib(n):	0,	1,	1,	2,	3,	5,	8,	13,	21,	... ,	9,227,465

```
def fib(n):
    if n == 0:
```

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```
def fib(n):
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```
def fib(n):
    if n == 0:
        return 0
    elif n == 1:
```

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```
def fib(n):
    if n == 0:
        return 0
    elif n == 1:
        return 1
    else:
        return fib(n-2) + fib(n-1)
```

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A Tree-Recursive Process

`The computational process of fib evolves into a tree structure`

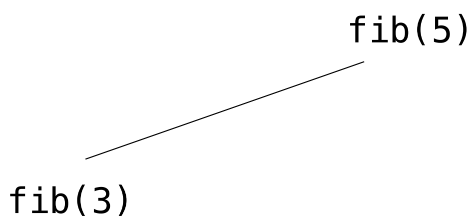
A Tree-Recursive Process

The computational process of fib evolves into a tree structure

fib(5)

A Tree-Recursive Process

The computational process of fib evolves into a tree structure



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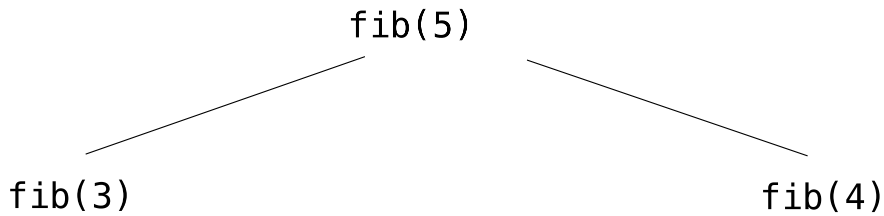

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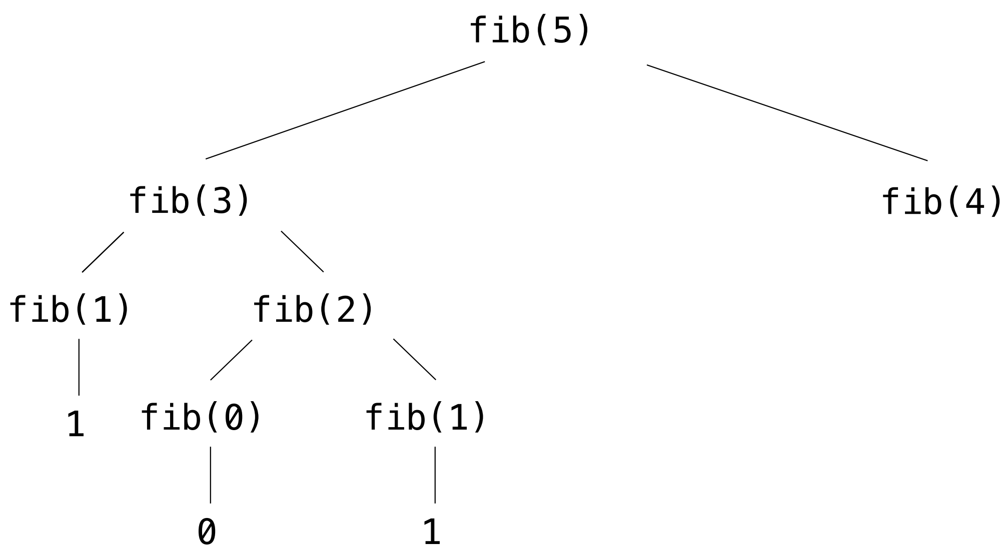

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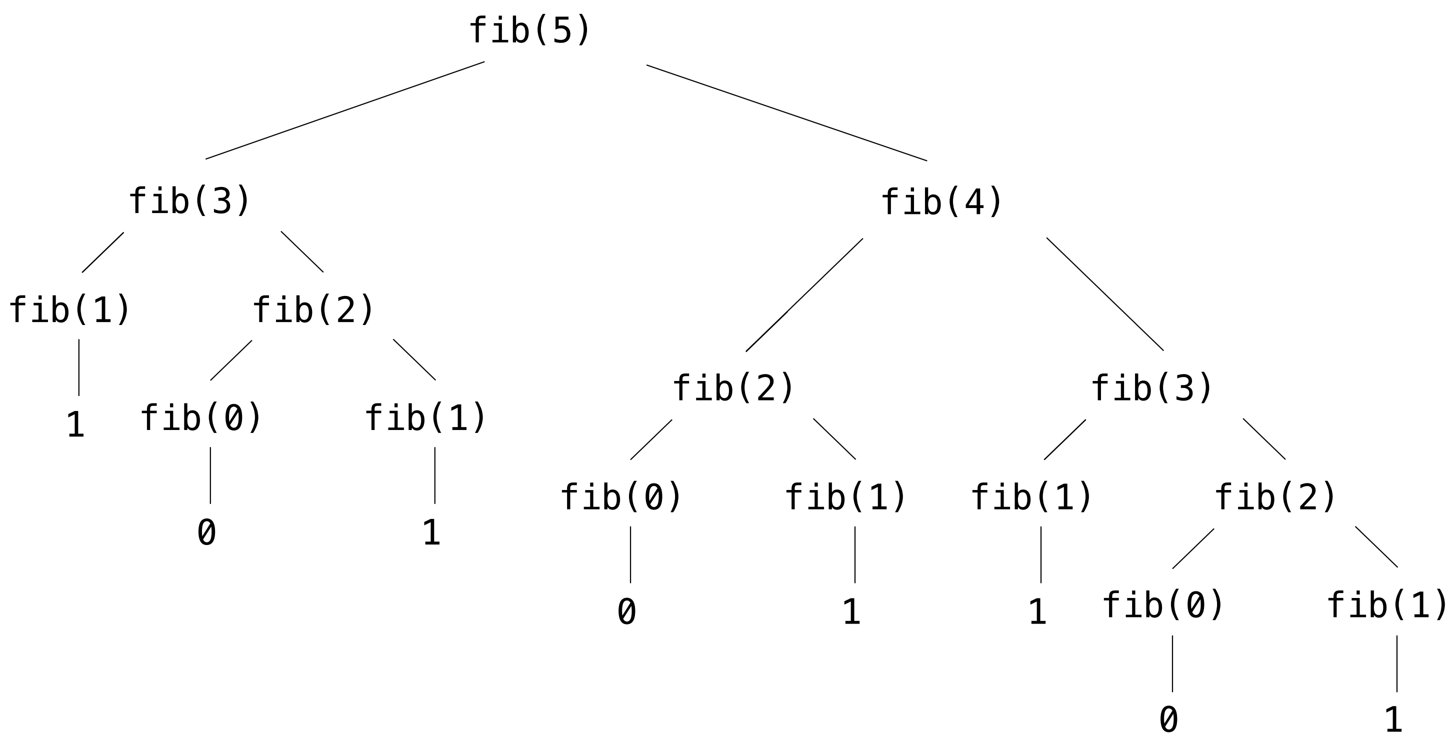

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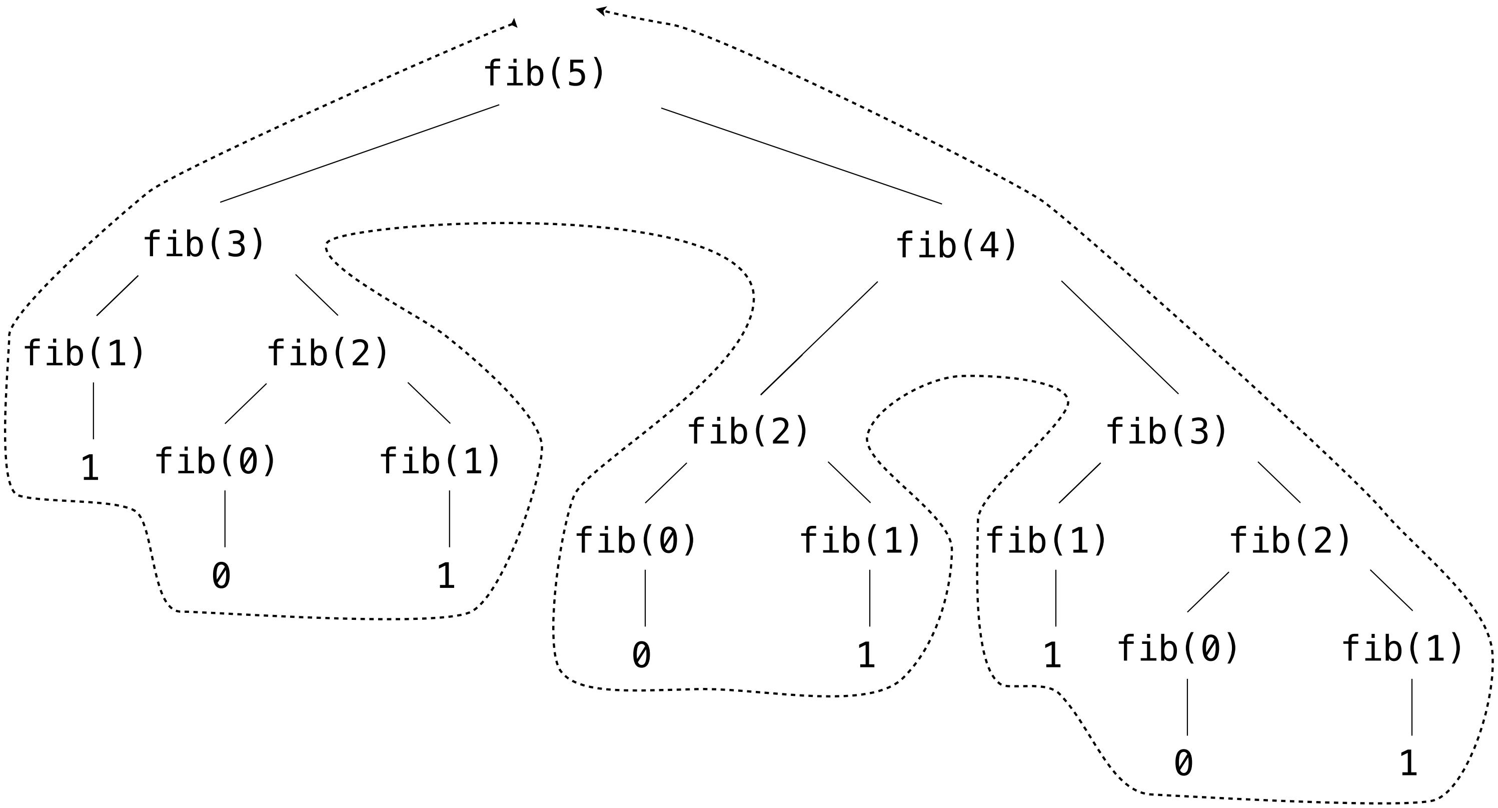

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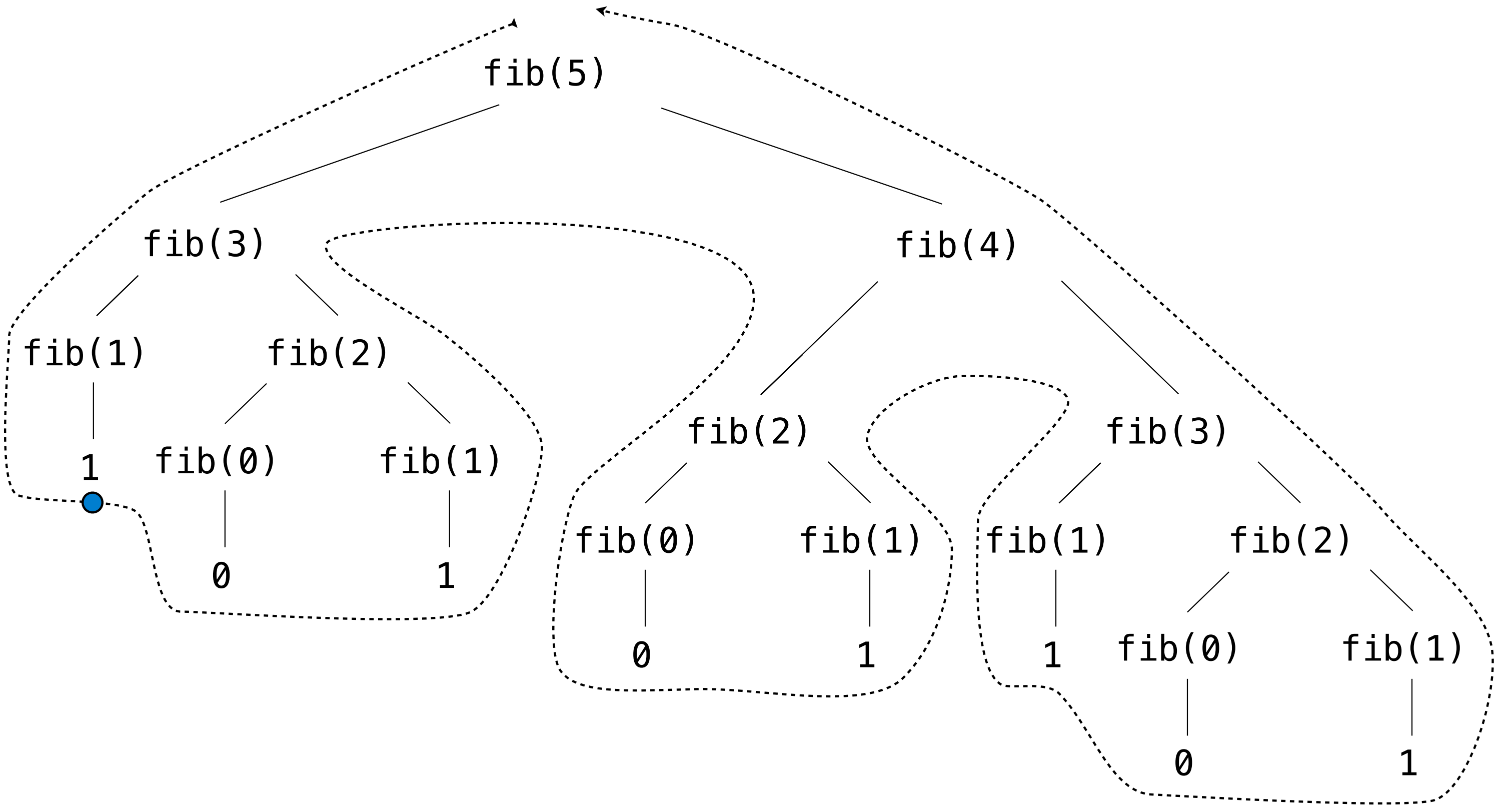

