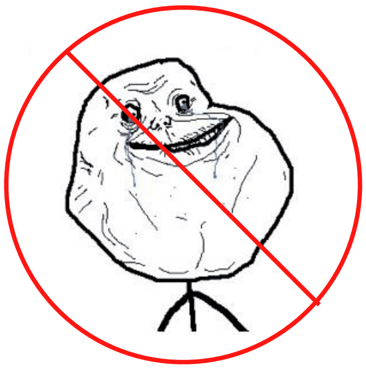
Higher-Order Functions

Announcements

Office Hours: You Should Go!

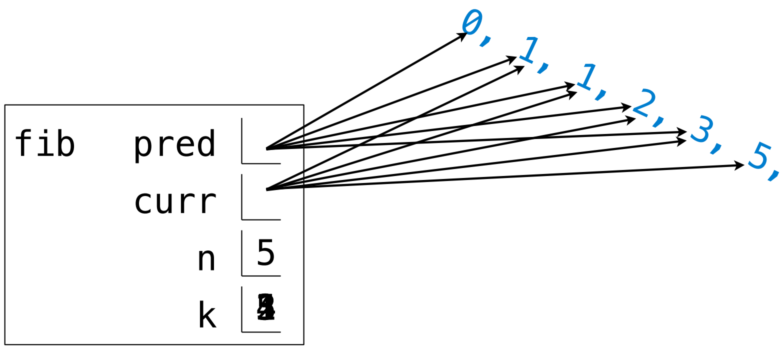
You are not alone!