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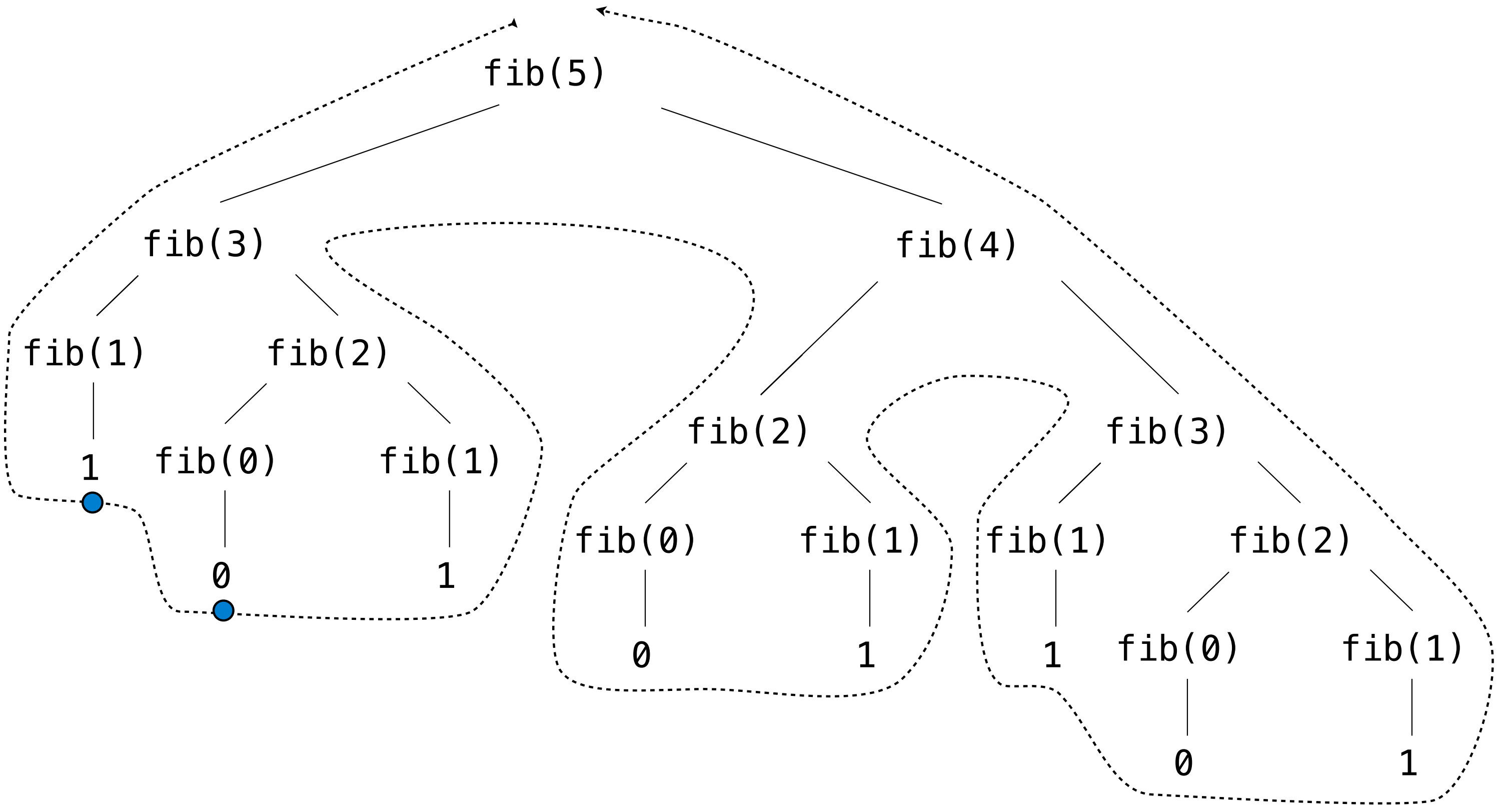

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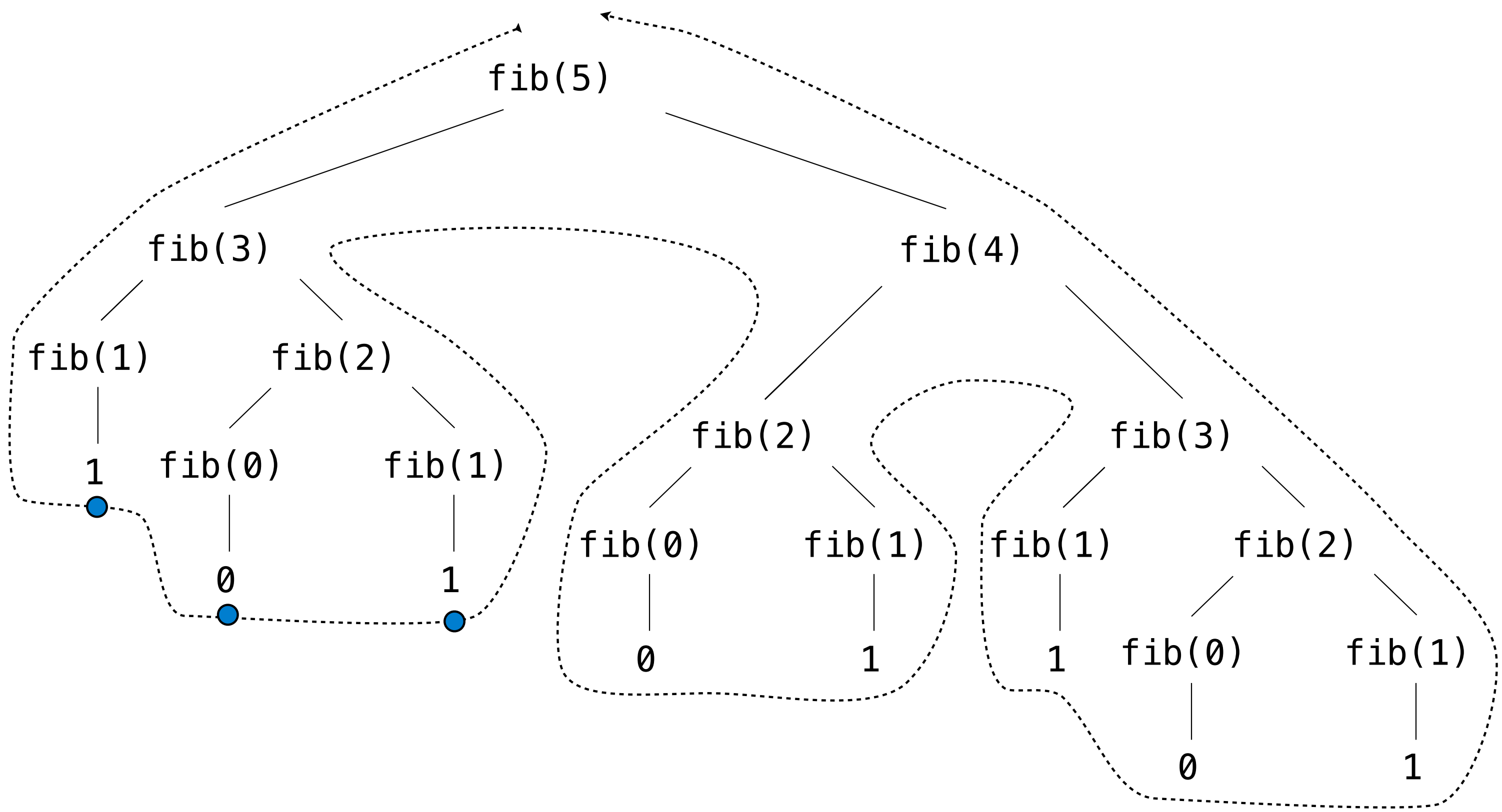

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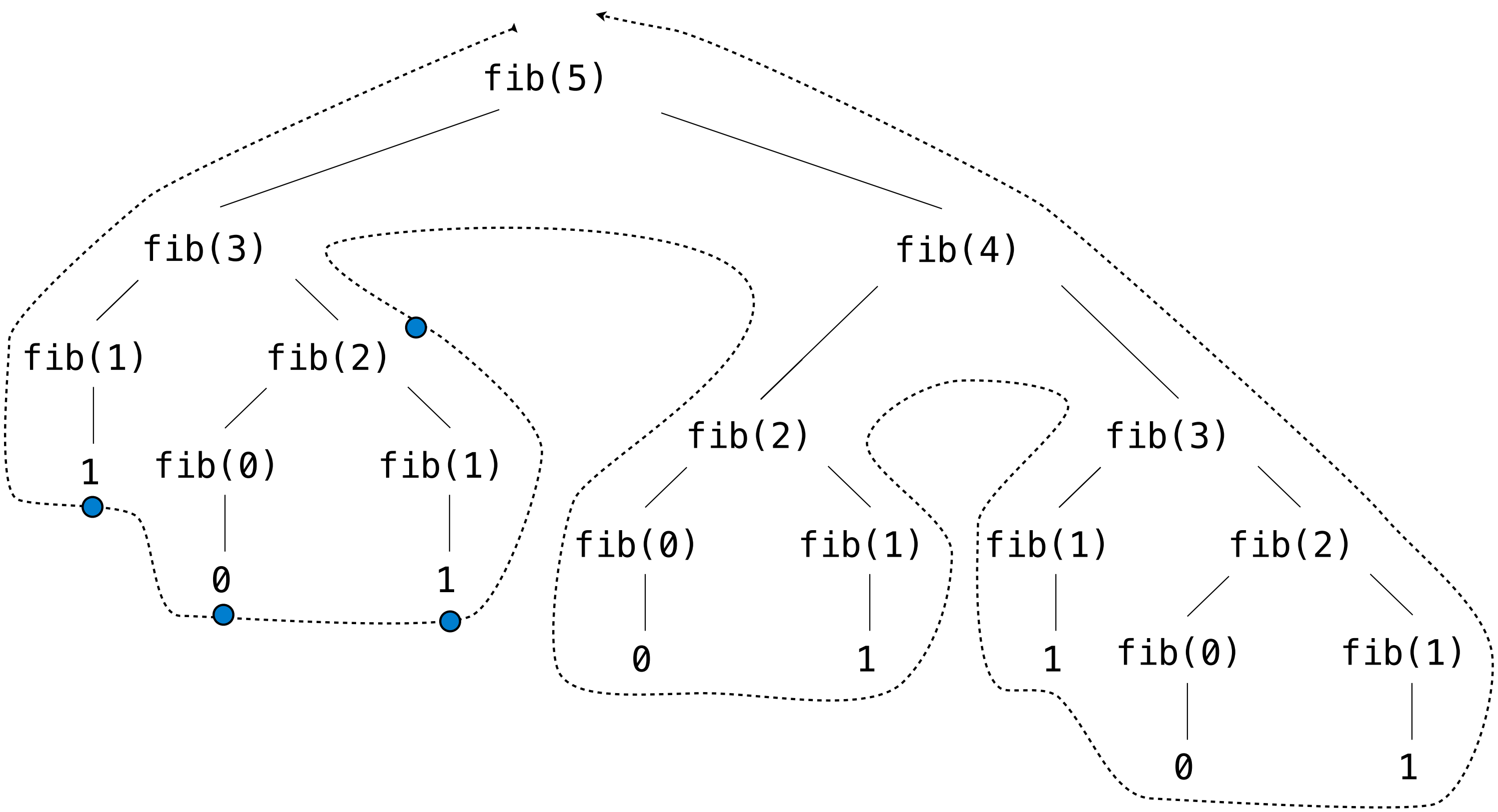

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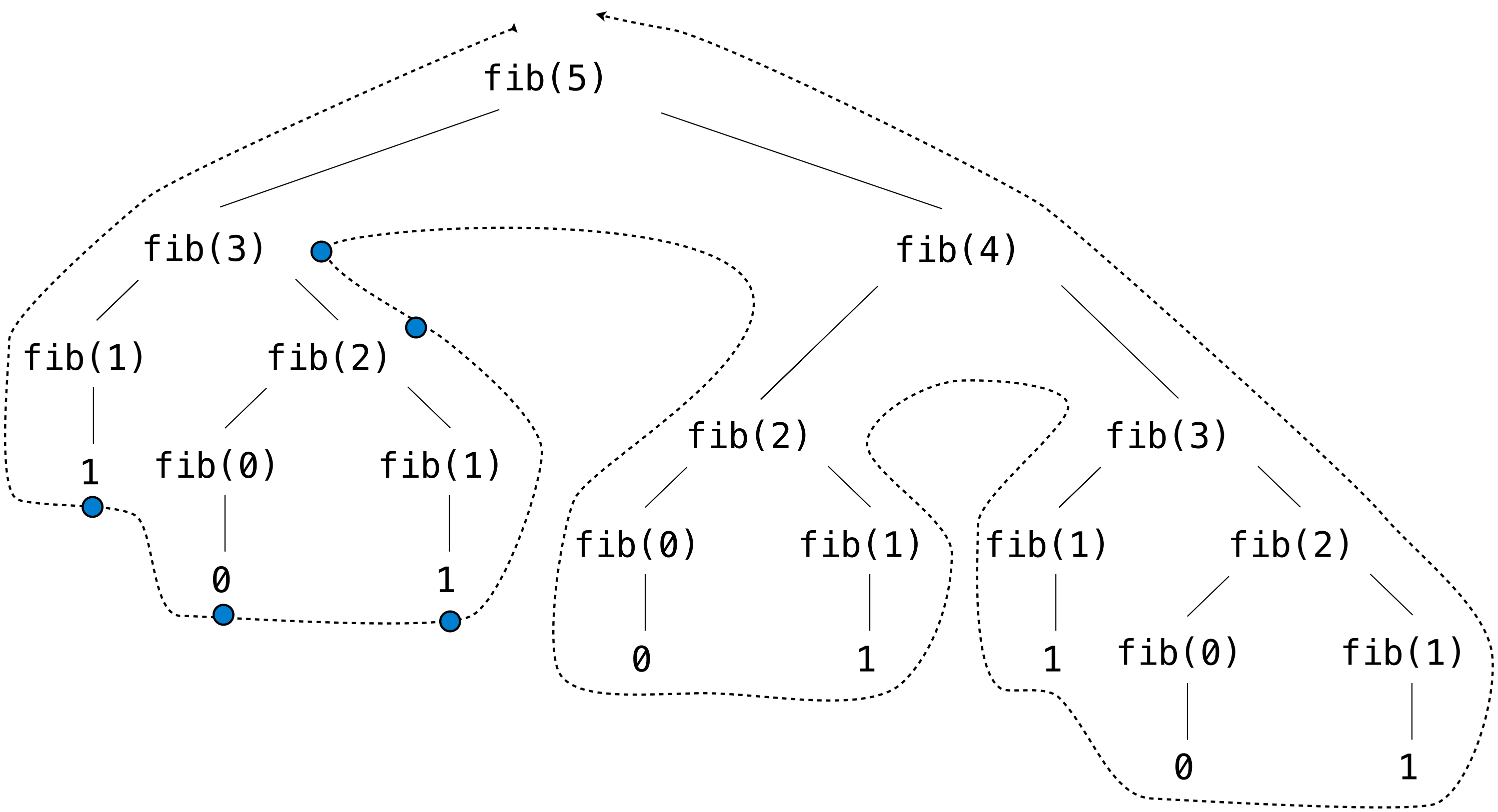

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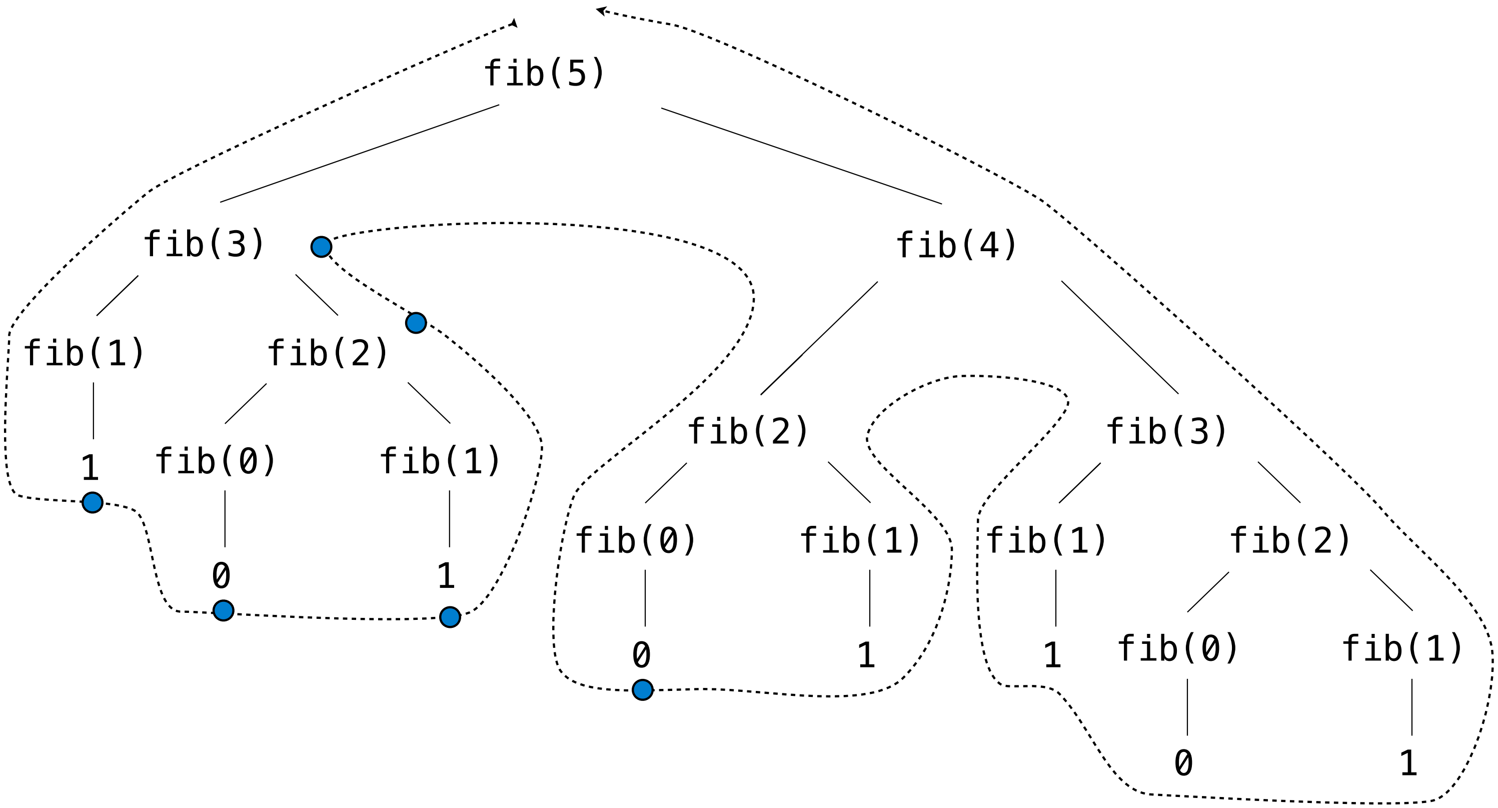

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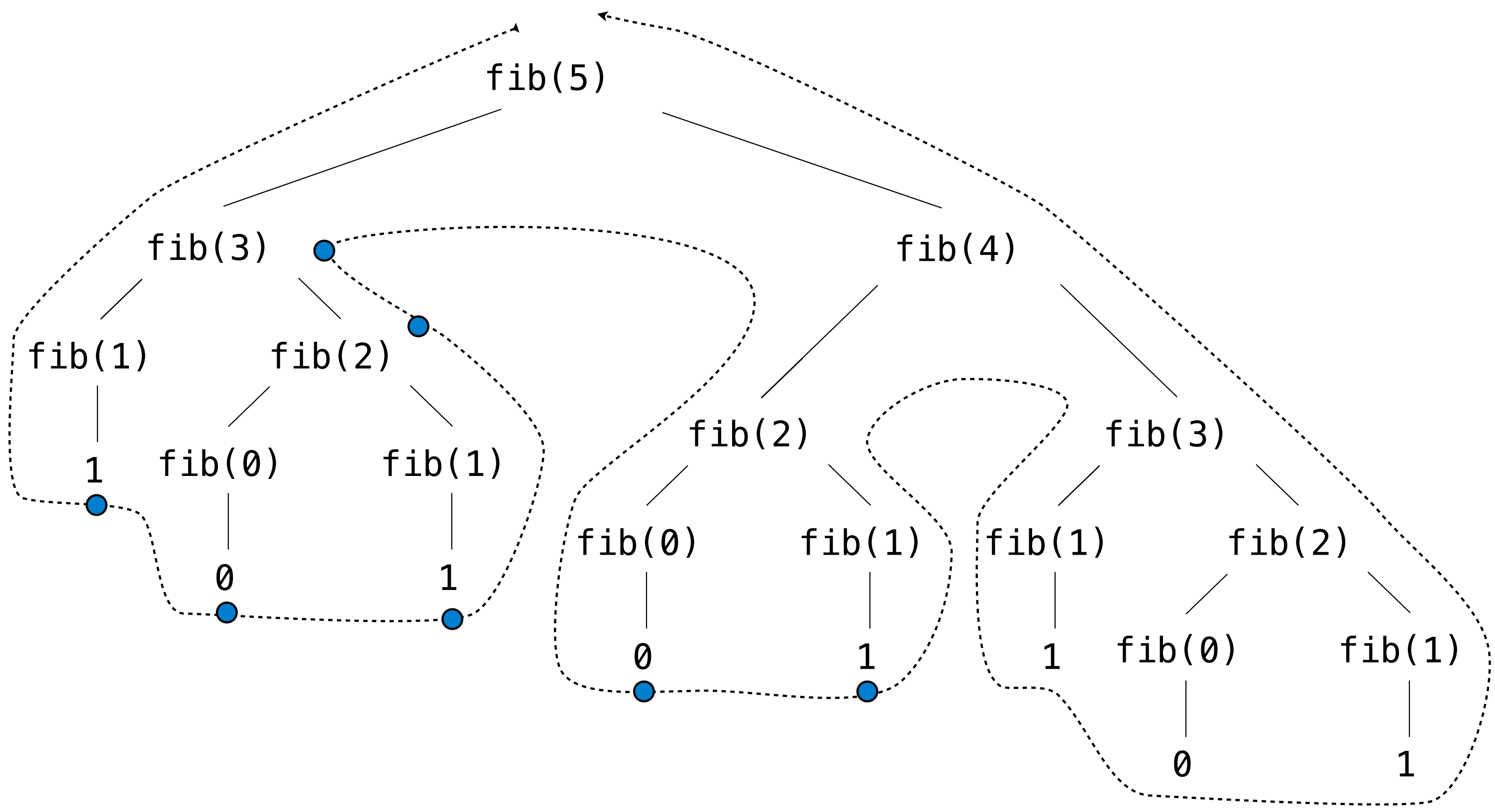

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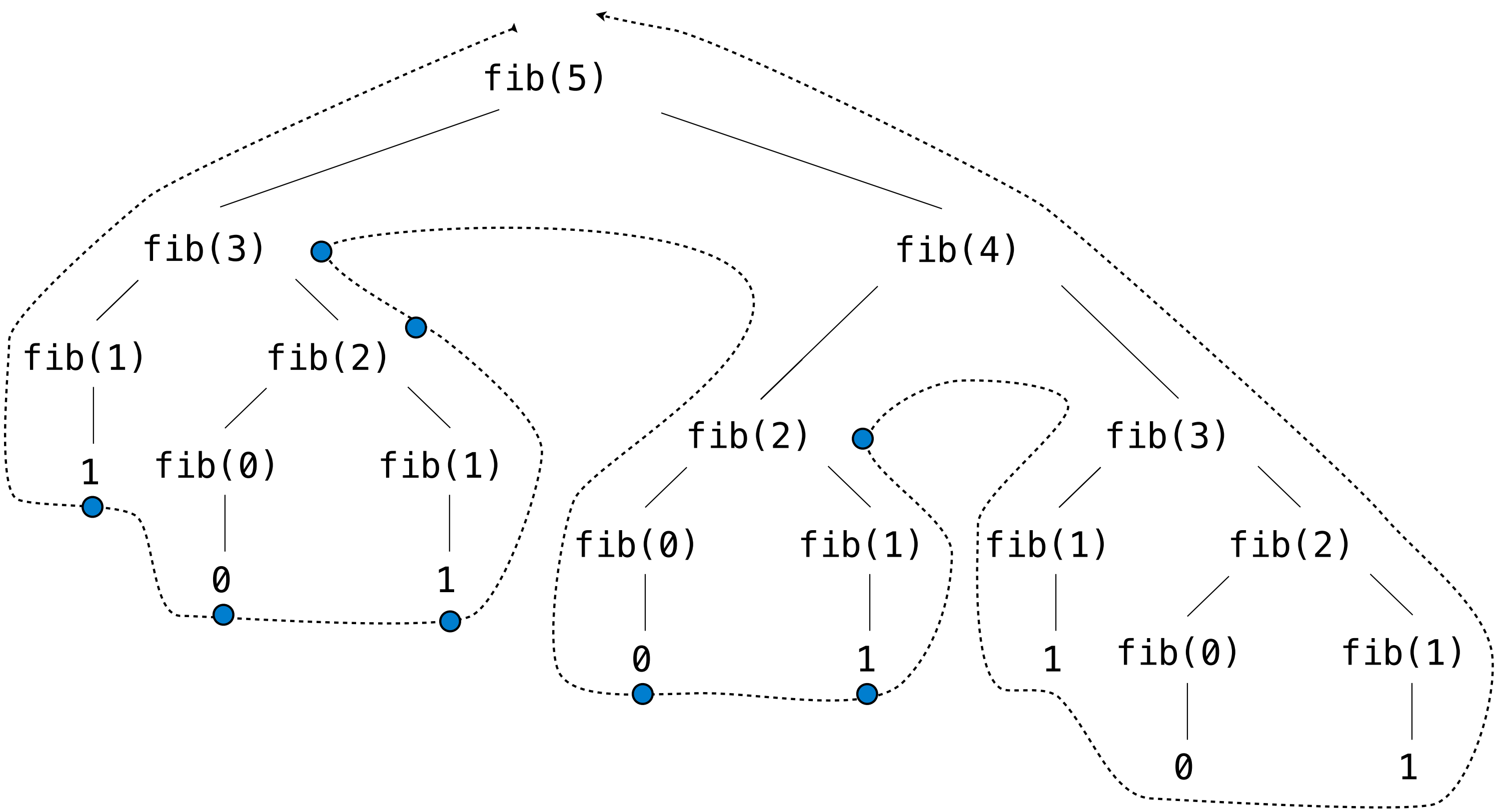