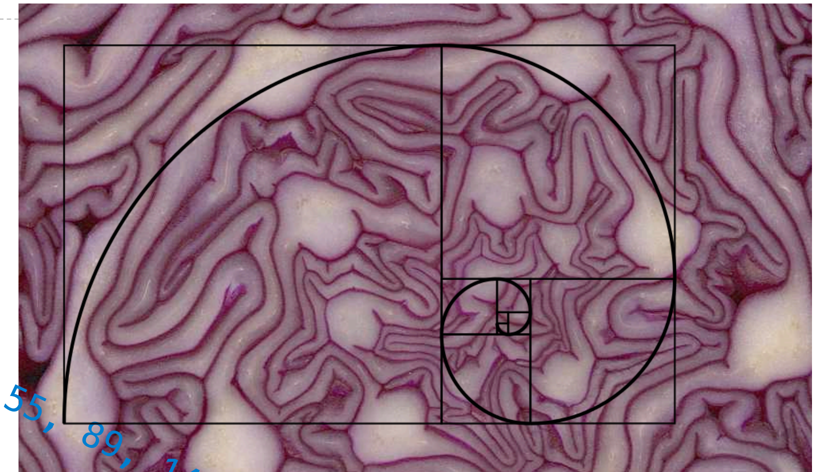
http://cs61a.org/office-hours.html

Iteration Example

The Fibonacci Sequence



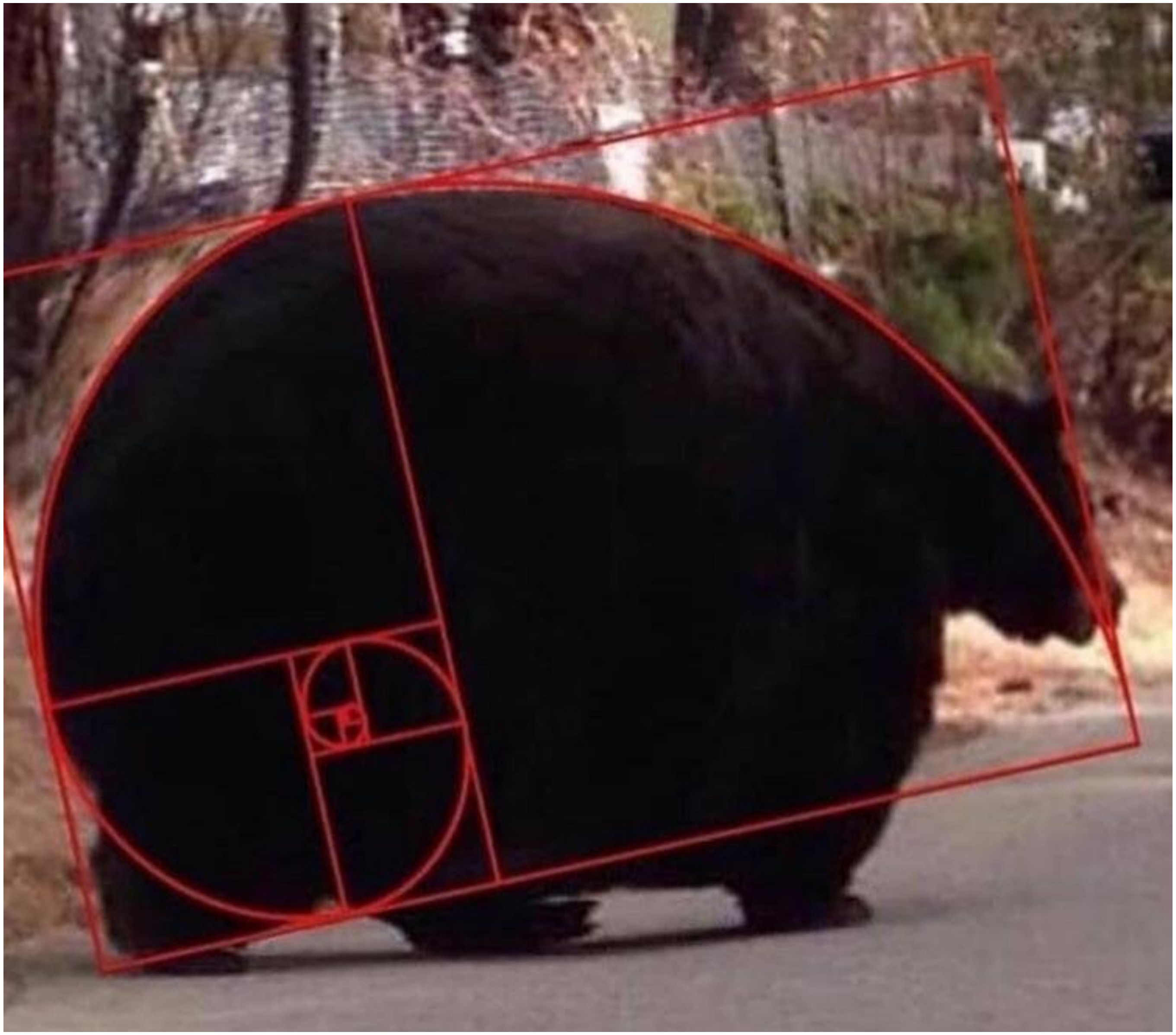
0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987



```
def fib(n):
    """Compute the nth Fibonacci number, for N >= 1."""
    pred, curr = 0, 1   # 0th and 1st Fibonacci numbers
    k = 1               # curr is the kth Fibonacci number
    while k < n:
        pred, curr = curr, pred + curr
        k = k + 1
    return curr
```

The next Fibonacci number is the sum of the current one and its predecessor

Go Bears!



Designing Functions

Describing Functions

```
def square(x):
    """Return X * X."""
```

A function's *domain* is the set of all inputs it might possibly take as arguments.

x is a number

A function's *range* is the set of output values it might possibly return.

square returns a non-negative real number

A pure function's *behavior* is the relationship it creates between input and output.

square returns the square of x

A Guide to Designing Function

Give each function exactly one job, but make it apply to many related situations

```
>>> round(1.23)
1
```

```
>>> round(1.23, 1)
1.2
```

```
>>> round(1.23, 0)
1
```

```
>>> round(1.23, 5)
1.23
```

Don’t repeat yourself (DRY): Implement a process just once, but execute it many times

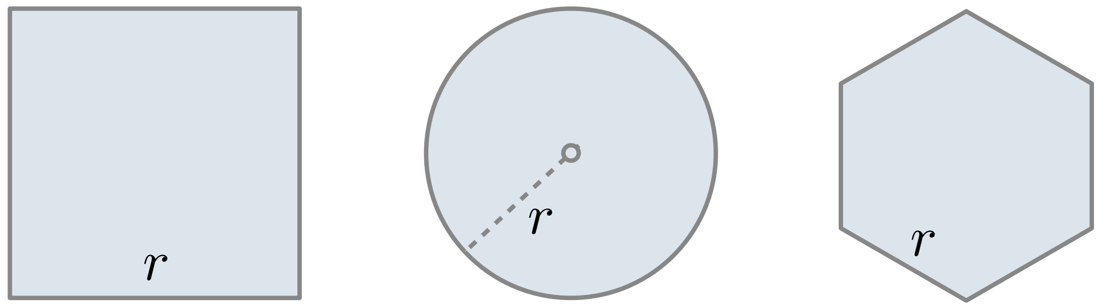
(Demo)

Generalization

Generalizing Patterns with Arguments

Regular geometric shapes relate length and area.