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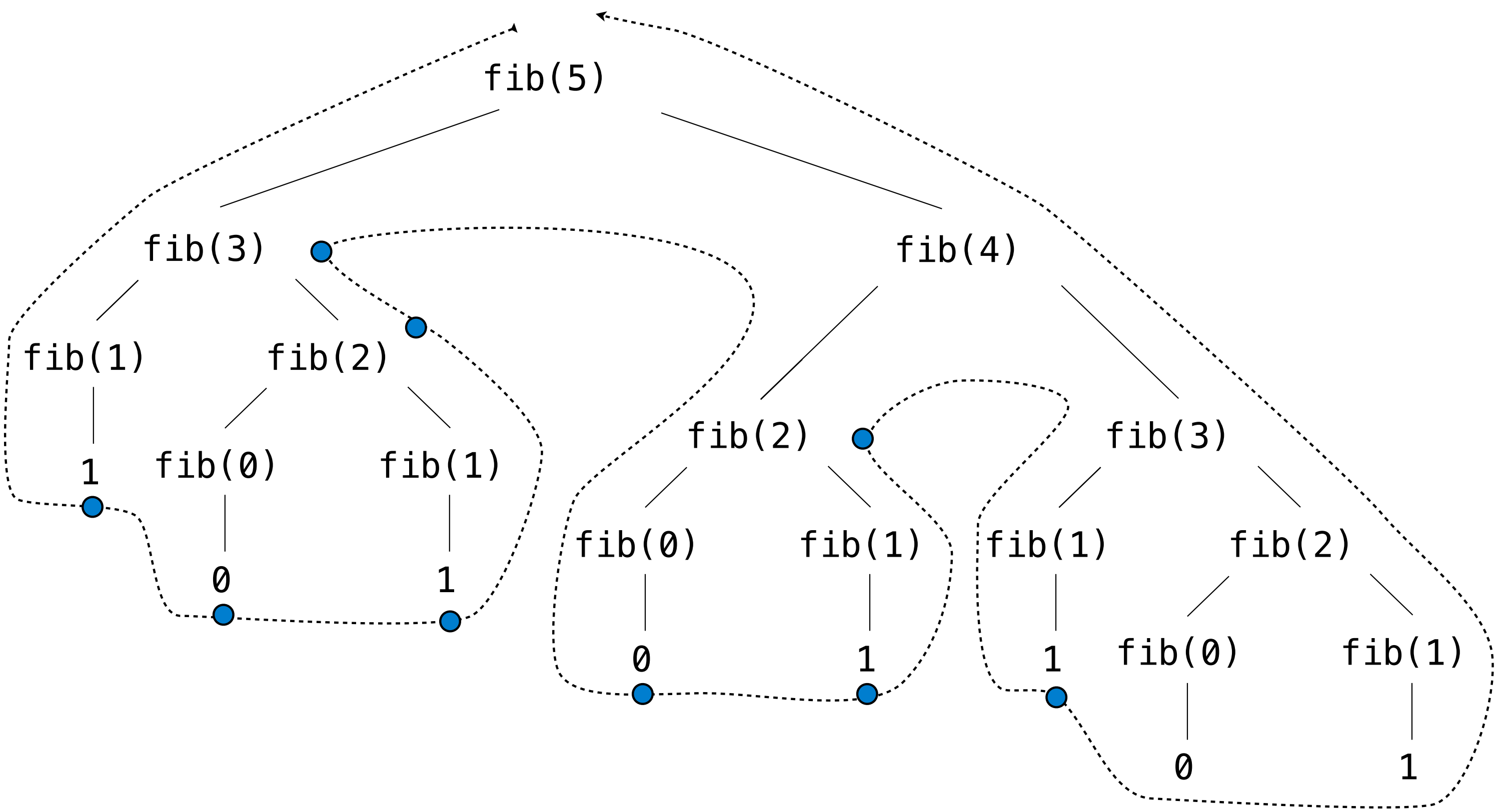

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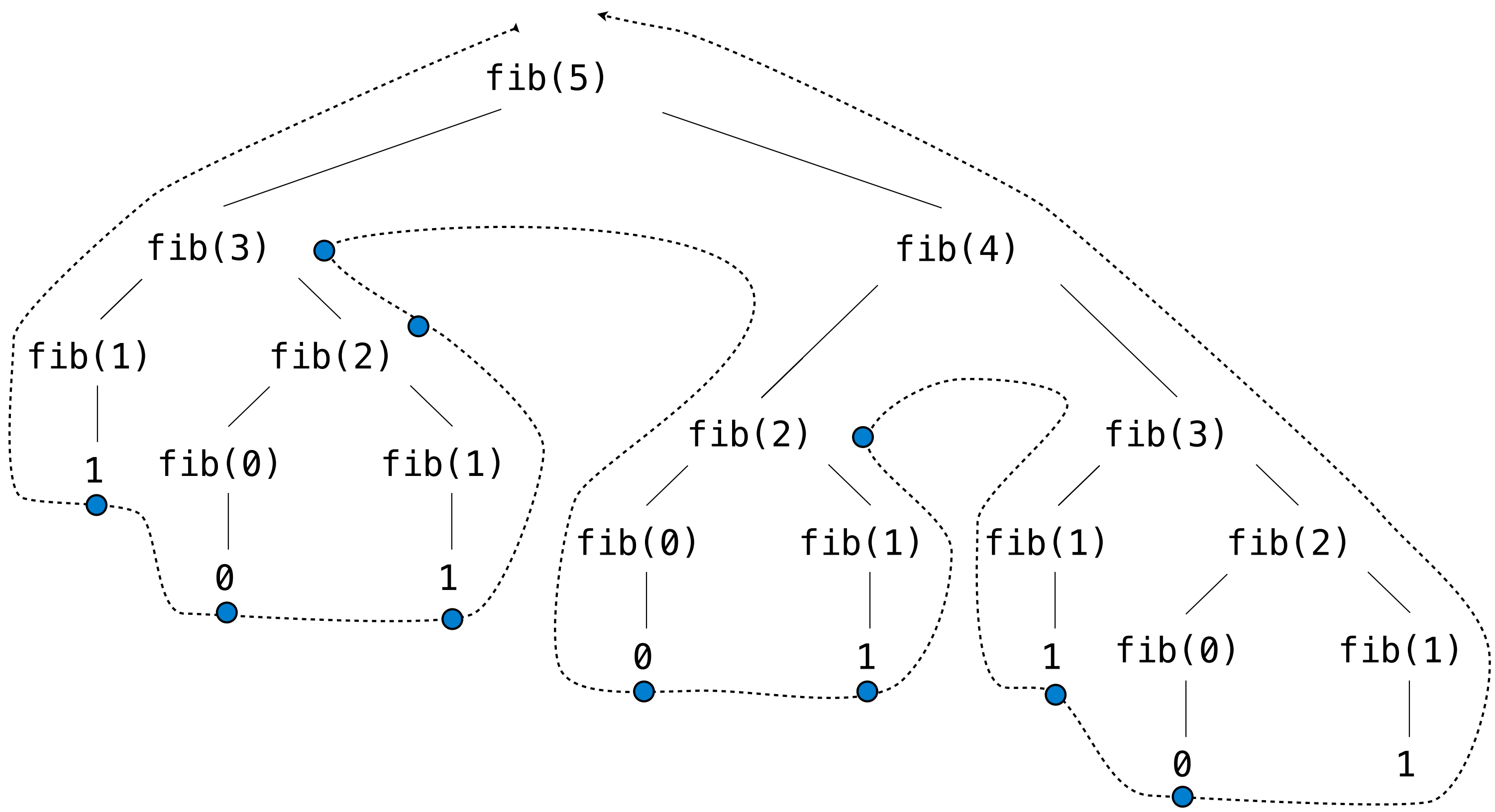

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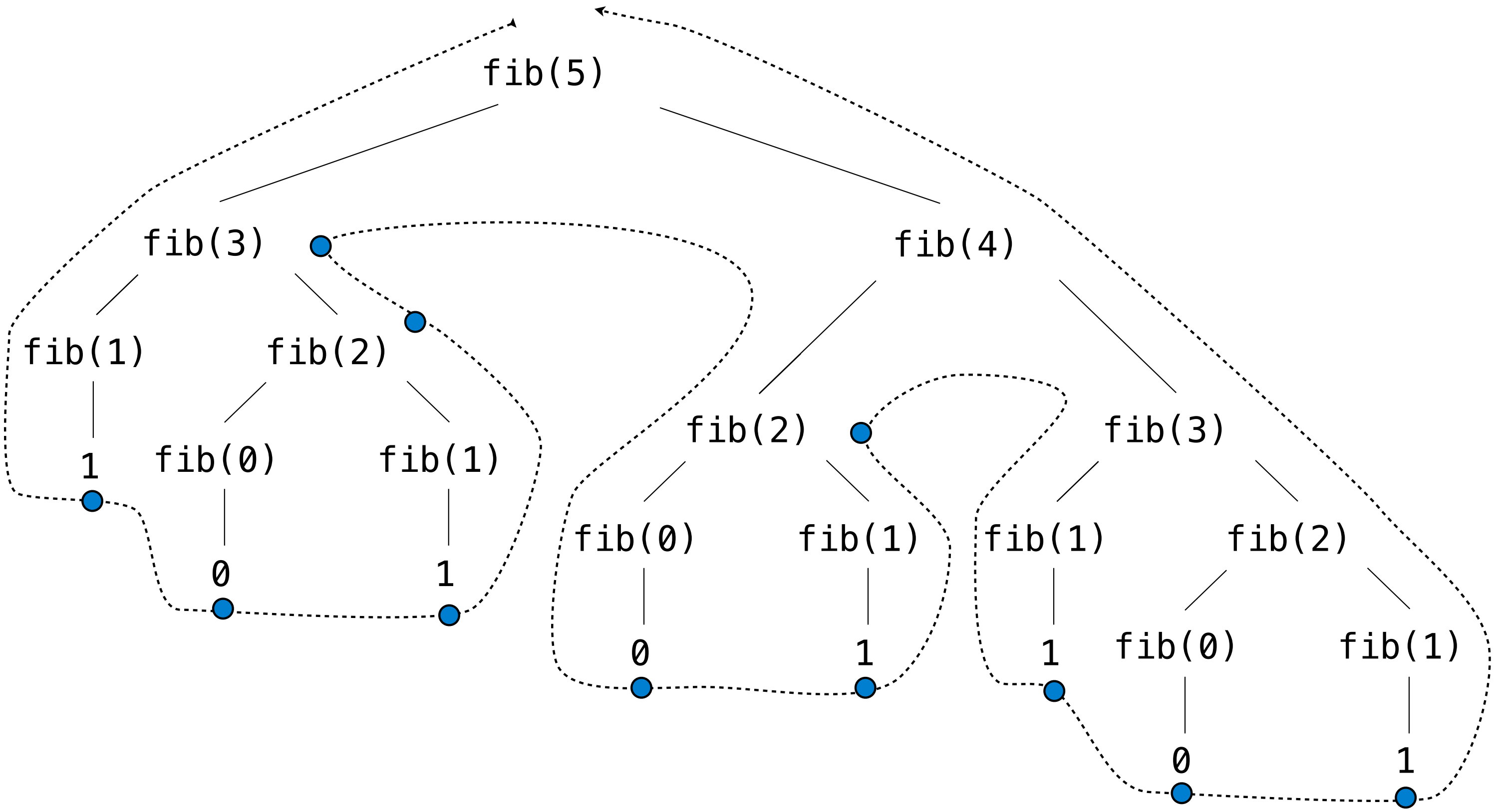

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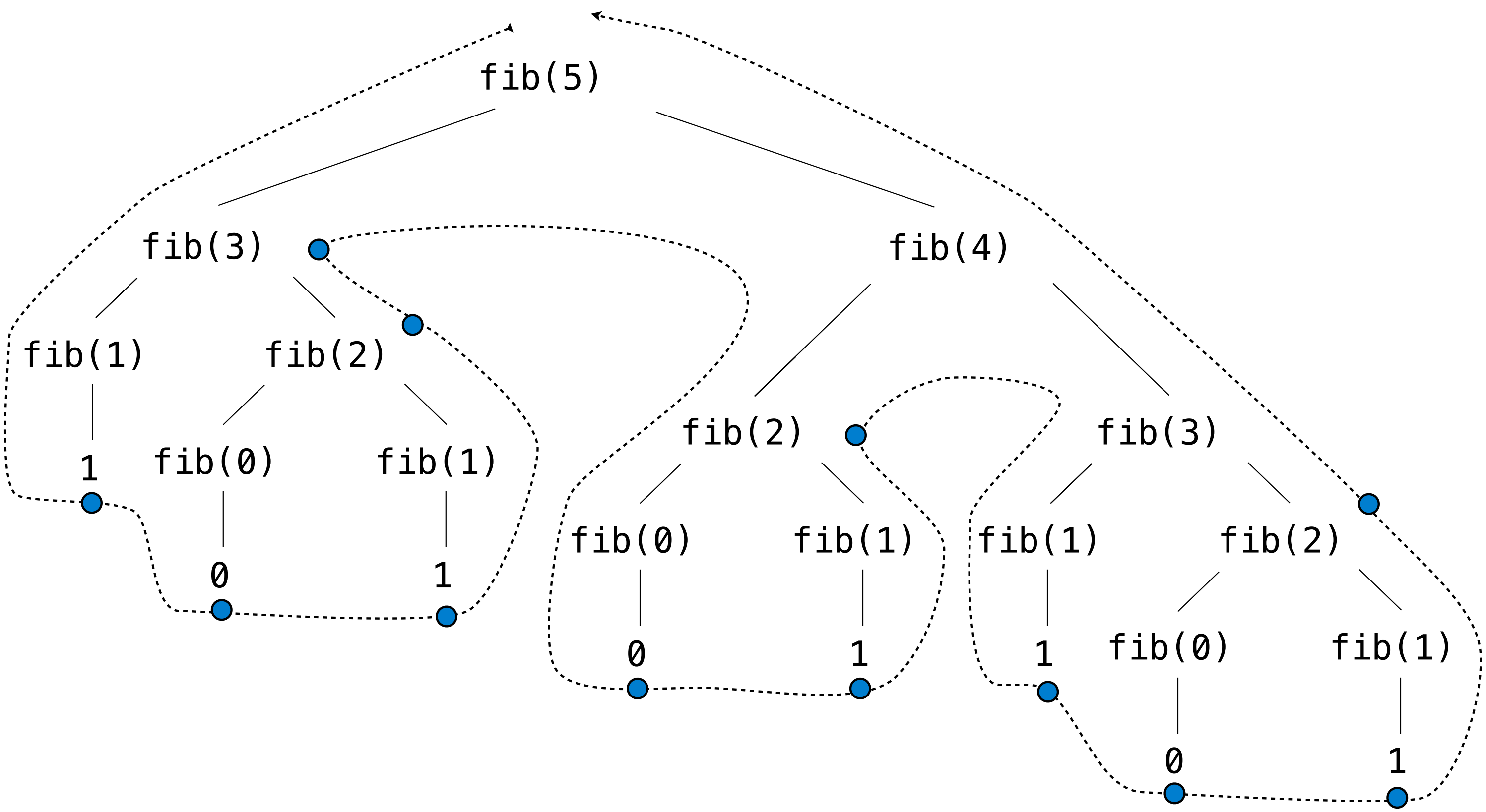