Shape:



Area: $\boxed{1} \cdot r^2$ $\quad$ $\boxed{\pi} \cdot r^2$ $\quad$ $\boxed{\frac{3\sqrt{3}}{2}} \cdot r^2$

Finding common structure allows for shared implementation

(Demo)

Higher-Order Functions

Generalizing Over Computational Processes

The common structure among functions may be a computational process, rather than a number.

$$\sum_{k=1}^{5} \boxed{k} = 1 + 2 + 3 + 4 + 5 \qquad = 15$$

$$\sum_{k=1}^{5} \boxed{k^3} = 1^3 + 2^3 + 3^3 + 4^3 + 5^3 \qquad = 225$$

$$\sum_{k=1}^{5} \boxed{\frac{8}{(4k-3)\cdot(4k-1)}} = \frac{8}{3} + \frac{8}{35} + \frac{8}{99} + \frac{8}{195} + \frac{8}{323} \qquad = 3.04$$

(Demo)

Summation Example

```
def cube(k):
    return pow(k, 3)

def summation(n, term):
    """Sum the first n terms of a sequence.

    >>> summation(5, cube)
    225
    """
    total, k = 0, 1
    while k <= n:
        total, k = total + term(k), k + 1
    return total
```

Function of a single argument (*not called* "*term*")

A formal parameter that will be bound to a function

The cube function is passed as an argument value

The function bound to term gets called here

0 + 1 + 8 + 27 + 64 + 125

Functions as Return Values

(Demo)

Locally Defined Functions

Functions defined within other function bodies are bound to names in a local frame

A function that returns a function

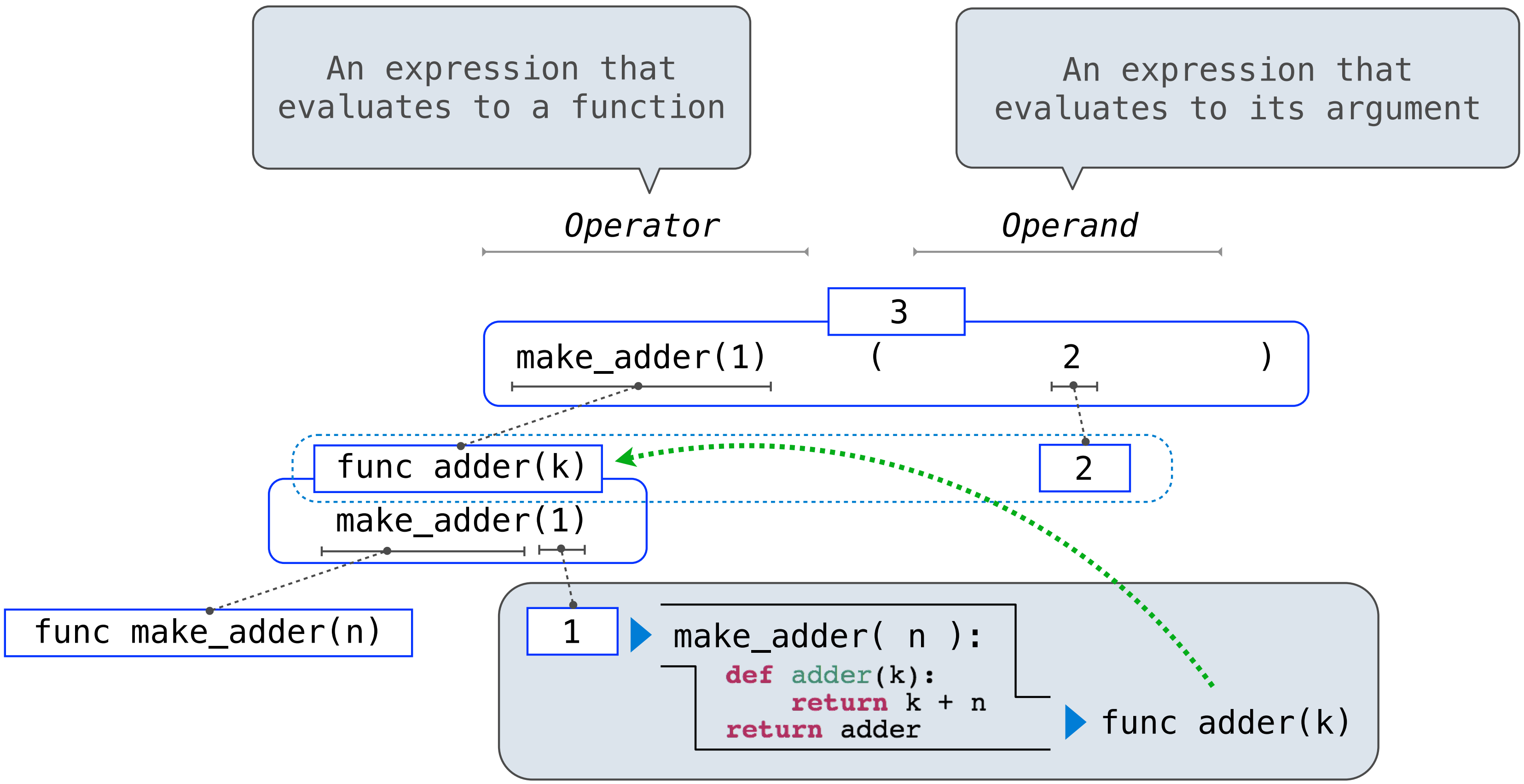
```
def make_adder(n):
    """Return a function that takes one argument k and returns k + n.

    >>> add_three = make_adder(3)
    >>> add_three(4)
    7
    """
    def adder(k):
        return k + n
    return adder
```