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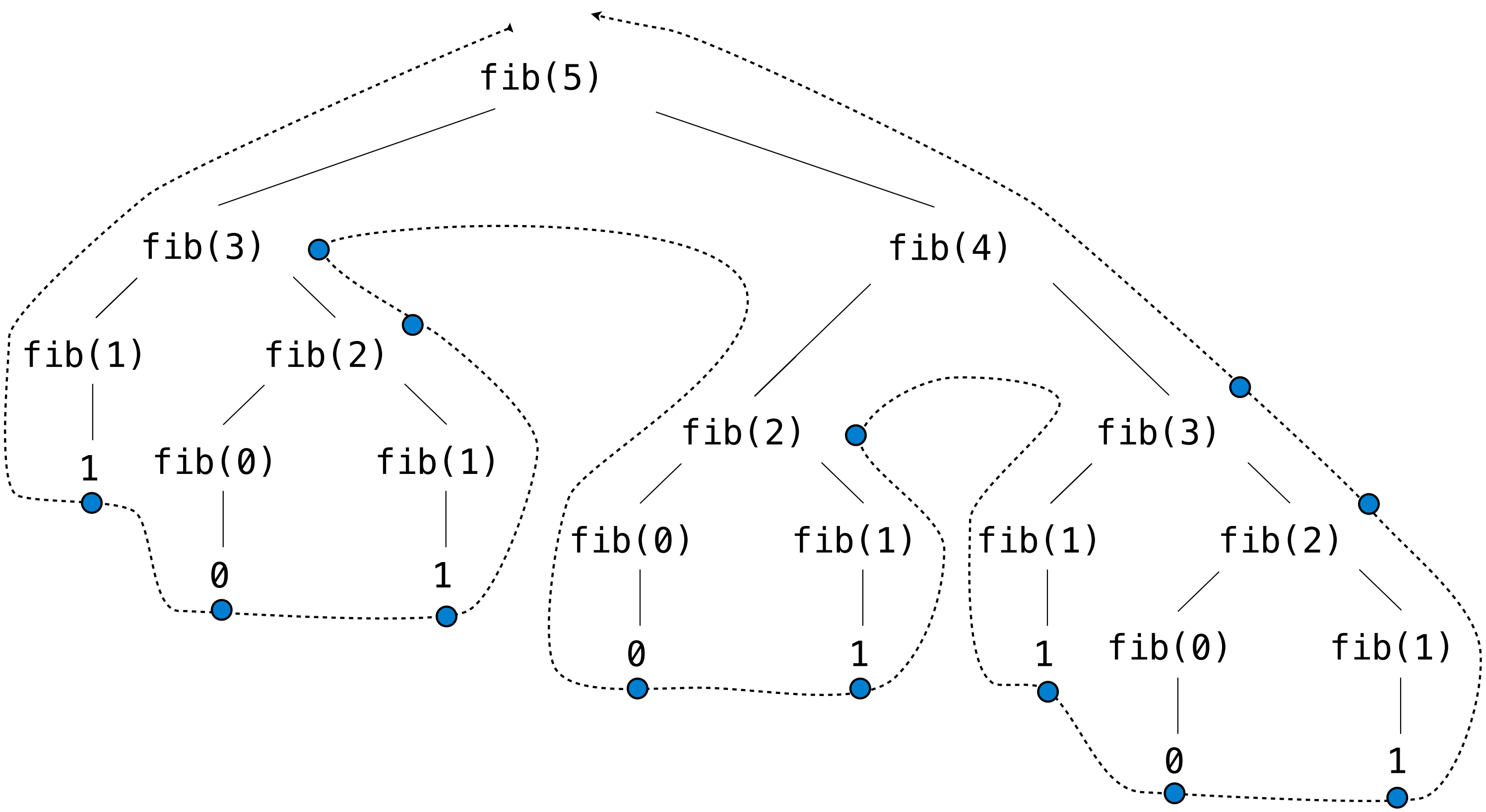

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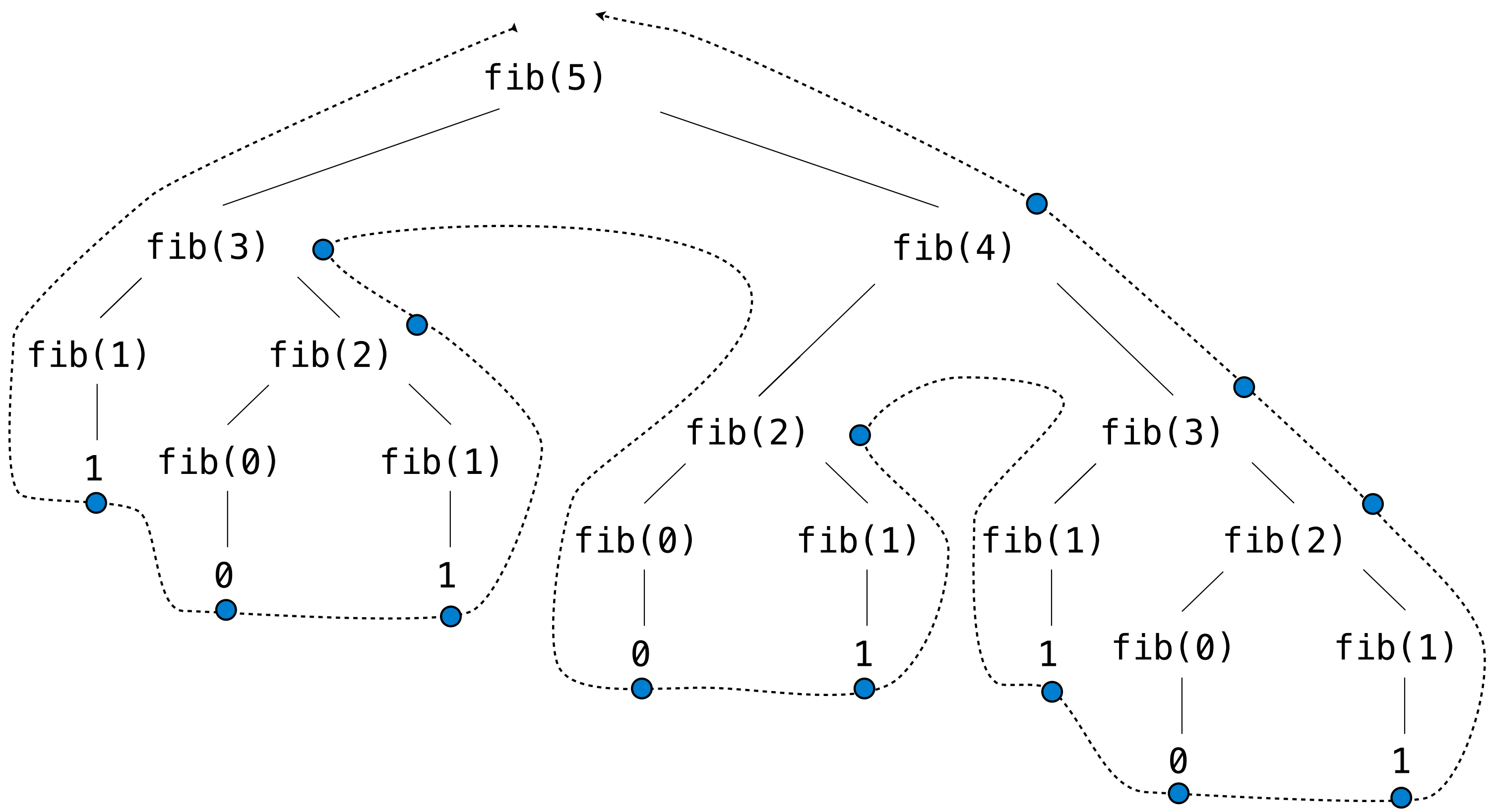

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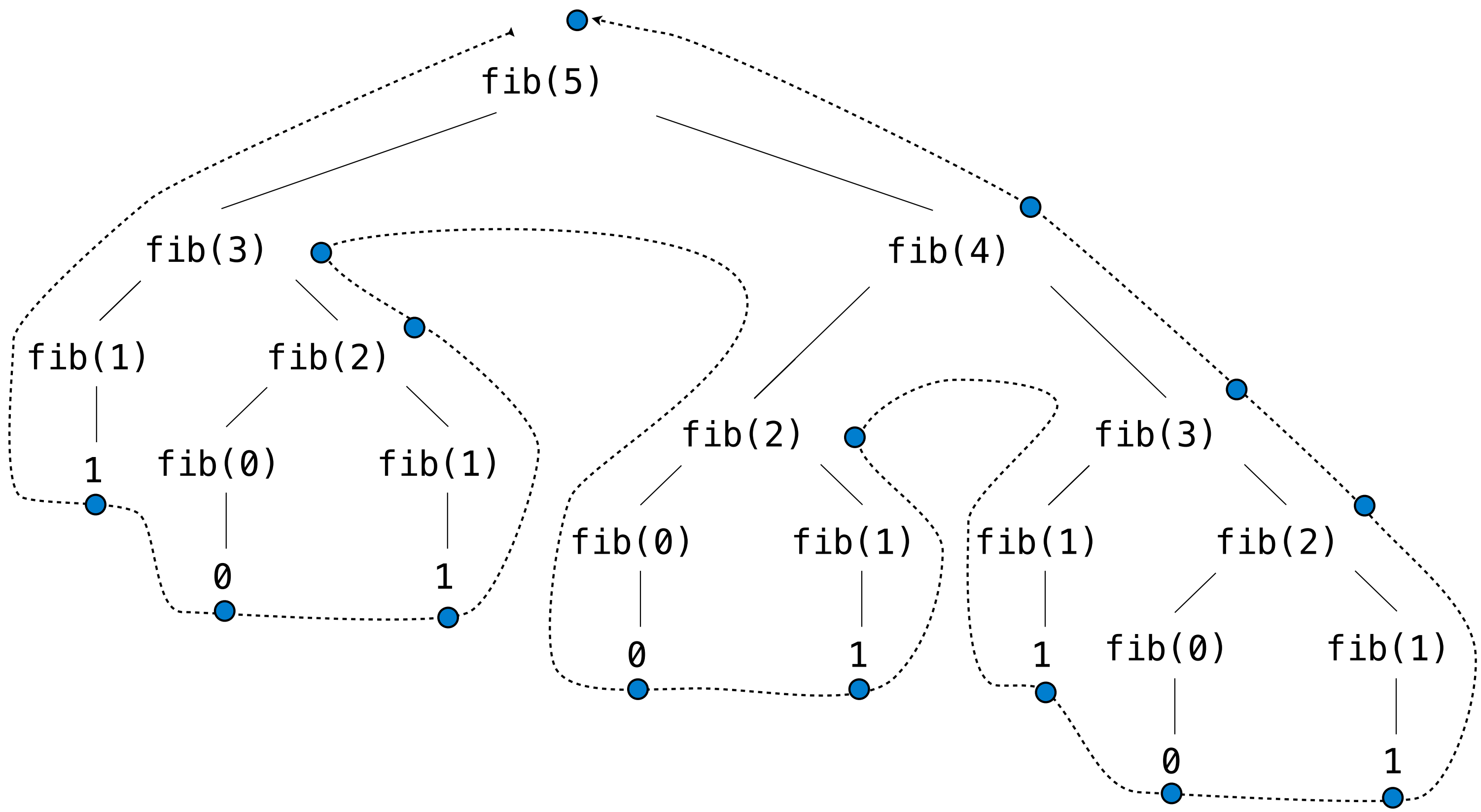

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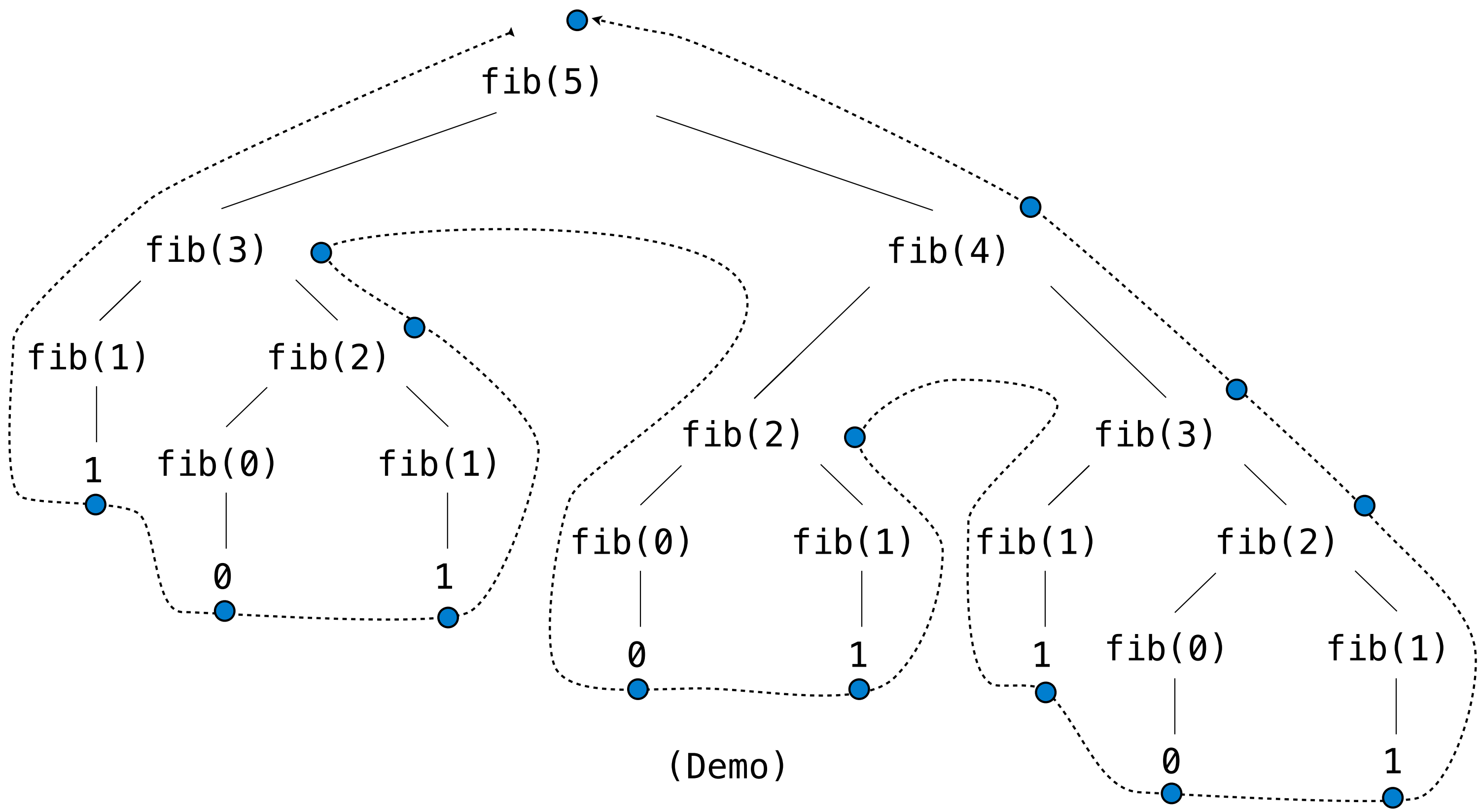