The name add_three is bound to a function

A def statement within another def statement

Can refer to names in the enclosing function

Call Expressions as Operator Expressions



Lambda Expressions

(Demo)

Lambda Expressions

```
>>> x = 10
```

An expression: this one evaluates to a number

```
>>> square = x * x
```

Also an expression: evaluates to a function

```
>>> square = lambda x: x * x
```

Important: No "return" keyword!

A function

with formal parameter x

that returns the value of "x * x"

```
>>> square(4)
16
```

Must be a single expression

Lambda expressions are not common in Python, but important in general

Lambda expressions in Python cannot contain statements at all!

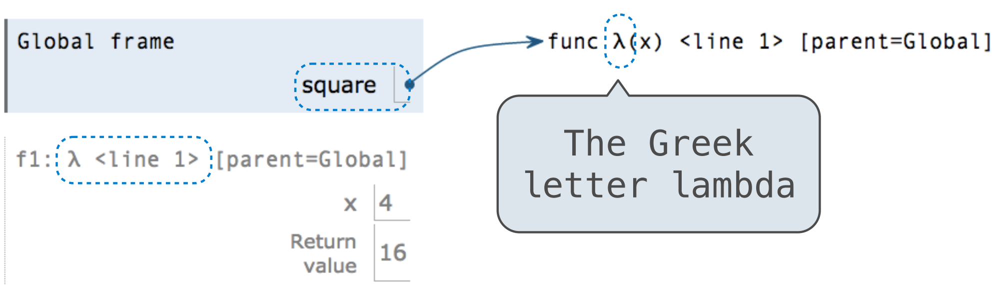
Lambda Expressions Versus Def Statements

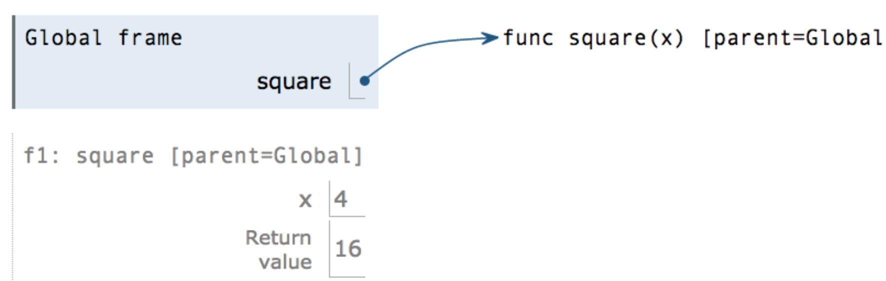
```
square = lambda x: x * x
```

VS

```
def square(x):
    return x * x
```

- Both create a function with the same domain, range, and behavior.
- Both bind that function to the name square.
- Only the def statement gives the function an intrinsic name, which shows up in environment diagrams but doesn't affect execution (unless the function is printed).