Repetition in Tree-Recursive Computation

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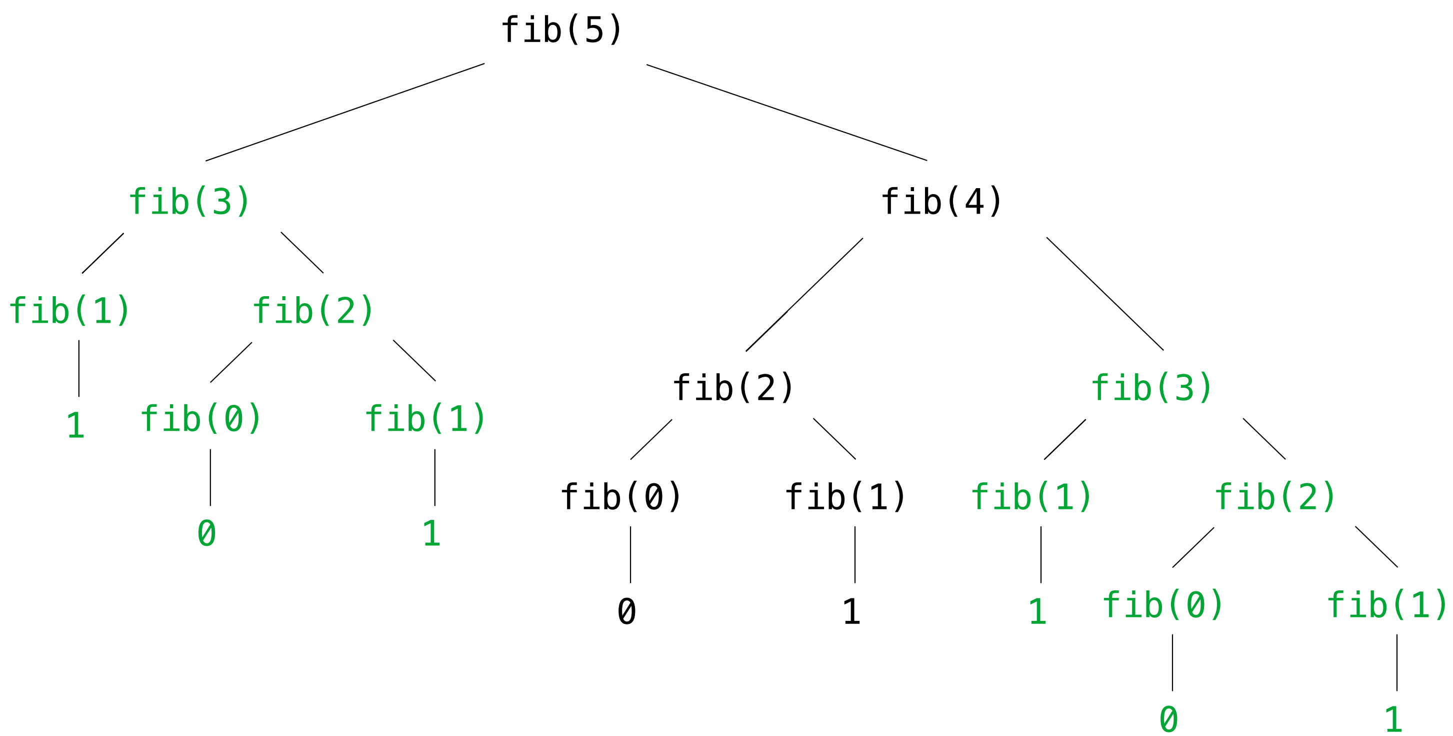

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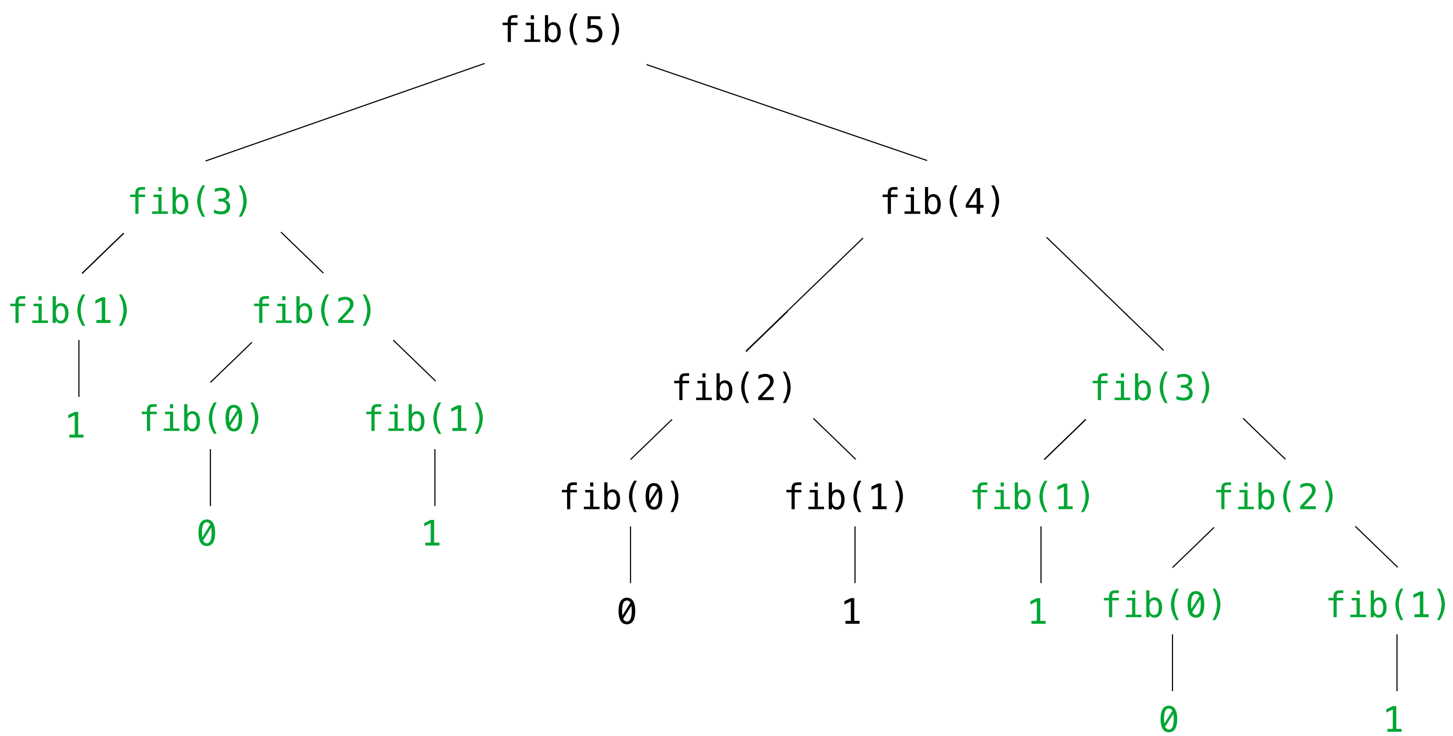
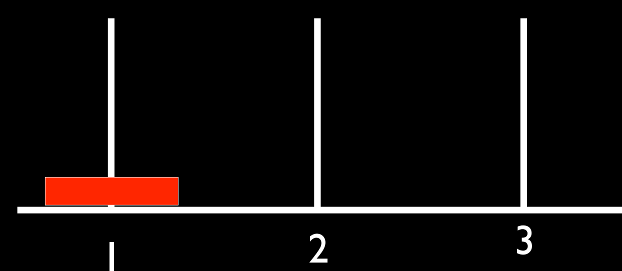


(We will speed up this computation dramatically in a few weeks by remembering results)

Example: Towers of Hanoi

Towers of Hanoi

n = 1: move disk from post 1 to post 2



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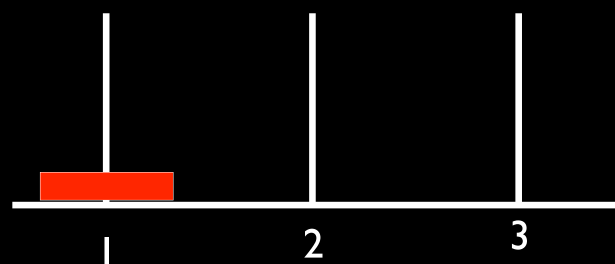

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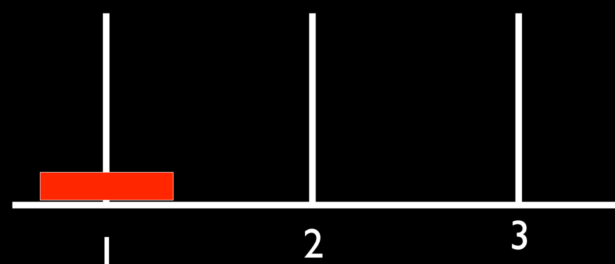

```
def move_disk(disk_number, from_peg, to_peg):
    print("Move disk " + str(disk_number) + " from peg " \
        + str(from_peg) + " to peg " + str(to_peg) + ".")


def solve_hanoi(n, start_peg, end_peg):
    if n == 1:
        move_disk(n, start_peg, end_peg)
    else:
```

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def solve_hanoi(n, start_peg, end_peg):
    if n == 1:
        move_disk(n, start_peg, end_peg)
    else:
        spare_peg = 6 - start_peg - end_peg
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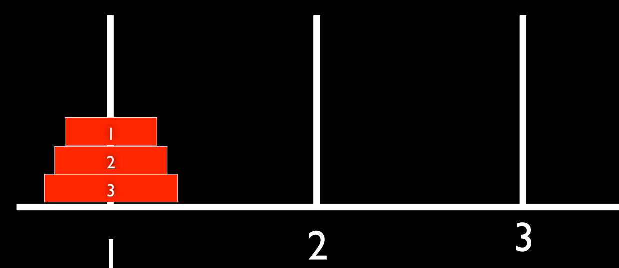
---

hanoi(3,1,2)



Example: Counting Partitions

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The number of partitions of a positive integer n, using parts up to size m, is the number of ways in which n can be expressed as the sum of positive integer parts up to m in increasing order.

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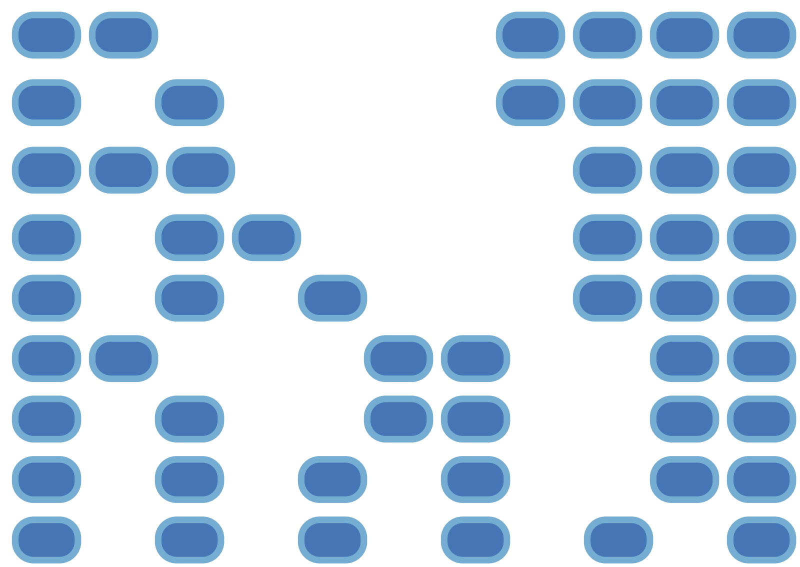

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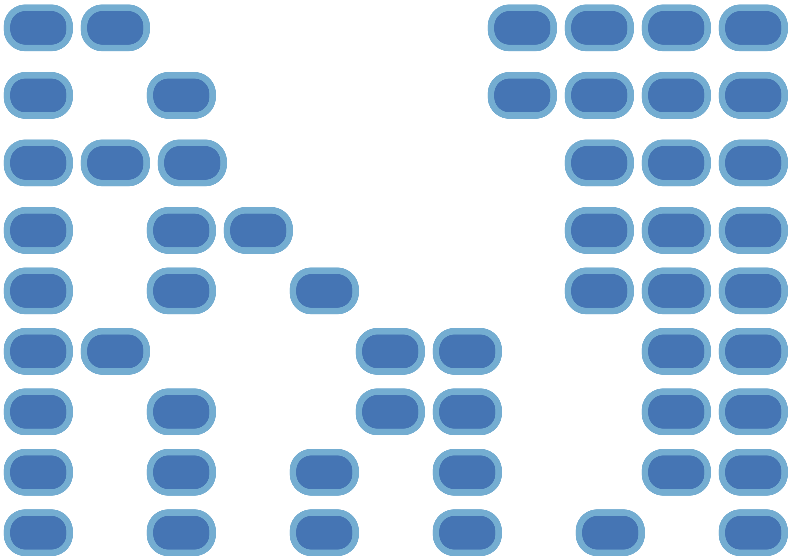

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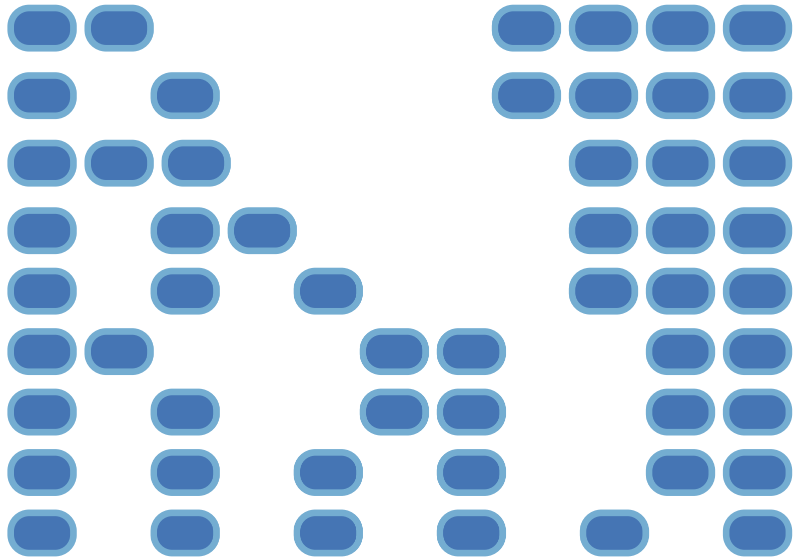

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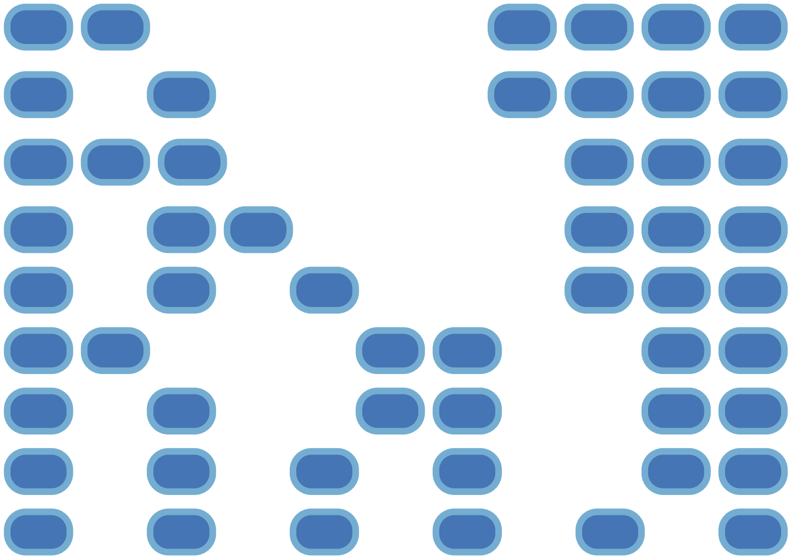

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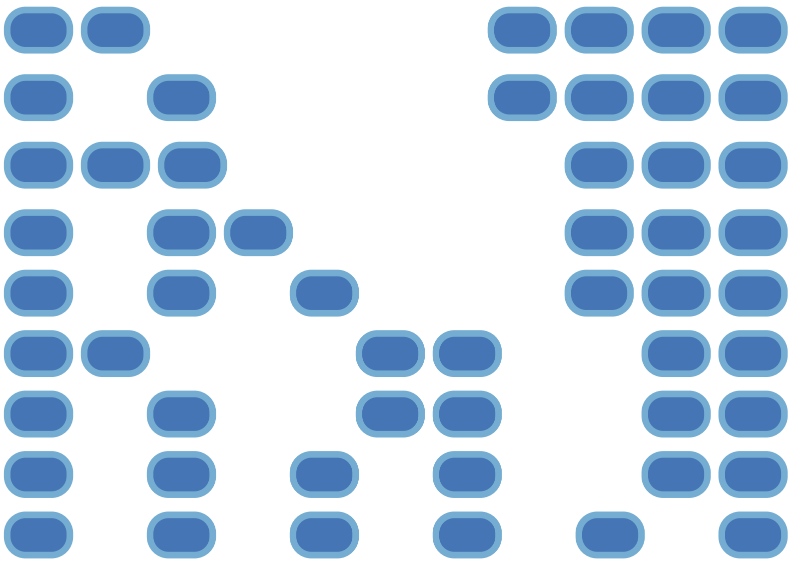

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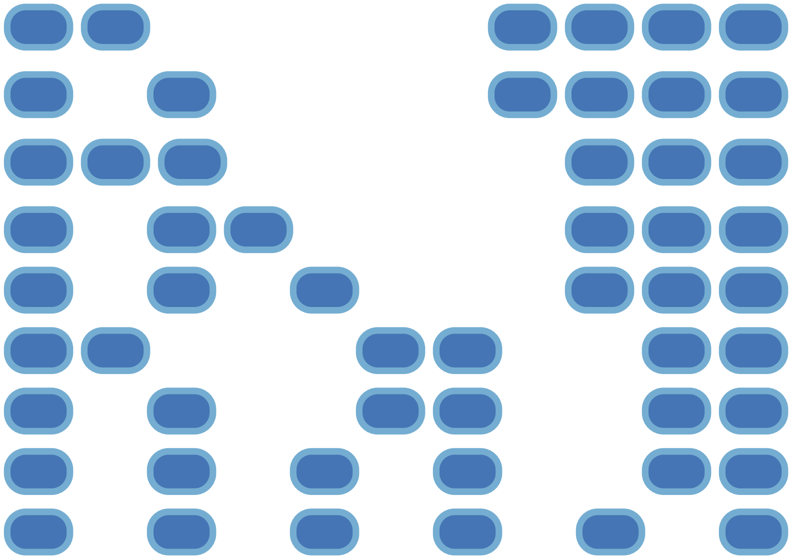

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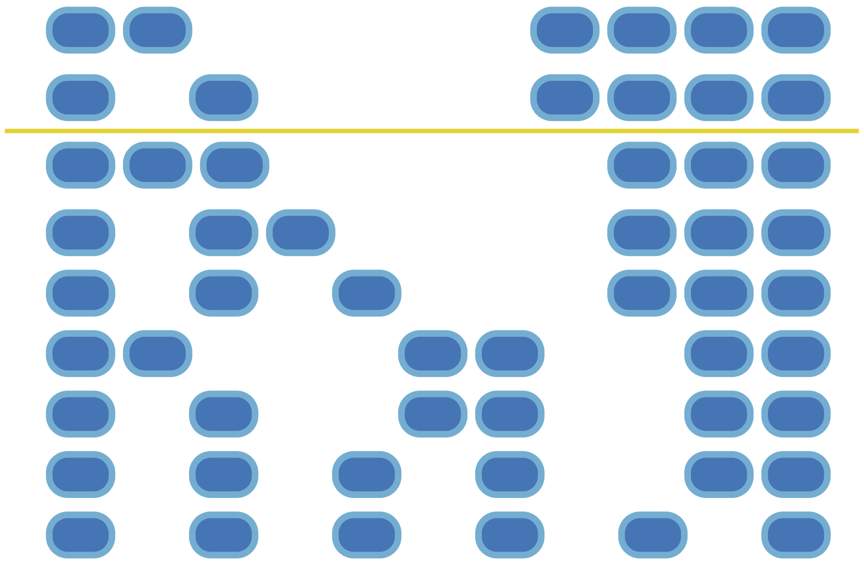

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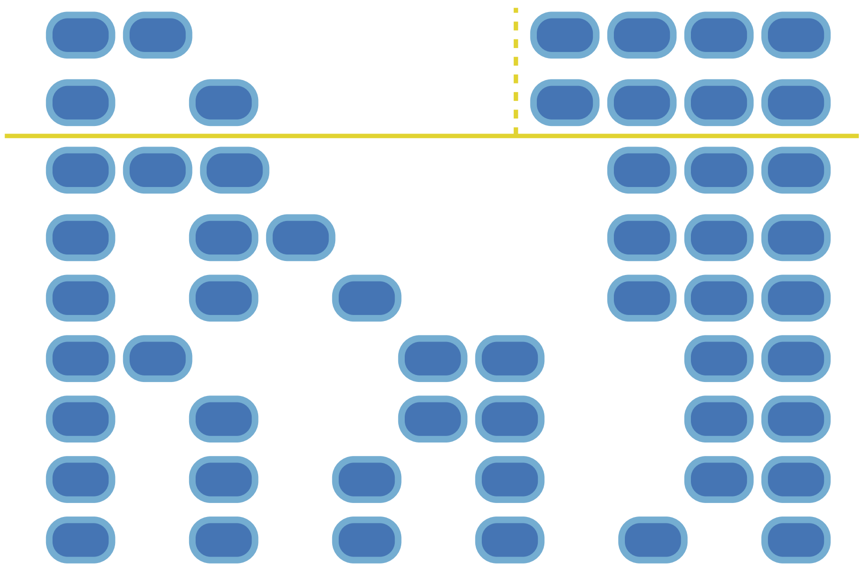

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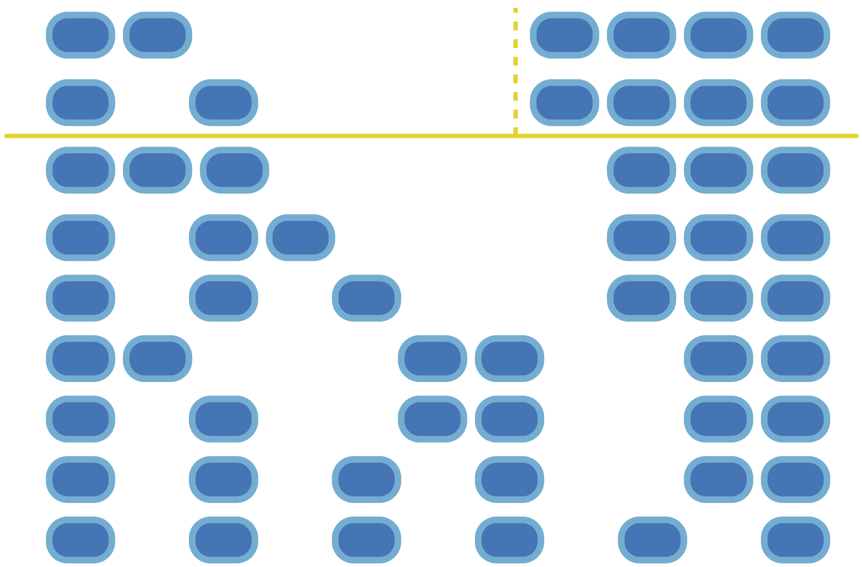

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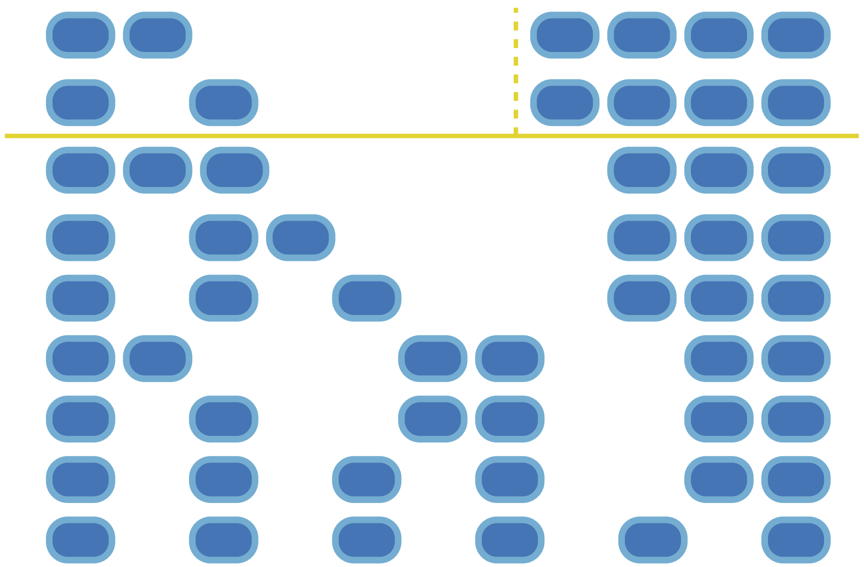

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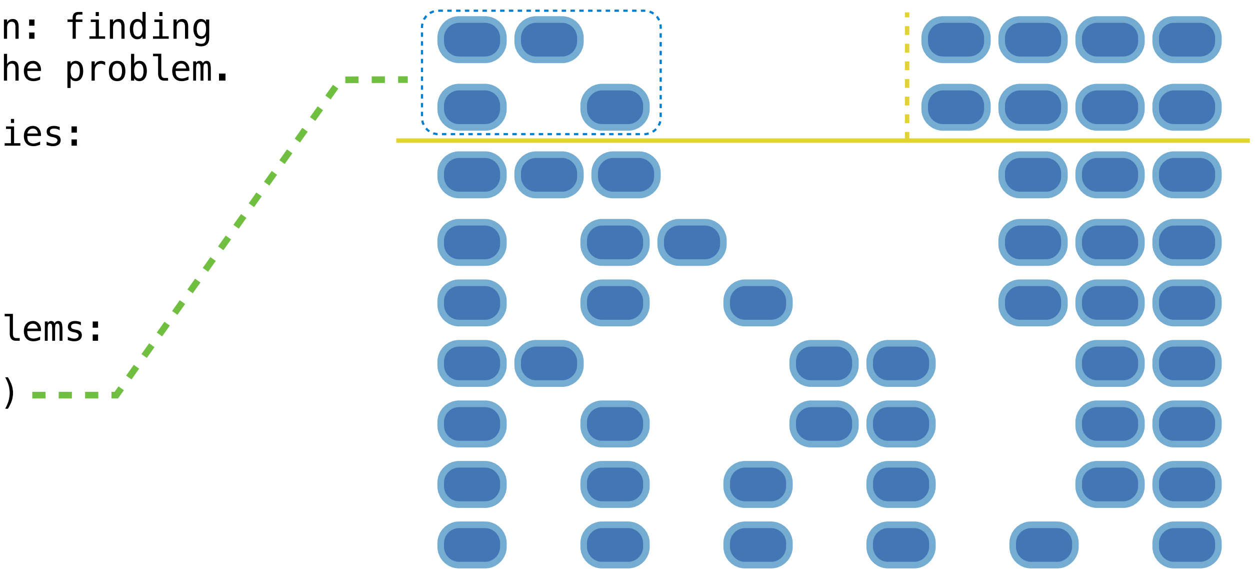