Return

Return Statements

A return statement completes the evaluation of a call expression and provides its value:

f(x) for user-defined function f: switch to a new environment; execute f's body

return statement within f: switch back to the previous environment; f(x) now has a value

Only one return statement is ever executed while executing the body of a function

```
def end(n, d):
    """Print the final digits of N in reverse order until D is found.

    >>> end(34567, 5)
    7
    6
    5
    """
    while n > 0:
        last, n = n % 10, n // 10
        print(last)
        if d == last:
            return None
```

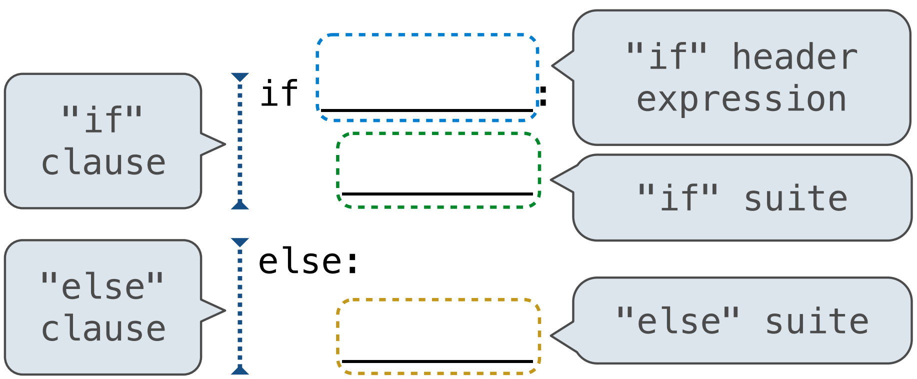
(Demo)

Control

If Statements and Call Expressions

Let's try to write a function that does the same thing as an if statement.

```
def if_(c, t, f):
    if c:
        return t
    else:
        return f
```



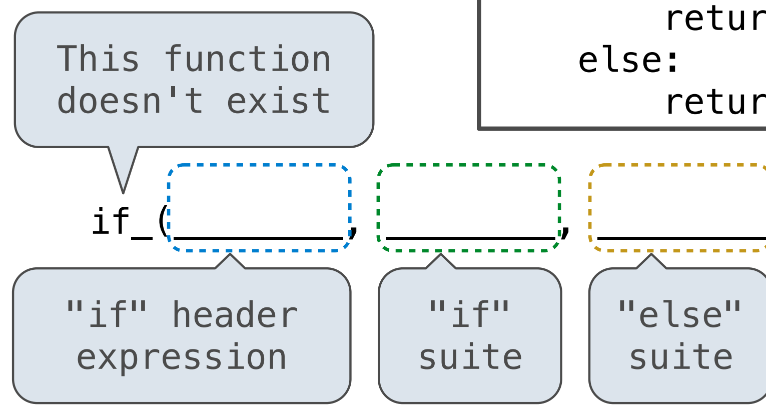
Execution Rule for Conditional Statements:

Each clause is considered in order.

1. Evaluate the header's expression (if present).

2. If it is a true value (or an else header), execute the suite & skip the remaining clauses.

(Demo)



Evaluation Rule for Call Expressions:

1. Evaluate the operator and then the operand subexpressions

2. Apply the function that is the value of the operator to the arguments that are the values of the operands

Control Expressions

Logical Operators

To evaluate the expression **<left>** **and** **<right>**:

1. Evaluate the subexpression **<left>.**
2. If the result is a false value **v,** then the expression evaluates to **v.**
3. Otherwise, the expression evaluates to the value of the subexpression **<right>.**

To evaluate the expression **<left>** **or** **<right>**:

1. Evaluate the subexpression **<left>.**
2. If the result is a true value **v,** then the expression evaluates to **v.**
3. Otherwise, the expression evaluates to the value of the subexpression **<right>.**

(Demo)

Conditional Expressions

A conditional expression has the form

<consequent> if <predicate> else <alternative>

Evaluation rule:

1. Evaluate the **<predicate>** expression.

2. If it's a true value, the value of the whole expression is the value of the **<consequent>**.

3. Otherwise, the value of the whole expression is the value of the **<alternative>**.

```
>>> x = 0
>>> abs(1/x if x != 0 else 0)
0
```