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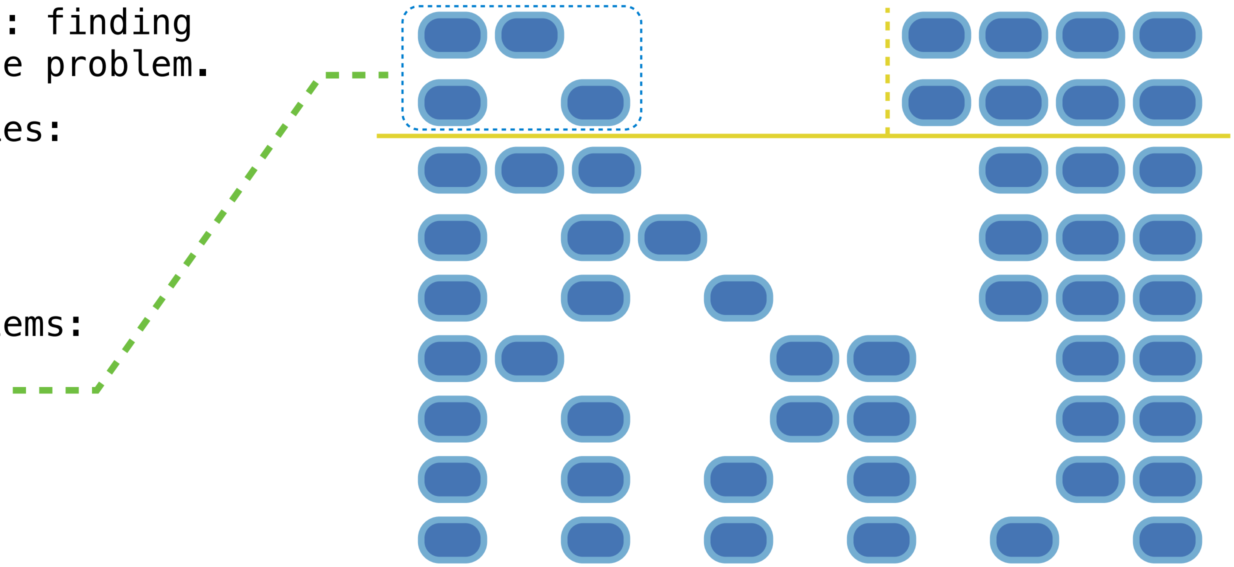

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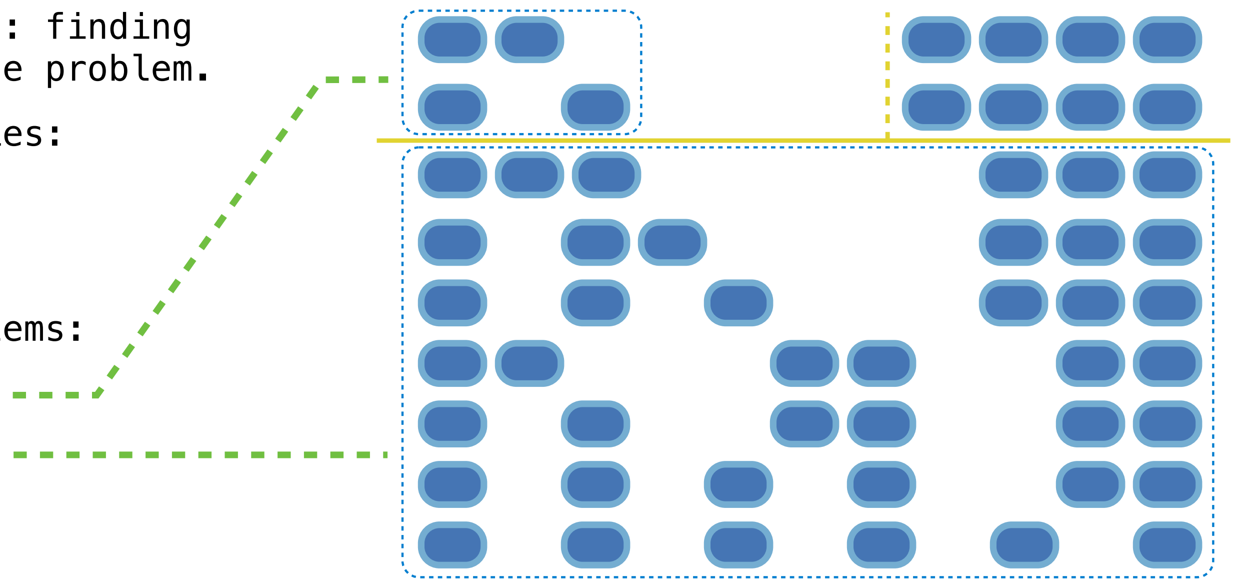

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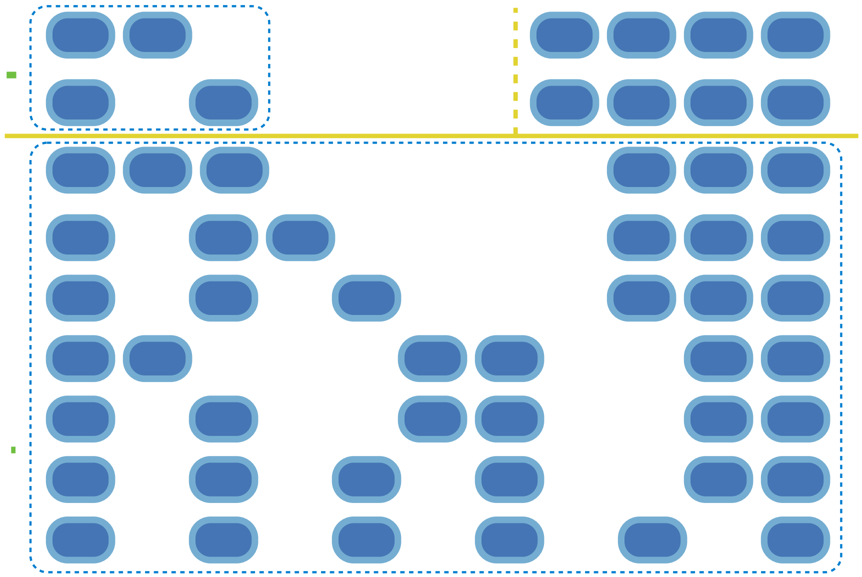

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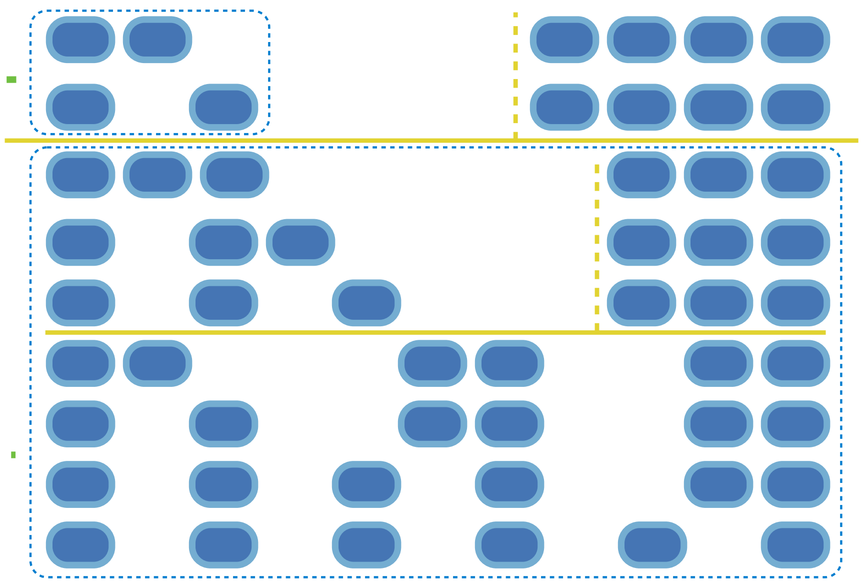

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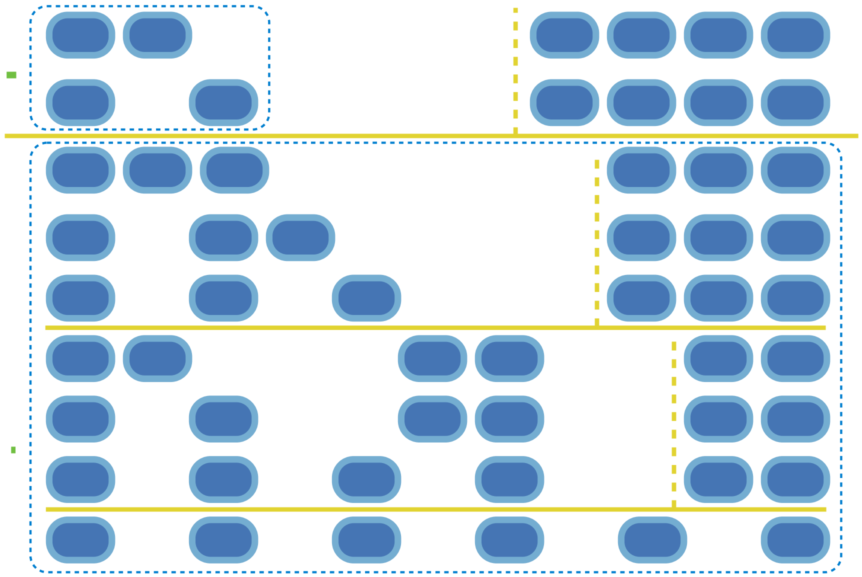

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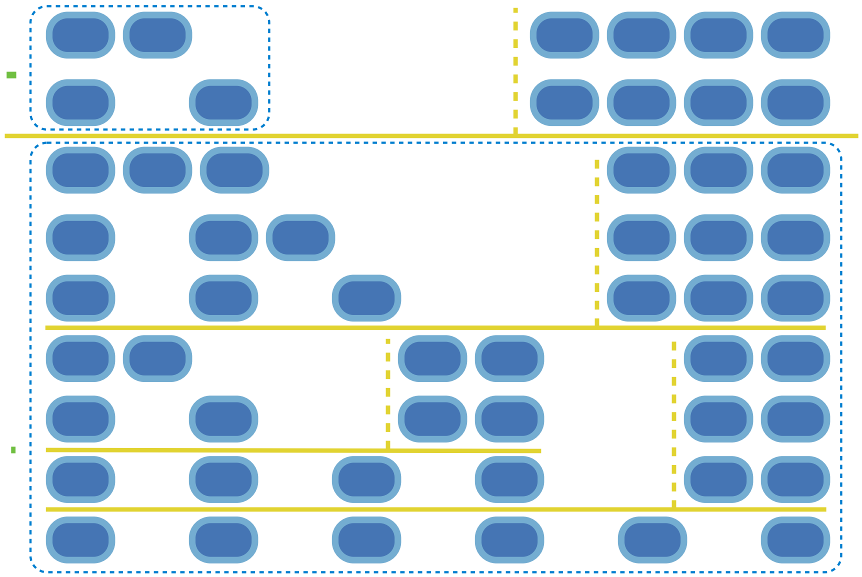

Counting Partitions

The number of partitions of a positive integer n, using parts up to size m, is the number of ways in which n can be expressed as the sum of positive integer parts up to m in increasing order.

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    else:
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```
def count_partitions(n, m):
    if n == 0:



    else:
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```
def count_partitions(n, m):
    if n == 0:
        return 1



    else:
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def count_partitions(n, m):
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(Demo)

pythontutor.com/composingprograms.html#code=def%20count_partitions%28n,%20m%29%3A%0A%20%20%20%20if%20n%20%3D%3D%200%3A%0A%20%20%20%20%20%20%20%20return%201%0A%20%20%20%20elif%20n%20<%200%3A%0A%20%20%20%20%20%20%20%20return%200%0A%20%20%20%20elif%20m%20%3D%3D%200%3A%0A%20%20%20%20%20%20%20%20return%200%0A%20%20%20%20else%3A%0A%20%20%20%20%20%20%20%20with_m%20%3D%20count_partitions%28n-m,
%20m%29%20%0A%20%20%20%20%20%20%20%20without_m%20%3D%20count_partitions%28n,
%20m-1%29%0A%20%20%20%20%20%20%20%20return%20with_m%20%2B%20without_m%0A%20%20%20%20%20%20%20%20%0Aresult%20%3D%20count_partitions%285,%203%29%0A%0A#%201%20%2B%201%20%2B%201%20%2B%201%20%2B%201%20%3D%205%0A#%201%20%2B%201%20%2B%201%20%2B%202%20%2B%20%20%20%3D%205%0A#%201%20%2B%202%20%2B%202%20%2B%20%20%20%20%20%20%20%3D%205%0A#%201%20%2B%201%20%2B%203%20%2B%20%20%20%20%20%20%20%3D%205%0A#
20%2B%203%20%2B%20%20%20%20%20%20%20%20%20%20%20%3D%205&mode=display&origin=composingprograms.js&cumulative=false&py=3&rawInputLstJSON=[]&curInstr=0